

LÉON WALRAS, ELEMENTS OF THEORETICAL ECONOMICS

In his fourth edition of *Éléments d'économie politique pure* (1900), Léon Walras introduced the device of written pledges to eliminate path dependency: sellers of products and services write out commitments to supply certain quantities at suggested prices. He tried unsuccessfully to show thereby that no commodities are actually produced and supplied until a set of prices is found at which supply and demand are equal simultaneously in every market. This brought about very serious alterations to the character of the book. Unfortunately, these changes resulted in an incomplete, internally contradictory, and occasionally incoherent text. This translation, therefore, by two leading scholars of Léon Walras's work, Donald Walker and Jan van Daal, revisits the third edition of his seminal work, including his brilliant explanation of his mature comprehensive model, with all its richness derived from reality. Growing research indicates that it was this third edition that contained his best theoretical research, so a translation of this edition of the book is therefore now a necessity.

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Léon Walras , Edited and translated by Donald A. Walker , Jan van Daal

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*To future generations of economists, whose understanding of
economic life and of the methods of economic theorizing
will be enriched by their study of this immortal work*

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Léon Walras, Elements of Theoretical Economics

or The Theory of Social Wealth

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Translators' introduction

Léon Walras was the founder of the modern theory of general economic equilibrium.¹ He was born on December 16, 1834, in Evreux, France, and christened Marie Esprit Léon. He had an excellent education during the period 1844 to 1854, covering such subjects as logic, philosophy, ethics, history, physical sciences, chemistry, mechanics, elementary mathematics, geometry, and calculus. After his *lycée* education, however, he failed the entrance examinations for the École Polytechnique, and did not make high enough grades to be retained in the program at the École impériale des Mines. Pursuing his predominant interests, in 1856 he abandoned the plan of becoming an engineer, began the pursuit of a literary career, and in fact published a novel and a short story. Revealing, however, an interest that had been latent but that had become even stronger, he made a promise in 1858 to his father, Auguste Walras, to give up his literary pursuits and to devote himself totally to the continuation of his father's economic studies.

Four phases of his subsequent scientific career can be distinguished: (1) the beginnings, (2) high creativity, (3) maturity, (4) decline. We deal with these phases in section 1 of this introduction. In section 2, we pay attention to the fourth phase, explaining that the fourth edition (1900) of Walras's *Éléments d'économie politique pure*, composed during that phase, contains new elements that are not improvements, but, on the contrary, that spoil his previous work. In section 3, we argue that the second and third editions of the *Éléments* are Walras's best theoretical work. In fact, there is so much original and creative

¹ For a biography of Walras, see Dockès and Potier 2001.

economic thought in these editions that has been neglected that we decided to prepare an English translation of the third edition.² In section 4, we explain that the third edition has an important place in the history of economic thought because many contemporaries and immediate followers of Walras took elements of the third edition that are not in the fourth as a basis of their research on general economic equilibrium. This constitutes another reason for our translation. In section 5, we make some remarks on translating in general and on translating Walras's writings in particular. We also discuss some aspects of Jaffé's translation of the fifth edition of the *Éléments*. A short conclusion follows in section 6. We then added four appendixes. Appendix A presents two tables that are relevant for our discussion in section 2. Appendix B deals with some of Walras's terminological idiosyncrasies regarding the factors of production. Appendix C contains a table of correspondence between the parts, lessons, and sections of edition 3 and editions 4 and 5. A table of the correspondence between the parts, lessons, and sections of all the editions of the *Éléments* has been provided by Jaffé in his translation (1954, pp. 559–63), and modified in Walker 1996, pp. 436–42; see also *CEÉC*, vol. VIII, pp. 733–7.³ Appendix D informs the reader about our editorial treatment of the translation.

Unlike his procedure in the first edition, Walras preceded the second and third editions of the *Éléments* with a nineteen-page presentation titled 'Des fonctions et de leur représentation géométrique. Théorie mathématique de la chute des corps'. His intention in doing so was to aid the reader to understand better the mathematics employed in his economic writings and to assert, in two sections, a parallelism of the results of the application of mathematics to physical sciences and their application to economic theory. We have not included this treatment of functions and of the law of gravity because it is an extraneous element bound in the books that contain the second or third editions but is not part of them, and it is not an exposition of economics. Walras placed it before the title page for Part I, and he numbered its sections separately from those of the book. He did not put it into the subsequent editions of the *Éléments*.

² See below for the differences between these two editions.

³ *CEÉC* stands for *Œuvres économiques complètes*, the collected works of Auguste and Léon Walras (1987–2005).

1. FOUR PHASES OF INTELLECTUAL ACTIVITY

The first phase. Walras's first phase of intellectual activity in regard to economic theory, beginning in 1859 and continuing until 1872, consisted largely of journalistic applications of his knowledge of existing theory but also included his study of social economics. Of course, Walras had to earn a living for himself and his family, so, during the years 1860 to 1870, he worked successively as a journalist, in a bank, in a railroad office, became managing director of a cooperative association bank, and worked for another bank. During this time, however, he began his lifelong study of economics. That was therefore a subject to which he devoted himself for many years, so it is not surprising that the degree of his technical sophistication and the character and quality of his theoretical work changed over time. His participation in 1860 in a conference on taxation in Lausanne (1861)⁴ and his publication of a long examination of the economic doctrines of P.-J. Proudhon in that year (1860) – one that mainly set forth the ideas of his father, Auguste Walras – are examples of his early interest in economic topics. Among his ideas was the argument that the state should own all land on the grounds that it is the patrimony of the entire nation because it is not produced as the result of the activity of any economic agent. Its value is given to it by the growth of the value of its services, which depends upon the growth of population and of the economy, but its increased value would occur whether the land is owned privately or by the state. Walras also wrote many analyses of economic problems, notably contributing articles to the journals *La Presse*, *L'Indépendant de la Moselle*, *Journal des Économistes*, and *Le Travail*, and gave and published public lectures on his ideas about an ideal society (1868). Those writings can be seen to be his early efforts to adopt an analytical approach to the study of practical economic issues, an approach that he refined in later years. Moreover,

⁴ A date without a name, sometimes followed by a page or section number, refers to Léon Walras's writings. See the references at the end of this introduction. Furthermore, in order to facilitate the comparison of the present translation with Jaffe's and with volume VIII of *CEÉC*, we use section numbers to refer to places in one or another of the editions of the *Éléments*. Such a section number always refers to the edition of the *Éléments* under discussion. In this respect, our table C1 will be helpful.

at the start and again toward the end of the first phase of his career, Walras began to use mathematics in the construction of rudimentary models dealing with exchange (1860, 1869, 1871; see *CEÉC*, vol. XI, pp. 315–409). He relates in his autobiography that the ‘idea of creating mathematical economics, which I had announced in my letter offering my services to the Council of State of Vaud, never ceased to occupy my mind after 1860’ (1965, vol. I, p. 5). He did not, however, have an adequate foundation in mathematics and lacked some of the concepts that he needed to establish the foundations of a theory of supply and demand and of the interrelationships of markets. Nothing that he wrote during the first phase of his career can be considered a valuable contribution to economic theory. On the other hand, some of his writings on moral philosophy, in particular his analysis of the human society (see above), are of enduring value and found a rightful place in his *Études d'économie sociale* (1898, chapters 2 and 3; 2010, pp. 19–113).

Nevertheless, Walras's early research made a valuable contribution to his career. His 1861 paper on taxation drew him to the attention of Louis Ruchonnet, a Swiss statesman who, nine years later, recommended successfully that Walras be offered an appointment at the Académie (subsequently Université) de Lausanne. Walras began his duties there as a professor without tenure in 1870, and the next year his appointment became permanent. His subsequent attempts to obtain a position in a French university were rebuffed because he lacked the necessary educational credentials and, moreover, was an exponent of mathematical economics, which was regarded unfavorably by the academic establishment. He therefore remained at Lausanne for his entire academic career, retiring because of bad health in 1892.

The second phase. Walras's second phase of theoretical activity, his period of high creativity and maximum theoretical prolificacy, spanned the period 1872 to 1877. It was during those years that he developed his initial comprehensive model of general equilibrium. By his comprehensive model is meant one that includes exchange, production, consumption, capital formation, credit, and money. He first presented it in four memoirs (1877a), the substance and much of the wording of which he put into the first edition of the *Éléments d'économie politique pure* (1874; 1877b). Walras published that edition in two installments, one in 1874 and the other in 1877. We chose

not to translate the first edition because, although it is a brilliant expression of pure originality, containing many theoretical innovations, it needed alteration and development in a variety of important respects.

The third phase. Walras's third intellectual phase, that of his maturity as an economic theoretician, began around 1877 and was given full expression in the second edition of the *Éléments*, published in 1889. In that edition he presented his mature comprehensive model of general equilibration and equilibrium in a purely competitive economy. Walras reaffirmed his satisfaction with that presentation in a third edition published in 1896. In it he also reissued the introduction and the body of the second edition, eliminated four lessons (37–40 of the second edition) that deal with the applied theory of money, and added 29 pages of appendixes of material previously published during his period of maturity. Henceforward, when we refer to the third edition, we will evidently mean the contents of the second edition reissued, along with the indicated deletions and additions, as a third edition. We will return very shortly to our discussion of the third edition.

The fourth phase. Walras's fourth intellectual phase began about 1896. His output of theoretical and other work had begun to diminish long before that year, but soon afterwards his powers of concentration and analysis weakened rapidly, and the lessening of his productivity and creativity became more marked (see Appendix A to this introduction). His only new research after 1898 was the article 'Équations de la circulation' (1899) and some revisions to the *Éléments* that resulted in a fourth edition in 1900. Although the new material in the fourth edition totaled only 35 pages, it profoundly affected the character of that edition and of the subsequent history of general equilibrium theory, as will be seen shortly. The major changes in that edition were Walras's displacement of his 1889 theory of money by the text of the 1899 article, and the introduction of a sketch of a pricing process that was outlined in a note appended to the article. Subsequently, making a few inconsequential alterations, he produced a fifth edition, published posthumously (1926).

A final phase of Walras's life spanned the years from 1901 to his death in 1910. That could be viewed as a period of repose rather than an intellectual phase, for after 1900 he wrote not another word of

theory and very little of anything else. Thus as a contributor to economic theory, he bloomed late – at the age of thirty-seven or thirty-eight – and faded early, writing less each decade after 1877, and writing little and adding even less of value after he was sixty, and nothing at all in the way of theoretical contributions during the last eleven years of his life, as has been demonstrated in a methodical and quantitative way (Walker 1999, 2006, pp. 183–92). He died on January 5, 1910, in Clarens, Switzerland, and the publication of the fifth edition was accomplished by his daughter Aline in 1926.

2. RESEARCH DURING THE FOURTH PHASE

Prior to issuing the fourth edition, Walras had been aware that the irrevocable disequilibrium transactions and disequilibrium production in his mature comprehensive model generated path dependency, which meant that the solutions to his systems of equations of general equilibrium were not the outcomes of the behavior of the model.⁵ He wanted those equations to be valid reflections and expressions of the interrelationships and equilibrium of a model that included durable goods and inventories. He realized that a virtual adjustment process would make it possible to have such a model. By a virtual economic process is meant one that is acted out without any changes in real economic magnitudes. If the model were complete, the only changes would be in suggested prices and desired supplies and demands. There would be no transactions, production, consumption, savings, or any other non-pricing economic behavior in disequilibrium, and thus no changes in the amount of capital goods or other asset holdings, and thus no changes in supply and demand functions during the equilibrating process of changing suggested prices.

⁵ He implied that it was a problem in 1889, in a revision of passages in the second edition, when he inserted the condition that exchange must be suspended if supply and demand are not equal (§ 42). He indicated in 1885, as a reaction to a comment made in 1883 by the mathematician Joseph Bertrand, that he had excluded non-virtual behavior from his model of exchange in order to eliminate path dependency (1896b, p. 352, n. 1; 2010, p. 250, n. 2; see also Walker 1996, pp. 95–100).

In 1899 he came to believe that that type of a model could be constructed by introducing the device of written pledges.⁶ That was the content of the above-mentioned note published in 1899 and elaborated slightly in the fourth edition. Walras specified that sellers of products and services write out commitments⁷ to supply certain quantities at suggested prices with no commodities actually produced and supplied until a set of prices is found at which supply and demand is equal simultaneously in every market (1926, §§ 207, 251). If that were true, the holdings of assets would be constants during the process of finding those prices, fulfilling the requirement that the holdings be constants in the equations of his model. The equilibrium prices would therefore be the solutions to his system of static equations of general equilibrium. The equilibrating changes of asset holdings, production, consumption, etc., would occur only at the equilibrium set of prices and would therefore leave those prices unchanged. The new set of magnitudes of stocks and flows would remain unchanged until some parametric change disrupted the equilibrium.

By thrusting the written pledges sketch into the fourth edition and retaining it in the fifth, Walras brought about very serious alterations of the character of the *Éléments*. He presented a text that has created much mischief by confusing the reader as to what Walras's theories are. In the first place, it became internally contradictory because in the majority of the pages of the fourth edition (and hence of the fifth) that deal with the question of the stability of equilibrium, Walras retained most of the depictions of the behavior of disequilibrium transactions and production that he had carefully developed in the third edition. That non-virtual behavior is incompatible with

⁶ So named after the term *engagements écrits* used in French securities markets. Walras used the word '*bons*' for the pledges written out by would-be suppliers, meaning that the seller pledges himself to be good for the quantity he offers. Jaffé translated Walras's word as 'tickets'. Walras did not, however, use a French word for 'tickets' (*ticket, billet*), and in English a ticket has no features in common with his '*bons*' or written pledges. A ticket has its conditions printed on it; it is not generated or written on by its holder but is bought by him. It is used to be admitted to a form of transportation or an event, or to be represented in a lottery; or, in other senses, is a tag or label, a license, a political slate, or a document issued by a police officer for a traffic violation. Thus Jaffé's translation conflicts with Walras's terminology, and with French and English language and practice.

⁷ Only the sellers make commitments; see below.

what he asserted to be true of the written pledges sketch. That is to say, Walras tried to construct a comprehensive model that is virtual because of the use of written pledges, but that has some markets that he asserted were of that type, some markets that he described as virtual without having written or oral pledges, and some markets that do not have written pledges and are non-virtual. In some passages he even described some commodities as being sold in written pledges and in other passages described those same commodities as being sold in non-virtual markets.⁸ The sub-models of particular markets are incompatible with each other and with his system of equations of general equilibrium, which require that all the markets be virtual.

Second, the sketch in the treatise is incomplete, illogical, superficial, and incoherent. Walras made his last mention of it and its supposed results less than two thirds of the way through the text (1900, 1926, § 274), and then apparently lost interest in that idea, not referring to it again, neither in the subsequent 185 pages of the book nor anywhere else. He was unable to devise a complete model of a virtual market, which is why we call it a sketch rather than a model. It lacks a demand side because there are no potential demanders who make written or oral pledges to purchase commodities, so they have no way of expressing demands. Since there are no equations of demand in the sketch, there are more unknowns than independent equations. This crucial defect is present in every one of Walras's presentations of the written pledges sketch. Moreover, it lacks the institutions and market rules and physical features and technology that would be necessary for the functioning of a pricing mechanism. There is no way for prices to be determined and announced, that is, no means of collecting information from would-be suppliers, and no market demands which could be used in conjunction with supply pledges in order for there to be excess demands. Since there are no excess demands, there is no behavior or information to serve as the basis for changing prices either up or down. Would-be suppliers and demanders do not meet, so there are no markets. Equilibrium does not exist in the sketch and it has no stability properties.

⁸ See Walker 1996, pp. 411–12.

Readers of Walras's fifth edition have unthinkingly added the missing demanders in their account of Walras's sketch, as did Jaffé when he asserted in what he believed was an explanatory note that 'the tickets issued *by buyers* of products as well as *by buyers* and sellers of services *are similarly worded*' (Walras 1954, p. 528, note [5]; emphasis added). Walras never wrote that buyers issue tickets. For example, he wrote that *suppliers* of new capital goods ('entrepreneurs') represent them with *bons* – he used that word for 'written pledges' with the thought that suppliers write out the amounts for which they are good – and that there are *suppliers* of services, namely 'landowners, workers, and capitalists representing similarly by *bons* the successive quantities of services at prices cried first at random...' (Walras 1926, § 251; emphasis added⁹). He did not mention buyers in those markets (in this case, the markets for newly constructed capital goods and services). Nevertheless, the innocent reader of Jaffé's translation finds that he represented Walras as writing that 'land-owners, workers and capitalists also use *tickets* to represent the successive quantities of *services which are offered and demanded* at prices first cried at random...' (Walras 1954, p. 282; boldface added to Jaffé's words; see the boldfaced passage in the preceding footnote). As has just been shown, Walras did not write the words in boldface; they were added by Jaffé. The 'landowners, workers, and capitalists' in Walras's sentence are service suppliers; he did not mention demanders of the services.

⁹ The original French passage is:

Un certain taux du revenu net et certains prix des services étant criés, et certaines quantités de produits et de capitaux neufs étant fabriquées, si ce taux, ces prix et ces quantités ne sont pas taux, prix et quantités d'équilibre, il faut non seulement crier un autre taux et d'autres prix, mais fabriquer d'autres quantités de produits et de capitaux neufs.

Nous résoudrons cette première difficulté en supposant les entrepreneurs de capitaux neufs représentant par des *bons* certaines quantités successives de ces *produits* déterminées d'abord au hasard, puis en augmentation ou diminution suivant qu'il y aura excédent du prix de vente sur le prix de revient ou réciproquement, jusqu'à égalité de ces deux prix; **et les propriétaires fonciers, travailleurs et capitalistes représentant de même par des bons des quantités successives de services à des prix criés d'abord au hasard**, puis en hausse ou baisse suivant qu'il y aura excédent de la demande sur l'offre du montant des capitaux neufs en numéraire, ou réciproquement, jusqu'à égalité de l'une et de l'autre (Walras 1926, pp. 259–60; boldface added).

Since a critique of the written pledges sketch is part of the rationale for our translation, it is relevant to remark upon yet another indication of Walras's incapacity to undertake sound modeling in the fifth edition. In the exposition just mentioned and similarly elsewhere, not only did he not equip demanders of services with pledges or even mention such participants, he did not represent changes of the prices of services as being a function of the supply and demand for them. Instead, he went on to assert that their 'prices cried first at random' are 'then raised or lowered according to whether there is an excess of demand over supply of the amount of new capital goods...' (1900, 1926, § 251; emphasis added; see the preceding footnote). That is, he asserted that prices of services are changed, not as a direct function of their excess demand, which actually does not exist, but as a direct and exclusive function of the excess demand *in another market*, namely the market for new capital goods, which is absurd. Compounding the confusion, he wrote in this, as in other circumstances, as though there is a single market for new capital goods, whereas there are many in reality and in his treatise, one for each kind.

Methodologically, Walras did not derive the features he attributes to the sketch from economic reality; what he describes as its outcomes are actually postulates that have no foundation in any conceivable institutional, technological, and behavioral phenomena. The material inserted into the fourth/fifth edition cannot be used in any empirical connection, no matter how the written pledges sketch is amended, because the real economy is not virtual. Most transactions and all production and consumption are irrevocable disequilibrium magnitudes, whereas none of the variables in the written pledges sketch are supposed to materialize in disequilibrium. Additionally, Walras's exposition of virtual behavior negatively affects the content of the theorization. It is disorganized, analytically confused, characterized by unclear definitions, badly arranged statements, lacunae, conflation of separate topics, contradictions, fragmentary constructions, long but elliptical sentences, rambling discourses, symbols and words crowded together in baffling sequences (Walker 1996, p. 400). Therefore, Walras's allegation that the behavior of the written pledges sketch eventuates in finding the set of equilibrium prices and quantities that are the solutions of his equations is not true. Even if the sketch is revised to enable demanders to make written pledges, its

problems are so fundamental that it cannot be made into a functioning system (see Walker 1996, pp. 379–95). It is defective in ways that cannot be rectified.

Some of the writings that Walras presented as a monetary theory are a mystery, centered as they are on incomprehensible equations.¹⁰ The text of the 1899 article on circulation and money, which contained his last theory of money and which he inserted verbatim into the fourth edition of the *Éléments*, made no mention whatsoever of pledges, and the monetary behavior in it contradicts the characteristics that he wanted to be true of the sketch. There are many other theoretical mistakes in the few lines that he added in the fifth edition but an exposition of them would require extensive contextualization and detailed economic analysis and is best read, like the whole of this critique, in a careful extended discussion (see Walker 1996, chapters 17 and 18).

With all respect for the Master of Lausanne, we have to conclude that the chaotic collection of incompatible non-virtual and incomplete written pledge markets does not constitute a functioning general equilibrium model. Of course, Walras stated that the sketch has an equilibrium (1900, 1926, §§ 207, 251, 274), but that is not true. His proof that equilibrium exists in each of his models is the same, namely that the number of independent equations and the number of unknowns are equal. That is not a proof regarding any economic model and especially not regarding the sketch because it has fewer equations than unknowns. The sketch is not sufficiently complete behaviorally, institutionally, and technologically to have an equilibrium. It has no path of variables and therefore no stability properties. It does not achieve Walras's objective of validating his system of equations.

By initiating the idea of a virtual general equilibrium model, however, Walras laid a basis for a subsequent type of modeling. Modifications of the written pledges sketch, themselves without institutional and technological foundations and based on implausible

¹⁰ Its characteristics prompted Pascal Bridel, an authority on Walras's treatment of money, to exclaim that 'there is no genuine monetary theory' in the fourth edition (Bridel 1997, p. 133). For a discussion of the merits of some of Walras's monetary innovations in the fourth edition, see Van Daal and Jolink 1993, ch. 16, and Van Daal 1994.

assumptions, constitute a branch of modern purely competitive general equilibrium theory, characterized by extreme unreality that has led to a scientific dead end. 'Suffice it to say, general equilibrium theory...had a very large burden to bear. It proved unequal to this task. Such became clear in a spectacular series of impossibility results that might be called Sonnenschein-Mantel-Debreu theorems after its main promulgators' (Rizvi 2003, p. 384). Their work 'showed that formalist general equilibrium theory had reached a dead end: no general results beyond existence of equilibrium were possible' (ibid.). 'Strictly, the arbitrariness results put an end to neoclassical general equilibrium theory of the Arrow-Debreu-McKenzie variety' (ibid., p. 385), that is to say, the virtual line of purely competitive general equilibrium models that were the descendants of the written pledges sketch in Walras's fourth and fifth editions.

The *Éléments* has been translated into Japanese (1933, 1953, and 1983), Chinese (1966), Italian (1974), and Spanish (1987); these are all translations of the fifth edition of the *Éléments* (1926). The previous English translation of the *Éléments*, namely, William Jaffé's (Walras 1954), is also a translation of that edition. That has been the unique source from which English-reading economists derived their knowledge of the *Éléments*. It has not even crossed the minds of most of them that the last edition of the *Éléments* may not be his best and definitive work on the theory of general equilibrium. Jaffé's translation of it has therefore led to a virtually complete neglect of his mature comprehensive model and therefore to much misunderstanding and under-valuation of Walras's contributions.

3. WORK DONE DURING THE MATURE PHASE

Fortunately, there is a magnificent alternative edition of the *Éléments*, namely the third, published in 1896. In it Walras presented the thoughts of his period of intellectual maturity as a theoretician, lasting from 1877 to about the middle of the 1890s. The defects of the fifth edition and the scientific and historical value of the third explain why we have chosen to translate the latter. We have made Walras's best theoretical work accessible to those who do not read French but who read English. We hope thereby to end, for the Anglophone economist, the monopolization of his writings by Jaffé's translation

and hence the domination of Walras's worst ideas in certain parts of his general equilibrium theory.

During the years after the publication in 1874 and 1877 of the two parts of the first edition of the *Éléments*, Walras altered, fleshed out, refined, and extended his concepts and theoretical tools, including the use of calculus, work that culminated in the third edition of the *Éléments*. He presented there his mature comprehensive model of general economic equilibration and equilibrium in a freely competitive economy, a reasonably complete, well-organized, and coherent exposition of a system containing many sub-models of aspects of economic behavior, woven into a seamless functioning whole. In that model, Walras not only expressed the belief that all economic phenomena are interrelated, which had been done by many economists before him, he also specified their interrelations, provided an account of their disequilibrium behavior, and described their conditions of equilibrium. Its various parts included models of consumers, workers, landlords, capitalists, and entrepreneurs, of exchange, production, capital formation and credit, a time-period analysis of monetary behavior, an analysis of arbitrage, an account and analysis of the operations of securities exchanges, and a treatment of economic growth. Walras included in his treatise his theory of monopoly and price fixing, and his analysis of taxation, and left his account of those matters unchanged in the fourth edition. Walras also added three brilliant and important appendixes to the third edition. He eliminated the third of these from the fourth/fifth edition, thus depriving the reader of the latter of that particular systematic treatment of the theory of marginal productivity; instead, he inserted some of the content of the appendix into the body of the text of the fourth and fifth editions of the *Éléments*. Fortunately, Jaffé provided his English translation of that appendix as an extra to his translation of the fifth edition of the *Éléments*.

Unlike the written pledges sketch, the mature model is logically valid in the important respect that the behavior it contains and its outcomes are generated by economic and social institutions, market procedures and rules, technologies, and other structural and behavioral underpinnings. That is not to say that its outcomes are the solutions to the system of static equations of general equilibrium,

as far as concerns his models with capital formation.¹¹ They are not, so there is an important respect in which the model is not logical if it is taken to be inclusive of the equations; or, to put differently, the model and equations are incompatible. The equations are inconsistent with the non-virtual behavior of the system in disequilibrium and the consequent path dependency of any equilibrium that it may have. Although Walras did not try to find a system of dynamic equations and their solutions that could complete the model, he identified its variables, its characteristics that determine the directions of their movements in disequilibrium, and their qualitative behavior.¹²

There is a sharp contrast between the degree of realism in the third and the fifth editions. In the fifth, the written pledges sketch has no connection with economic reality. The characteristics Walras assumed it to have are simply figments of his imagination. In the third, he tried to make sure that his assumptions are connected to reality, although they are abstract and general. To do so, Walras drew from the economy of his day the economic elements that he considered important and put them into his model. For example, believing that the majority of real markets in the economy of his day were freely competitive, Walras assumed that that is true of all the markets in his general equilibrium model. He observed that there is irrevocable disequilibrium economic behavior in the real world, and he assumed that it occurred in his model. Likewise, he inferred from his experience the assumption that all economic variables are interdependent. He even stated (erroneously) that his equations could, in principle, be converted into empirical statements by using statistical research to obtain numerical values for their coefficients.

Three major questions that arise concerning a model of general equilibrium are the existence of equilibrium in it, the uniqueness of equilibrium, and the stability of equilibrium. Walras was the first to consider these questions with reference to that type of model, thereby

¹¹ One aspect of the written pledges device is that Walras's mature consumer commodities model, into which he introduced it, has no need of such a feature because the model is not path dependent (Walker 1996, ch. 7), and another is that in his subsequent models, which are path dependent, the device does not work.

¹² Like all the great theorists, Walras recognized the truth of Claude Bernard's maxim that 'The *qualitative* study of phenomena must necessarily precede their *quantitative* study' (quoted in Robert 2002, p. 2129).

initiating the central agenda of general equilibrium research during the one hundred years that followed the first edition of his *Éléments*. We give a brief account of his research on these matters to alert the reader to the remarkably rich content of what is to be found in the third edition.

The existence of general equilibrium. Walras indicated the conditions of a Walrasian equilibrium, that is to say, that the supply and demand are equal simultaneously for each good and service at positive prices. He counted the number of variables and the number of independent equations in his model, established that the two are equal, and concluded from that equality that he had proved that an equilibrium exists. His immediate successors used the same procedure and arrived at the same conclusion. In regard to the uniqueness of equilibrium, Walras studied only the case of a model in which two commodities are exchanged for each other, concluding that there could be multiple equilibria and that there could be an unstable equilibrium. In the case of exchange of many commodities, he believed that 'generally' multiple equilibria are not possible, but he did not offer a proof of that assertion.

The stability of the mature comprehensive model. Reference to the stability of a model is a brief way of referring to the way that the model functions in disequilibrium. A stable model moves from disequilibrium to equilibrium. That is the principal topic that Walras takes up in the third edition. He wrote that the economy moves toward equilibrium by a process of *tatonnement*; that is, by groping its way. In what follows we briefly point out the main facets of economic behavior regarding which Walras explained how *tatonnement* and other adjustment processes occur, thereby presenting analyses and descriptions that pulsate with economic life (see, for example, lesson 37).

After having identified the market participants, the market institutions, the firms, the goods, the services, the manner of operating of the entrepreneurs, etc., Walras modeled their interrelated behavior in disequilibrium and in equilibrium. The sequence of prices quoted by the buyers and sellers in each market and the consequent sequence of quantities produced, sold, and used, influences the set of equilibrium prices and quantities. Walras believed that the sequences move toward equilibrium; that is, that stability of equilibrium is 'probable'

(§ 213) in the purely hypothetical case of a model in which all commodities are non-durable and are used as soon as they are produced. There are no asset holdings, so supply and demand functions do not change during the equilibrating process. In a model in which durable goods are produced, he stated that its stability is 'evident' (§ 254). As is evident from his conclusions regarding the processes of tatonnement, which occupy a great deal of the *Éléments*, he was convinced that his mature comprehensive model is stable. The reader of this book will see that Walras argued that the stability is brought about principally by a number of non-virtual aspects of the disequilibrium adjustment process.

First, Walras analyzed *homo œconomicus*, showing that he engages in tatonnements, that is, in successive adjustments in disequilibrium of the quantities he purchases and consumes of different products, including savings instruments, in order to maximize his utility.

Second, the entrepreneur (*'l'entrepreneur qui tâtonne'* (Walras 1896a, p. 490; this book, Appendix III)) makes a series of adjustments of the quantities of inputs into the process of production in his firm in the mature model, as distinct from the written pledges sketch in which he has no functions of any kind and does not make an appearance. In the mature model, the entrepreneur undertakes non-virtual adjustments in order to come progressively closer to the minimum cost of production and finally to reach it, and to maximize the output made by a given combination of inputs.

A third aspect of tatonnement is the changing of prices. This is done following a rule that Walras drew from his observations of real markets, namely that the price of each commodity in disequilibrium is changed in the same direction as the sign of the market excess demand until the price that equates supply and demand is reached. Suppliers and demanders meet in markets. They do not know the market excess demand, but suppliers lower the price when they find that they do not receive orders to sell all that they wish at the prevailing price, and demanders raise the price when they find that they do not receive commitments to provide all the amounts they wish. It will be noted that the participants change prices, so there is no mention of an auctioneer. Walras departed from reality by assuming that on any given market day there are no transactions at disequilibrium prices, and that trade occurs only at the price at which supply and

demand are equal. He asserted that that was a hypothesis that no reasonable scientist would hesitate to concede to the theoretician. In his model, after trade occurs, the traders in each market consider the prices that have been found for commodities other than their own and react to them, changing their desired supply and demand quantities. The market day prices at which trade occurs are disequilibrium ones from the point of view of the general economy if the first round of pricing has not led all markets to be simultaneously in equilibrium. Normally they do not immediately reach general equilibrium, so additional rounds of pricing and irrevocable transactions occur within each market. Walras presented a detailed account of the paths of the prices, that is to say, of the economic behavior underlying and being expressed in the price changes, thus explaining why the prices in the many markets tend to an equilibrium set.

Fourth, in accordance with the behavior of real markets, Walras modeled the *tatonnements* in production. If the price of a product exceeds its average cost, the entrepreneurs increase the quantity produced in order to make more profits. That leads to the increase of the prices of services used in its production. Since the supply of an output increases, its price falls. The difference between the price and average cost, namely the profit, therefore decreases. That aspect of the overall *tatonnement* continues until the equalization of the two magnitudes reduces profits and the increase of output to zero. If the average cost is less than the price, the reverse process occurs.

A fifth aspect of *tatonnement*, also performed by the entrepreneurs, is their activity of directing economic resources in disequilibrium toward economic activities that are profitable and away from those in which there are losses. That has the effect of lowering prices in the lines of economic activity in which output is increased, diminishing their profitability, and the reverse in those in which output is decreased. The reallocations of resources cease when profits and losses are zero in all lines of activity. The profits to which Walras referred in the mature comprehensive model are really made, whereas in a virtual model profits are not actually made in any state of the market.

A sixth aspect of the *tatonnement* process occurs in the formation of new capital. This occurs until eventually the ratio of the net revenue per unit of each type of capital good and its price are the same for all capital goods, the price being equal to its average cost.

This same ratio is the equilibrium rate of interest that makes the consumers' and entrepreneurs' aggregate demand for money equal to its aggregate supply.

Some other contributions. Among the many other economic processes that Walras identified and explained in the third edition is his brilliant non-virtual theory of money, a theory which he had the bad judgment to eliminate from the fourth/fifth edition, thus leading its readers (in both French and English versions) to be unaware of that remarkable achievement. He examined monetary expenditures by means of a non-virtual time-period analysis, tracing a series of adjustments of payments, of cash balances held by the participants in the economy, and of changes of rates of interest. He thus anticipated the work of J.M. Keynes and D.H. Robertson on these matters. His theory of entrepreneurial profits is the foundation of Continental thinking on that subject. He anticipated (§ 370) the heart of the treatment of product differentiation and price discrimination that E.H. Chamberlin subsequently presented in his theory of monopolistic competition. Walras was the first to deal with the problem of bimetallism by noting that, in the case of a gold and silver standard, the problem of the ratio of the value of the two metals is not completely determined economically, and that the legislator can intervene to determine arbitrarily one of the six unknowns or introduce in one way or another a sixth equation (lesson 34). The third edition shows that Walras had a perfect grasp of the static theory of the firm (§ 362 ff.). He also studied the implications in disequilibrium and in equilibrium of Walras's law, of the budget of the firm and of the consumer, and of the theorem of equivalent distributions of commodities.

4. THE PLACE OF THE SECOND/THIRD EDITION IN THE HISTORY OF ECONOMIC THOUGHT

Another reason for providing Anglophone students of the history of economic thought with our translation is that the second and third editions were important in that history. Those editions were responsible for the origins and lines of development of the non-virtual branch of purely competitive general equilibrium theory. The treatise we have translated is the one that taught Walras's contemporaries and immediate successors their general equilibrium theory and in