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Glossary of Notation

Relations, operators, positional notation

$\{\}$ - set
$\cap, \cup, \in, \triangle$ - intersection, union, membership, symmetric difference
$\lfloor x \rfloor, \lceil x \rceil$ - floor, ceiling (nearest integer at most or at least x)
$[k]$ - $\{1, \ldots, k\}$
k-object - "object" with parameter value k
$[a, b]$ - $\{a, \ldots, b\}$, if a, b are integers
$[x, y]$ - $\{z\colon x \le z \le y\}$, if x, y are real numbers or poset elements
$[x^k]$ - coefficient of x^k in a formal power series in x
$[S, T]$ - cut consisting of all edges from S to T
$\langle v_1, \ldots, v_n \rangle$ - path with vertices $v_1, \ldots, v_n$ in order
$[v_1, \ldots, v_n]$ - cycle with vertices $v_1, \ldots, v_n$ in order
$n!$ - $\prod_{i=1}^{n} i$
$n!!$ - $\prod_{i=0}^{\lfloor (n-1)/2 \rfloor} (n - 2i)$
$n_{(r)}$ - $n(n-1)\cdots(n-r+1)$
$n^{(r)}$ - $n(n+1)\cdots(n+r-1)$
$d \mid n$ - d divides n
$d \nmid n$ - d does not divide n
$|S|$ - size of set S
$\#\{i\colon i \in S\}$ - size of described set
$\#(G)$ - number of components of a graph
A^T - transpose of a matrix A
$\overline{S}$ - complement of set S (within a given universe)
$\overline{G}$ - complement of graph G
G^*, P^*, M^* - dual (of a planar graph, poset, or matroid)
$\leftrightarrow$ - adjacency relation on vertices of graph
$\rightarrow$ - succession relation on nodes (vertices) of digraph
$\|$ - incomparability relation on elements of poset
$\prec, \succ$ - cover relation on elements of poset
$a_n \sim b_n$ - asymptotic to (ratio approaching 1)
$\lambda \vdash n$ - λ partitioning the integer n, with $\lambda_1 \ge \cdots \ge \lambda_k$ and $\sum \lambda_i = n$

$G - v$, $G - S$, $G - e$, $G - F$ - deletion of vertex, vertex set, edge, edge set
$G \cdot e$, $M \cdot e$ - contraction of edge or matroid element
G^- - graph formed by deleting one edge from an edge-transitive graph G
G^+ - graph formed by adding one edge to a graph G with edge-transitive $\overline{G}$
$M|F$, $M.F$ - restriction or contraction of a matroid to the elements F
$\mathbf{0}$, $\mathbf{1}$ - unique lower and upper bounds of a poset
$\mathbf{1}_n$ - column vector of n 1s
$\mathbf{2}$, $\mathbf{n}$ - two-element, n-element chains in posets
$\mathbf{2}^n$ - subset poset of order n
$\mathbf{2}^E$ - subsets of E, ordered by inclusion
$\binom{n}{k}$ - binomial coefficient counting k-subsets of the set $[n]$
$\binom{n}{k_1,\ldots,k_m}$ - multinomial coefficient
$\binom{S}{k}$ - the collection of k-subsets of set S
$G + H$ - disjoint union of graphs or posets
mG - disjoint union of m copies of graph or poset
$G \oplus H$ - join (disjoint union plus all edges between)
$G[S]$ - subgraph of G induced by S
$G[H_1, \ldots, H_n]$, $G[H]$ - lexicographic product, composition (graphs or posets)
$G \,\square\, H$ - Cartesian product of graphs
$G \boxtimes H$ - strong product
$G * H$ - weak product, tensor product
$P \times Q$ - Cartesian product of sets or posets
P^k - Cartesian product of k copies of poset P
G^k - graph on $V(G)$ with $u \leftrightarrow v$ if $d_G(u, v) \le k$
P_k - kth rank of a ranked poset P
$x \vee y$ - least upper bound in poset (join)
$x \wedge y$ - greatest lower bound in poset (meet)
X^-, X^+ - down-set and up-set generated by set X in poset

Usage of Roman alphabet

$A(G)$ - adjacency matrix of a graph G
$\mathbf{A}(P)$ - incidence algebra of a poset P
A_n - n-element antichain
$A(n, k)$ - Eulerian number counting permutations of $[n]$ with k runs
$A(x)$, $B(x)$ - typical generating functions with formal variable x
AP - arithmetic progression
box G - boxicity of a graph G
$B(G)$ - bandwidth of a graph G, block graph of G
B_n - Bell number counting partitions of $[n]$
$\mathbf{B}(M)$ - bases of a matroid M
Bin (n, p) - binomial distribution (n trials, success probability p)
BIBD - balanced incomplete block design
$c(G)$ - circumference of a graph G
$c(n, k)$ - number of permutations of $[n]$ with k cycles
ch(G) - choosability (list chromatic number) of a graph G

$\mathrm{cr}(G)$ - crossing number of a graph G
$\mathrm{cr}'(G)$ - (recti)linear crossing number of a graph G
$\mathrm{cr}_k(G)$ - crossing number when each edge is at most k segments
$\mathbb{C}$ - complex numbers
C_n - cycle of length n, nth Catalan number
$C(P)$ - set of critical (unforced) incomparable pairs of a poset P
$\mathbf{C}(M)$ - circuits of a matroid M
$d(v)$, $d_G(v)$ - degree, valence (of a vertex v in G)
$\overline{d}(G)$ - average vertex degree in a graph G
$d(u,v)$, $d_G(u,v)$ - distance between u and v (in a graph G)
$d_1, \ldots, d_n$ - degree list
$d_k(P)$ - size of maximum k-family in a poset P
$\hat{d}_k(P)$ - size of maximum k-cofamily in a poset P
$d_{m,n}$ - Delannoy number counting up/right/diagonal paths from $(0,0)$ to (m,n)
$\mathrm{def}(S)$ - deficiency of a vertex set S (given by $o(G-S)-|S|$)
$\mathrm{df}(S)$ - defect of a set S in a bipartite graph (given by $|S|-|N(S)|$)
det - determinant of a matrix
$\mathrm{diam}\,(G)$ - diameter of a graph G
$\dim(P)$ - order dimension of a poset P
$\partial_k(m)$ - numerical shadow of k-binomial expansion of an integer m
$\partial(F)$ - shadow of a set-family F
$D[S]$ - down-set (ideal) of poset, generated by set S, often $D[x]$
D_n - number of derangements of $[n]$
D_N - divisors of N, ordered by divisibility
e - base of natural logarithm, 2.71828...
$e(P)$ - number of linear extensions of a poset P
$e_k(x)$ - number of linear extensions with x at height k
$\mathrm{ex}\,(n;H)$ - maximum number of edges in n-vertex graph not containing H
E - typical set of elements of a matroid
$E(G)$ - set of edges in a (hyper)graph G
$\mathbb{E}(X)$ - expectation of random variable X
EGF - exponential generating function
f_k - rank parameters of family F in a ranked poset
f_λ - number of Young tableaux with shape λ
$f(T)$ - in inclusion-exclusion, #elements whose properties are indexed by T
F_n - nth classical Fibonacci number
$\hat{F}_n$ - nth adjusted Fibonacci number (equal to F_{n+1})
$\mathbb{F}_q$ - q-element field
F, G, H - typical graphs, digraphs, or hypergraphs
$g(G)$ - girth
G^p - random graph in Model A (edge-probability model)
G^m - random graph in Model B (fixed-size model)
$\mathbb{G}(n,p)$ - random graph model with edge-probability p
$G(P)$ - comparability graph of a poset P
$h(P)$ - height of a poset P
$h(x)$ - height of poset element x, or expected height of element
H_n - nth harmonic number, $\sum_{i=1}^{n} 1/i$
$H_{k,n}$ - k-connected Harary graph with n vertices

$i(G)$ - interval number of a graph G
$\mathbf{I}(M)$ - independent sets of a matroid M
$I(P)$ - set of incomparable pairs of poset P
$\mathrm{Int}(P)$ - family of intervals in a poset P
$I(x)$ - set of elements incomparable to x
I - identity matrix (diagonal matrix with 1s on diagonal),
J - square matrix with every entry equal to 1
$J(P)$ - lattice of order ideals of a poset P, ordered by inclusion
K_n - complete graph of order n
$K_{m,n}$ - complete bipartite graph with parts of sizes m and n
$K_{S,T}$ - complete bipartite graph with parts S and T
$K(n, k)$ - Kneser graph with vertex set $\binom{[n]}{k}$
$l(P)$ - length of a poset P (length of longest chain)
lg - logarithm in base 2
ln - natural logarithm
$L(G)$ - line graph of a graph G
$L(v)$ - list of colors at a vertex in a list assignment
$L(m, n)$ - poset of m-tuples a with $0 \le a_1 \le \cdots \le a_m$, componentwise ordered
$L_n(q)$ - lattice of subspaces of finite vector space of dimension n over $GF(q)$
m - often the number of edges in a graph
$m_k(\mathbf{C})$, $m_k(a)$ - k-norm of chain partition or sequence $[= \sum_i \min\{k, a_i\}]$
M - a matching, a matroid, etc.
M^e - chain product $\underline{e_1} \times \cdots \times \underline{e_n}$
$M(G)$ - cycle matroid of a graph G
$\mathrm{Mad}(G)$ - maximum average degree among subgraphs of a graph G
$\mathrm{MOLS}(n, k)$ - family of k pairwise (mutually) othogonal Latin squares of order n
M_5 - the 5-element modular non-distributive lattice
n - typically the number of vertices or elements in a (hyper)graph or poset
N_5 - the 5-element non-modular non-distributive lattice
$N_k(P)$ - size of kth rank of a poset P
$N(x)$, $N_G(x)$ - (open) neighborhood of x in a graph G
$N[x]$ - $N(x) \cup \{x\}$ (closed neighborhood)
$N(S)$ - $\cup_{x \in S} N(x)$
$N^+(x)$ - out-neighborhood (successor set) of x in a digraph
$N^-(x)$ - in-neighborhood (predecessor set) of x in a digraph
$N(m)$ - maximum number of pairwise orthogonal Latin squares of order m
$\mathbb{N}$ - set of natural numbers
$\mathbb{N}_0$ - $\mathbb{N} \cup \{0\}$
$o(G)$ - number of odd components
$o(f(n))$ - functions whose ratio to $f(n)$ approach 0
$O(f(n))$ - functions bounded (for large n) by a constant multiple of $f(n)$
O_k - Odd graph (disjointness graph on k-sets and $(k+1)$-sets in $[2k+1]$)
$\mathbf{O}(Q)$ - order polytope of a poset Q
OGF - ordinary generating function
OA - orthogonal array
$p(n, \varepsilon)$ - threshold probability function
per - permanent of a matrix
$\mathbb{P}(A)$ - probability of event A

P_n - path with n vertices
$P(n, k)$ - generalized Petersen graph
PIE - inclusion-exclusion principle
$P(L)$ - poset of join-irreducibles of a lattice L
PBD - pairwise balanced design
$P(\sigma)$ - P-symbol of a permutation σ
$Q(\sigma)$ - Q-symbol of a permutation σ
$\mathbb{Q}$ - rational numbers
Q_k - hypercube of dimension k (as a graph)
$r(M)$, $r(P)$ - rank of a matroid or poset
$r(x)$, $r(X)$ - rank of an element in a poset, rank of a set in a matroid
r_M - rank function of matroid M
$\mathbb{R}$ - real numbers
$R_B(x)$ - rook polynomial of the board B
$R(p_1, \ldots, p_k; r)$ - Ramsey number for k-coloring r-sets
$R_k(p; r)$ - Ramsey number for k-coloring r-sets with common threshold p
$R(p, q)$ - Ramsey number for 2-coloring 2-sets
$R(G_1, \ldots, G_k)$ - graph Ramsey number
$\mathbb{S}_n$ - symmetric group on n elements (set of permutations of $[n]$)
S_g - surface with g handles
$S(n, k)$, $s(n, k)$ - Stirling numbers of second and first kinds
SDR - system of distinct representatives
STS(v) - Steiner triple system of order v
T - typical tree or tournament
$T_{n,r}$ - Turán graph (equipartite complete r-partite n-vertex graph)
$U[S]$ - up-set in poset generated by set S, typically $U[x]$
$V(G)$ - set of vertices of a graph or hypergraph G
$w(l, k)$ - van der Waerden number (guaranteeing l-term AP in k-coloring)
$w(P)$ - width of a poset P
$W_k(P)$ - size of kth rank of a poset (Whitney numbers)
X - typical random variable
X^+, X^- - positive and negative parts in a signed involution
X, Y - typical bipartition of a graph into two independent sets
Y - Young lattice of all partitions of integers, ordered componentwise
$\mathbb{Z}$ - integers

Usage of Greek alphabet

$\alpha(G)$ - maximum size of independent set in a graph G
$\alpha'(G)$ - maximum size of a matching in G (pairwise independent edges)
$\beta(G)$ - minimum number of vertices covering all edges in G
$\beta'(G)$ - minimum number of edges covering all vertices in G
$\gamma(G)$ - genus, domination number of a graph G
$\delta_{i,j}$ - Kronecker delta (1 if $i = j$, 0 if $i \neq j$)
$\delta(G)$ - minimum vertex degree of a graph G
$\Delta(G)$ - maximum vertex degree of a graph G
Δ - a simplex

$\Delta_k(P)$ - $d_k(P) - d_{k-1}(P)$ for a poset P
$\hat{\Delta}_k(P)$ - $\hat{d}_k(P) - \hat{d}_{k-1}(P)$ for a poset P
$\varepsilon(v)$ - eccentricity of a vertex v
$\zeta, \eta, \mu, \kappa, \lambda$ - incidence functions on posets
$\theta(G)$ - minimum number of cliques to cover all vertices in a graph G
$\kappa(G)$ - connectivity of a graph G
$\kappa'(G)$ - edge-connectivity of a graph G
$\kappa(x, y)$ - local connectivity for vertices in a graph
$\kappa'(x, y)$ - local edge-connectivity for vertices in a graph
$\lambda(x, y)$ - maximum number of independent x, y-paths
$\lambda'(x, y)$ - maximum number of edge-disjoint x, y-paths
$\lambda_1, \ldots, \lambda_n$ - eigenvalues (of adjacency matrix)
Λ_n - partitions of integer n, ordered by refinement
$\mu(n)$ - Möbius function of an integer n
$\mu(x, y)$ - Möbius function on interval $[x, y]$
$\mu_1, \ldots, \mu_n$ - Laplacian eigenvalues of a graph
π - circumference of circle with unit diameter, 3.1415926...
π, σ, τ - typical permutations
Π_n - lattice of partitions of an n-set, ordered by refinement
ρ - density
$\sigma(X)$ - span of a set X in a matroid
Σ - universe of sets for intersection classes, surface
$\sum$ - summation
$\tau(G)$ - number of spanning trees in a graph G
$\tau(G)$ - minimum partition into complete bipartite subgraphs
$\Upsilon(G)$ - arboricity of a graph G
ϕ - a mapping, often an isomorphism, coloring, or homomorphism
$\varphi(m)$ - Euler totient function (numbers in $[m]$ relatively prime to m)
$\chi(G)$ - chromatic number of a graph G
$\chi'(G)$ - edge-chromatic number of G
$\chi''(G)$ - total chromatic number of G
$\chi_\ell(G)$ - list-chromatic number of G
$\chi'_\ell(G)$ - list-edge-chromatic number of G
$\omega(G)$ - clique number of a graph G
ω_n - unbounded sequence
$\Omega(\mathbf{A})$ - intersection graph of collection $\mathbf{A}$
$\Omega(\sum)$ - intersection class for subsets of $\sum$
$\Omega(f(n))$ - functions at least a constant multiple of $f(n)$ (for large n)

Subject Index

A page number in italics indicates a definition. A single listing in italics may indicate the definition for a concept so prevalent (such as "graph") that it would not be productive to list its occurrences. An item that appears on few pages may have none italicized for the definition.

Page ranges in bold indicate material such as the proof of a major result or the main treatment of the concept; this may also include a definition. Pages ranges may also include isolated pages where the term does not appear.

Parenthetic clarifiers act as subheadings. Terms consisting of a word modified by prefatory notation are alphabetized according to the root word.