Optimization Methods in Finance

Optimization methods play a central role in financial modeling. This textbook is devoted to explaining how state-of-the-art optimization theory, algorithms, and software can be used to efficiently solve problems in computational finance. It discusses some classical mean–variance portfolio optimization models as well as more modern developments such as models for optimal trade execution and dynamic portfolio allocation with transaction costs and taxes. Chapters discussing the theory and efficient solution methods for the main classes of optimization problems alternate with chapters discussing their use in the modeling and solution of central problems in mathematical finance.

This book will be interesting and useful for students, academics, and practitioners with a background in mathematics, operations research, or financial engineering.

The second edition includes new examples and exercises as well as a more detailed discussion of mean–variance optimization, multi-period models, and additional material to highlight the relevance to finance.

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Second Edition

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Preface

The use of sophisticated mathematical tools in modern finance is now commonplace. Researchers and practitioners routinely run simulations or solve differential equations to price securities, estimate risks, or determine hedging strategies. Some of the most important tools employed in these computations are optimization algorithms. Many computational finance problems ranging from asset allocation to risk management, from option pricing to model calibration, can be solved by optimization techniques. This book is devoted to explaining how to solve such problems efficiently and accurately using the state of the art in optimization models, methods, and software.

Optimization is a mature branch of applied mathematics. Typical optimization problems have the goal of allocating limited resources to alternative activities in order to maximize the total benefit obtained from these activities. Through decades of intensive and innovative research, fast and reliable algorithms and software have become available for many classes of optimization problems. Consequently, optimization is now being used as an effective management and decision-support tool in many industries, including the financial industry.

This book discusses several classes of optimization problems encountered in financial models, including linear, quadratic, integer, dynamic, stochastic, conic, and nonlinear programming. For each problem class, after introducing the relevant theory (optimality conditions, duality, etc.) and efficient solution methods, we discuss several problems of mathematical finance that can be modeled within this problem class.

The second edition includes a more detailed discussion of mean–variance optimization, multi-period models, and additional material to highlight the relevance to finance.

The book's structure has also been clarified for the second edition; it is now organized in four main parts, each comprising several chapters. Part I guides the reader through the solution of asset liability cash flow matching using linear programming techniques, which are also used to explain asset pricing and arbitrage. Part II is devoted to single-period models. It provides a thorough treatment of mean–variance portfolio optimization models, including derivations of the one-fund and two-fund theorems and their connection to the capital asset pricing model, a discussion of linear factor models that are used extensively in risk and portfolio management, and techniques to deal with the sensitivity of mean–variance models to parameter estimation. We discuss integer programming formulations for portfolio construction problems with cardinality constraints, and we explain how this is relevant to constructing an index fund. The final chapters of Part II present a stochastic programming approach to modeling measures of risk other than the variance, including the popular value at risk and conditional value at risk.

Part III of the book discusses multi-period models such as the iconic Kelly criterion and binomial lattice models for asset pricing as well as more elaborate and modern models for optimal trade execution, dynamic portfolio optimization with transaction costs and taxes, and asset–liability management. These applications showcase techniques from dynamic and stochastic programming.

Part IV is devoted to more advanced optimization techniques. We introduce conic programming and discuss applications such as the approximation of covariance matrices and robust portfolio optimization. The final chapter of Part IV covers one of the most general classes of optimization models, namely nonlinear programming, and applies it to volatility estimation.

This book is intended as a textbook for Master's programs in financial engineering, finance, or computational finance. In addition, the structure of chapters, alternating between optimization methods and financial models that employ these methods, allows the book to be used as a primary or secondary text in upper-level undergraduate or introductory graduate courses in operations research, management science, and applied mathematics. A few sections are marked with a '$*$' to indicate that the material they contain is more technical and can be safely skipped without loss of continuity.

Optimization algorithms are sophisticated tools and the relationship between their inputs and outputs is sometimes opaque. To maximize the value from using these tools and to understand how they work, users often need a significant amount of guidance and practical experience with them. This book aims to provide this guidance and serve as a reference tool for the finance practitioners who use or want to use optimization techniques.

This book has benefited from the input provided by instructors and students in courses at various institutions. We thank them for their valuable feedback and for many stimulating discussions. We would also like to thank the colleagues who provided the initial impetus for this book and colleagues who collaborated with us on various research projects that are reflected in the book. We especially thank Kathie Cameron, the late Rick Green, Raphael Hauser, John Hooker, Miroslav Karamanov, Mark Koenig, Masakazu Kojima, Vijay Krishnamurthy, Miguel Lejeune, Yanjun Li, François Margot, Ana Margarida Monteiro, Mustafa Pınar, Sebastian Pokutta, Sanjay Srivastava, Michael Trick, and Luís Vicente.