

Quantum Mechanics

An Experimentalist's Approach

Eugene D. Commins takes an experimentalist's approach to quantum mechanics, preferring to use concrete physical explanations over formal, abstract descriptions to address the needs and interests of a diverse group of students. Keeping physics at the foreground and explaining difficult concepts in straightforward language, Commins examines the many modern developments in quantum physics, including Bell's inequalities, locality, photon polarization correlations, the stability of matter, Casimir forces, geometric phases, Aharonov-Bohm and Aharonov-Casher effects, magnetic monopoles, neutrino oscillations, neutron interferometry, the Higgs mechanism, and the electroweak standard model. The text is self-contained, covering the necessary background on atomic and molecular structure in addition to the traditional topics. Developed from the author's well-regarded course notes for his popular first-year graduate course at the University of California, Berkeley, instruction is supported by over 160 challenging problems to illustrate concepts and provide students with ample opportunity to test their knowledge and understanding, with solutions available online for instructors at www.cambridge.org/commins.

EUGENE D. COMMINS is Professor Emeritus at UC Berkeley's Department of Physics, where he has been a faculty member since 1960. His main area of research is experimental atomic physics. He is a member of the National Academy of Sciences, a Fellow of the American Association for the Advancement of Science, a Fellow of the American Physical Society, and he has been awarded several prizes for his teaching, including the American Association of Physics Teachers Ørsted Medal in 2005, its most prestigious award for notable contributions to physics teaching. He is the author (with Philip H. Bucksbaum) of the monograph *Weak Interactions of Leptons and Quarks* (Cambridge University Press, 1983).

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Preface

This book developed from lecture notes that I wrote and rewrote while teaching the graduate course in quantum mechanics at Berkeley many times and to many hundreds of students between 1965 and 2010. It joins a crowded field of well-established quantum mechanics texts. I hope that by virtue of its contents and approach, this book may add something distinctive and be of use to physics students and to working physicists.

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