

A Student's Guide to Infinite Series and Sequences

Why study infinite series? Not all mathematical problems can be solved exactly or have a solution that can be expressed in terms of a known function. In such cases, it is common practice to use an infinite series expansion to approximate or represent a solution. This informal introduction for undergraduate students explores the numerous uses of infinite series and sequences in engineering and the physical sciences. The material has been carefully selected to help the reader develop the techniques needed to confidently utilize infinite series. The book begins with infinite series and sequences before moving onto power series, complex infinite series, and finally Fourier, Legendre, and Fourier-Bessel series. With a focus on practical applications, the book demonstrates that infinite series are more than an academic exercise and helps students to conceptualize the theory with real-world examples and to build their skill set in this area.

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To Angie, Tim, Brian, Aaron, and Cam,
for their advice and support along the way.

Contents

<i>Preface</i>	<i>page ix</i>
1 Infinite Sequences	1
1.1 Introduction to Sequences	1
1.2 Notation	1
1.3 Example Sequences	3
1.4 Limits and Convergence	7
1.5 Examples	18
2 Infinite Series	25
2.1 Introduction to Series	25
2.2 Convergence and the Sequence of Partial Sums	26
2.3 Testing Infinite Series for Convergence	32
2.4 Alternating Series	43
2.5 Conditionally Convergent Series	44
2.6 Examples	46
3 Power Series	50
3.1 Interval of Convergence	50
3.2 Properties of Power Series	52
3.3 Power Series Expansions of Functions	57
3.4 Other Methods for Constructing Power Series Expansions	62
3.5 Accuracy of Series Approximations	71
3.6 Asymptotic Series Expansions	74
3.7 Examples	79
4 Complex Infinite Series	92
4.1 Complex Numbers	92
4.2 Complex Infinite Series	118

4.3	Determining the Disk of Convergence	123
4.4	Functions of Complex Variables	126
4.5	Laurent Series	128
4.6	Examples	132
5	Series Solutions for Differential Equations	142
5.1	Introduction	142
5.2	Series Solutions for Differential Equations	143
5.3	Generalized Series Solutions and the Method of Frobenius	150
5.4	Introduction to Special Functions: Bessel, Hermite, and Legendre	156
5.5	Examples	163
6	Fourier, Legendre, and Fourier-Bessel Series	174
6.1	Introduction	174
6.2	Fourier Series	175
6.3	Legendre Series	177
6.4	Fourier-Bessel Series	178
6.5	Examples	178
	<i>References</i>	185
	<i>Index</i>	186

Preface

Why study infinite series? Not all mathematical problems can be solved exactly or have a solution that can be expressed in terms of a known function. In such cases, it is common practice to use an infinite series expansion to approximate or represent a solution. For example, many differential equations have solutions that cannot be expressed in terms of known or elementary functions, yet their solutions can be written out as infinite series of terms.

Infinite series are also used to approximate the numerical values of specific functions or integrals. For example, the value of a transcendental function can be calculated using an algorithm in which the transcendental is represented as an infinite series of terms. Representing a function as an infinite series is a technique that is widely used in the sciences and engineering. The ability to expand a function as an infinite series along with proficiency in manipulating such a series is a useful skill set for those pursuing careers in the sciences or engineering.

Most of us are introduced to infinite series in a second-semester calculus course, where the material is usually treated as cursory, leaving us with only a vague sense of infinite series. Furthermore, this treatment gives little indication of the practical applications of infinite series. While the present text is certainly incomplete, the material it contains has been selected to help the reader develop the skills needed to confidently create and manipulate infinite series as well as appreciate their wide range of applications in science and engineering.