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A Guide to NIP Theories

The study of NIP theories has received much attention from model theorists in the last decade, fuelled by applications to o-minimal structures and valued fields. This book, the first to be written on NIP theories, is an introduction to the subject that will appeal to anyone interested in model theory: graduate students and researchers in the field, as well as those in nearby areas such as combinatorics and algebraic geometry. Without dwelling on any one particular topic, it covers all of the basic notions and gives the reader the tools needed to pursue research in this area. An effort has been made in each chapter to give a concise and elegant path to the main results and to stress the most useful ideas. Particular emphasis is put on honest definitions, handling of indiscernible sequences and measures. The relevant material from other fields of mathematics is made accessible to the logician.

PIERRE SIMON is Chargé de recherche, CNRS at Université Lyon 1 in France. He completed his PhD at Université Paris-Sud, Orsay under the supervision of Elisabeth Bouscaren. His thesis, “Ordre et stabilité dans les théories NIP”, received the 2012 Sacks Prize for the best thesis in logic that year as well as the Perrissin-Pirasset/Schneider prize from the Chancellerie des Universités de Paris.

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CNRS, Université Lyon 1



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