A Student's Guide to Infinite Series and Sequences

Why study infinite series? Not all mathematical problems can be solved exactly or have a solution that can be expressed in terms of a known function. In such cases, it is common practice to use an infinite series expansion to approximate or represent a solution. This informal introduction for undergraduate students explores the numerous uses of infinite series and sequences in engineering and the physical sciences. The material has been carefully selected to help the reader develop the techniques needed to confidently utilize infinite series. The book begins with infinite series and sequences before moving onto power series, complex infinite series, and finally Fourier, Legendre, and Fourier-Bessel series. With a focus on practical applications, the book demonstrates that infinite series are more than an academic exercise and helps students to conceptualize the theory with real-world examples and to build their skill set in this area.

BERNHARD W. BACH JR. is the Director of Undergraduate Laboratories at the University of Nevada, Reno. His research interests focus on gamma ray, X-ray and UV spectroscopy, and the manufacture of diffractive optics and spectroscopic instrumentation. He has contributed to the design and construction of scientific instruments for numerous space-flight missions and synchrotron light sources around the world.

Other Books in the Student Guide Series

A Student's Guide to Atomic Physics, Mark Fox
A Student's Guide to Waves, Daniel Fleisch, Laura Kinnaman
A Student's Guide to Entropy, Don S. Lemons
A Student's Guide to Dimensional Analysis, Don S. Lemons
A Students Guide to Numerical Methods, Ian H. Hutchinson
A Student's Guide to Langrangians and Hamiltonians, Patrick Hamill
A Student's Guide to the Mathematics of Astronomy, Daniel Fleisch, Julia Kregonow
A Student's Guide to Vectors and Tensors, Daniel Fleisch
A Student's Guide to Maxwell's Equations, Daniel Fleisch
A Student's Guide to Fourier Transforms, J. F. James
A Student's Guide to Data and Error Analysis, Herman J. C. Berendsen

A Student's Guide to Infinite Series and Sequences

BERNHARD W. BACH JR.
University of Nevada, Reno

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107059825
DOI: 10.1017/9781107446588

First published 2018

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Bach, Bernhard W., Jr., 1965– author.
Title: A student's guide to infinite series and sequences / Bernhard W. Bach, Jr. (University of Nevada, Reno).
Other titles: Infinite series and sequences
Description: Cambridge : Cambridge University Press, 2018. | Includes bibliographical references and index.
Identifiers: LCCN 2017061456 | ISBN 9781107059825 (alk. paper)
Subjects: LCSH: Series, Infinite. | Calculus.
Classification: LCC QA295 .B2245 2018 | DDC 515/.243–dc23
LC record available at https://lccn.loc.gov/2017061456

ISBN 978-1-107-05982-5 Hardback
ISBN 978-1-107-64048-1 Paperback

To Angie, Tim, Brian, Aaron, and Cam,

for their advice and support along the way.

Contents

Preface

Why study infinite series? Not all mathematical problems can be solved exactly or have a solution that can be expressed in terms of a known function. In such cases, it is common practice to use an infinite series expansion to approximate or represent a solution. For example, many differential equations have solutions that cannot be expressed in terms of known or elementary functions, yet their solutions can be written out as infinite series of terms.

Infinite series are also used to approximate the numerical values of specific functions or integrals. For example, the value of a transcendental function can be calculated using an algorithm in which the transcendental is represented as an infinite series of terms. Representing a function as an infinite series is a technique that is widely used in the sciences and engineering. The ability to expand a function as an infinite series along with proficiency in manipulating such a series is a useful skill set for those pursuing careers in the sciences or engineering.

Most of us are introduced to infinite series in a second-semester calculus course, where the material is usually treated as cursory, leaving us with only a vague sense of infinite series. Furthermore, this treatment gives little indication of the practical applications of infinite series. While the present text is certainly incomplete, the material it contains has been selected to help the reader develop the skills needed to confidently create and manipulate infinite series as well as appreciate their wide range of applications in science and engineering.