Complexity Dichotomies for Counting Problems

Volume 1: Boolean Domain

Complexity theory aims to understand and classify computational problems, especially decision problems, according to their inherent complexity. This book uses new techniques to expand the theory for use with counting problems. The authors present dichotomy classifications for broad classes of counting problems in the realm of P and NP. Classifications are proved for partition functions of spin systems, graph homomorphisms, constraint satisfaction problems, and Holant problems. The book assumes minimal prior knowledge of computational complexity theory, developing proof techniques as needed and gradually increasing the generality and abstraction of the theory.

This volume presents the theory on the Boolean domain, and includes a thorough presentation of holographic algorithms, culminating in classifications of computational problems studied in exactly solvable models from statistical mechanics.

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Volume 1: Boolean Domain

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Preface

The raison d'être of computational complexity theory is to understand and classify computational problems according to their inherent complexity. In the past fifteen years or so, there have been enormous advances in the study of the complexity of counting problems. A substantial theory has been developed that classifies every problem in a class of Sum-of-Product computations as being either polynomial-time computable or #P-hard. Such theorems are called complexity dichotomies.

This book presents the dichotomy theory for counting problems, expressible as Sum-of-Product computations. More specifically, it presents dichotomy theorems for partition functions of Spin Systems, Graph Homomorphisms, Constraint Satisfaction Problems, and Holant Problems. The theorems have the consequence that, assuming the complexity theory hypothesis P $\neq$ #P, which is implied by P $\neq$ NP, in broad classes of counting problems, up to polynomial-time equivalence, there are only exactly two distinct levels for exact counting complexity.

Volume I presents the theory on the Boolean domain; Volume II presents the theory on general domains. In Volume I a particular feature is a thorough presentation of holographic algorithms, culminating in classifications of computational problems studied in Exactly Solvable Models from statistical mechanics.

Much of the material in the book is presented in book form for the first time. The book assumes minimal prior knowledge of computational complexity theory. The proof techniques are developed as needed. The dichotomy theorems are carefully organized, so that the theory is presented in gradually increasing generality and abstraction.

We want to thank many colleagues who have read parts of the manuscript and shared their thoughts on the subject. Their comments and criticisms have been a source of inspiration, and we wish to express our gratitude for their help. They are Manuel Blum, Andrei Bulatov, Sitan Chen, Martin Dyer, Alan Frieze,

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