

## Index

---

- absolute value, 97, 102
- absolutely convergent, 36, 41, 44
  - geometric series, 42
- absolutely convergent series, 45
- alternating harmonic series, 43, 45,
- alternating series, 43, 44, 45
- analytic functions, 126
- analytic, 123
- angle, 97
- approximations
  - asymptotic, 81
  - functions, 82
  - integrals, 80
  - least-squares fit, 183
  - Stirling, 81
- Argand diagram, 94
- argument, 97
- asymptotic
  - expansions, 77
- asymptotic series, 74
  - theorems, 79
- branch points, 129
- Bessel functions, 157, 163, 174, 178
  - of first kind, 162
  - of second kind, 162
- Bessel's equation, 157
  - of order  $p$ , 161
- binomial series, 87
- breeding season, 24
- chain reactions, 22
- complex
  - numbers, 92
- convergent series
  - addition of, 43
- common difference, 3
- common ratio, 5, 6, 19, 21, 22
  - chain reactions, 22
  - f-stops, 1, 2, 5, 19
  - genetic complexity, 21
- comparison test, 34, 35, 37, 38, 39
  - $p$ -series, 37
- complex
  - absolute value of, 96
  - addition, 100
  - angle, 133
  - argument, 96
  - conjugate, 99, 103
  - division, 101
  - equations, 105
  - Fourier series, 175, 179
  - functions, 92, 126
  - geometric series, 120
  - imaginary part, 94
  - index of refraction, 133
  - modulus, 96
  - multiplication, 100
  - number, 94
  - phase, 96
  - power series, 92, 121, 125, 126
  - powers, 113
  - real part, 94
  - roots, 112, 115, 116
  - series, 118, 119
  - subtraction, 100
  - vector, 100, 110
- complex conjugate, 103
- complex plane, 94
- compounding of interest, 24
- conditional convergence, 45
- conditionally convergent, 45

- conjugate pairs, 100
- contact angle, 134
- convergence, 7, 8, 9, 10, 11, 13, 16
  - geometric sequence, 5
- convergent series
  - alternating series, 44
  - geometric, 39
  - $p$ -series, 38
  - subtraction of, 43
  - neglected terms, 43
- damped oscillator, 46
- differential equations, 142
- definite integrals, 79
- degrees, 98
- DeMoivre's theorem, 113
- differential equation
  - homogenous, 144
  - linear, 144
  - order of, 143
  - ordinary, 142
  - power series solution, 143
  - homogeneous, 143
  - linear, 143
  - nonhomogeneous, 143
  - nonlinear, 143
  - regular, 152
- differentiation
  - of Fourier series, 177
- limits
  - difficult, 13
- diffraction grating, 139
- discriminant, 93
- disk of convergence, 121, 122, 123, 124, 128, 130
- distinct roots, 116, 118
- divergence, 8, 10, 13
  - geometric sequence, 11
- divergence test, 35
  - of  $p$ -series, 33
  - $p$ -series, 33
- divergent series
  - geometric, 39
  - harmonic series, 35
  - neglected terms, 43
  - $p$ -series, 33
- dominance, 37
- dominated, 36, 38
- doubling time, 82
- error, 44, 75
  - in alternating series, 72
  - in asymptotic approximation, 76
  - in filtering process, 48
  - in non-alternating series, 74
- singularity
  - essential, 129, 130, 132
- Euler formula, 127
- Euler's equation, 97
- expansion
  - asymptotic, 75
- expansions
  - asymptotic, 76
  - complex series, 126
  - Fourier series, 176
  - Fourier-Bessel series, 178
  - geometric, 131
  - Laurent series, 128, 139
  - Legendre series, 178, 181, 182
  - power series, 56
  - Taylor series, 57
- exponential growth, 24
- Fibonacci sequence, 134
- filtering, 47
- finite sequence, 2
- fission, 22
- Fourier series, 50, 61, 175
  - coefficients, 176, 178
  - complex, 175, 179
  - expansions, 175
  - numerical summation, 180
  - solutions, 177
  - square-wave, 179
- Fourier-Bessel series, 178
- Frobenius, 151, 153
  - method, 166
  - series, 155, 162, 163, 167, 168, 172
- Fuchs's Theorem, 159, 163
- general term, 2, 3, 6, 8, 9, 10, 13, 16
- generalized complex power series, 123
- generalized power series, 152, 155
  - solution, 151
  - solution at infinity, 171
- genetic complexity, 21
- geometric growth, 1, 3, 5, 6, 7, 10, 12, 13, 19,
  - 20, 21, 22, 23, 24
  - virus, 22, 23
  - avalanche, 22
  - breeding season, 24
  - genetic complexity, 21
  - snowballing, 23

- geometric sequence, 1, 5, 6, 7, 10, 12, 13, 19,
  - 20, 21, 22, 24
  - shutter speed, 20
  - f-stops, 19
  - genetic complexity, 23
  - chain reactions, 22
- geometric series
  - limit comparison test, 41
  - ratio test, 40
  - absolute convergence, 42
  - convergent, 31
  - damped oscillator, 46
  - divergent, 31
  - filtering, 47
  - interval of convergence, 51
  - partial sum of, 31
  - sum of, 30
- going viral, 23
- growth factor, 22, 23, 24
  - plutonium, 22
  - uranium-235, 22, 23
  - virus, 23
  - water lily, 23
- harmonic analysis, 175
- harmonic series, 35
  - integral test, 35
- Hermite polynomials, 157, 169
- Hermite's differential equation, 157
- Hermite's equation, 169
- imaginary axis, 94
- imaginary number, 93
- indeterminate forms, 13, 14, 15, 88
- index notation, 2
- index of refraction, 92, 132
- indicial equation, 154, 158, 161, 168
- infinite sequence, 25
- Infinite Series, 25
  - sum of, 27
- integral test, 34, 35
  - of irregular series, 43
- interval notation, 55
- interval of convergence, 51
  - power series, 52, 53
- L'Hôpital's Rule, 13, 14, 15, 88
- Laplace transforms, 143
- lattice sum, 90
- Laurent series, 128, 129, 130, 131, 132, 140, 141
  - expansion, 140
- least-squares fit, 184
- Legendre differential equation, 51
  - or order  $l$ , 160
- Legendre polynomials, 51, 137, 138, 156, 157, 160, 177, 184
- Legendre series, 50, 156, 161, 177, 178, 183
  - expansion, 181
  - step function, 182
- Legendre's equation, 156, 158
- length, 96, 97
- limit, 7
- limit comparison test, 40
- limits
  - of factorials, 17
- limit theorems, 11
- Maclaurin series, 146, 148, 149, 153, 158, 169
- Maclaurin series expansion, 61
- Mariner's Rule of Twelfths, 1, 18
- mean value theorem, 71
- Frobenius
  - method, 153
- method of Frobenius, 156, 157, 171
- method of Frobenius, 151, 158
- modulus, 97, 102
- singularity
  - nonessential, 151, 152
- numerical
  - computations, 91
  - series, 89
- numerical summation
  - of Fourier series, 180
- optical cavity, 48
- order, 1, 3, 10, 16, 21
- ordinary point, 144, 146, 170
  - Hermite equation, 169
- orthogonal polynomials, 177
- partial differential equation, 142
- partial differential equations, 143
- partial sum, 27, 28, 30, 31, 32, 44
- periodic functions, 175, 177
- phase, 97
- Plutonium-239, 22
- polar form, 96
- pole of order  $n$ , 129
- positive term series, 34, 37
  - absolute convergence, 36

- comparison test, 35
- tests, 34
- theorems, 43
- positive term series test
  - integral test, 34
- positive term test
  - limit comparison test, 40
- power series, 50, 122, 123, 128, 130, 134, 135, 137, 144, 145, 146, 150, 156, 157, 164, 166
  - addition, 69
  - differentiation, 67
  - division, 66
  - expansions, 57, 62
  - integration, 68
  - interval of convergence, 52
  - multiplication, 63
  - partial fraction decomposition, 70
  - properties, 52, 55, 56
  - representations, 56
  - solution, 145, 146, 148, 150, 151, 152, 158, 163, 164, 166, 168, 169,
  - substitution, 63
  - subtraction, 69
  - theorems, 57
- $p$ -series, 33, 35, 38
  - comparison test, 38
  - convergence, 33, 37
  - divergence, 33
  - harmonic series, 33
  - theorems, 33
- pure imaginary numbers, 94
- quadratic equation, 92
- radians, 98
- radius of convergence, 124
  - geometric series, 51
  - power series, 53
- ratio test, 39, 57, 120, 123, 125, 126
  - geometric series, 40
  - inconclusive, 40, 53
  - power series, 52
- rational
  - expression, 70
  - function, 70
- rationalizing the denominator, 101
- real axis, 95
- rectangular form, 96
- recurrence relation, 134
- recursion relations, 145, 148, 154, 155, 159, 161, 165, 170
- regular point, 132, 140
- remainder, 44
  - of a series after  $n$  terms, 44
  - of the series after  $n$  terms, 75
- singularity
  - removable, 130
- road surveying, 85
- Russian roulette, 84
- rule of, 72, 82
- rules of thumb
  - rule of 72, 82
- Schrödinger equation, 166
- semi-convergent, 26
- sequence
  - alternating, 2
  - arithmetic, 5, *See* vibrating string
  - Fibonacci, 134, 135, 136
  - $f$ -stops, 19
  - geometric, 5
  - harmonic, 11, *See* vibrating string
  - limit theorems, 11
  - Mariner's Rule of Twelfths, 1, 18
  - of allowed frequencies. *See* vibrating string
  - of allowed wavelengths. *See* vibrating string
  - positive integer, 2
  - prime number, 3
- sequence of allowed sequence
  - of allowed frequencies, 9
- sequence of partial sums, 27, 28, 29, 32
- series
  - alternating, 42
  - asymptotic, 26, 29, 74
  - convergent, 26, 28, 36, 38
  - divergent, 26, 29
  - Fourier-Bessel, 178
  - geometric, 26, 29, 30, 38
  - harmonic, 35, 38, 40
  - Legendre, 177, 181
  - semi-convergent, 29
  - telescoping, 29, 30
- series approximation
  - accuracy, 71
  - remainder, 71
- sigma, 25
- simple pole, 132
- singular point, 144, 146, 150, 151, 153, 154, 158, 161, 170, 171
- singularities, 129, 131

190

singularity  
    isolated, 129  
    essential  
snowballing  
    water lily  
square-wave, 179, 180  
singular points, 129  
squeeze theorem, 12  
step function, 181  
Stirling approximation, 81  
summation notation, 25  
  
Taylor polynomial, 71  
Taylor series, 57, 123, 126, 127, 128  
    expansion, 59  
    formula, 59  
Taylor series expansion, 61

Index

telescope mirror, 86  
    Madelung constant, 90  
theorems  
    asymptotic series, 79  
    complex power series, 122  
    DeMoivre, 113  
    Fuchs's, 155  
theorem  
    squeeze, 12  
  
uranium-235, 22  
  
vibrating string, 4  
virus, 23  
  
water lily, 23  
wires in conduit, 20