Symplectic geometry has its origin in physics, but has flourished as an independent subject in mathematics, together with its offspring, symplectic topology. Symplectic methods have even been applied back to mathematical physics; for example, Floer theory has contributed new insights to quantum field theory. In a related direction, noncommutative geometry has developed an alternative mathematical quantization scheme, based on a geometric approach to operator algebras. Deformation quantization, a blend of symplectic methods and noncommutative geometry, approaches quantum mechanics from a more algebraic viewpoint, as it addresses quantization as a deformation of Poisson structures.

This volume contains seven articles based on lectures given by invited speakers at two May 2010 workshops held at MSRI: *Symplectic and Poisson Geometry in Interaction with Analysis, Algebra and Topology* (honoring Alan Weinstein, one of the key figures in the field) and the *Hayashibara Forum on Symplectic Geometry, Noncommutative Geometry and Physics*. Both workshops were jointly sponsored by MSRI, the Research Institute of Mathematical Sciences at Kyoto University (RIMS), and the Hayashibara Foundation. The articles include presentations of previously unpublished results and comprehensive reviews including recent developments in these areas.

Mathematical Sciences Research Institute
Publications

62

Symplectic, Poisson, and Noncommutative Geometry

Mathematical Sciences Research Institute Publications

1 Freed/Uhlenbeck: *Instantons and Four-Manifolds*, second edition
2 Chern (ed.): *Seminar on Nonlinear Partial Differential Equations*
3 Lepowsky/Mandelstam/Singer (eds.): *Vertex Operators in Mathematics and Physics*
4 Kac (ed.): *Infinite Dimensional Groups with Applications*
5 Blackadar: *K-Theory for Operator Algebras*, second edition
6 Moore (ed.): *Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics*
7 Chorin/Majda (eds.): *Wave Motion: Theory, Modelling, and Computation*
8 Gersten (ed.): *Essays in Group Theory*
9 Moore/Schochet: *Global Analysis on Foliated Spaces*, second edition
10–11 Drasin/Earle/Gehring/Kra/Marden (eds.): *Holomorphic Functions and Moduli*
12–13 Ni/Peletier/Serrin (eds.): *Nonlinear Diffusion Equations and Their Equilibrium States*
14 Goodman/de la Harpe/Jones: *Coxeter Graphs and Towers of Algebras*
15 Hochster/Huneke/Sally (eds.): *Commutative Algebra*
16 Ihara/Ribet/Serre (eds.): *Galois Groups over* $\mathbb{Q}$
17 Concus/Finn/Hoffman (eds.): *Geometric Analysis and Computer Graphics*
18 Bryant/Chern/Gardner/Goldschmidt/Griffiths: *Exterior Differential Systems*
19 Alperin (ed.): *Arboreal Group Theory*
20 Dazord/Weinstein (eds.): *Symplectic Geometry, Groupoids, and Integrable Systems*
21 Moschovakis (ed.): *Logic from Computer Science*
22 Ratiu (ed.): *The Geometry of Hamiltonian Systems*
23 Baumslag/Miller (eds.): *Algorithms and Classification in Combinatorial Group Theory*
24 Montgomery/Small (eds.): *Noncommutative Rings*
25 Akbulut/King: *Topology of Real Algebraic Sets*
26 Judah/Just/Woodin (eds.): *Set Theory of the Continuum*
27 Carlsson/Cohen/Hsiang/Jones (eds.): *Algebraic Topology and Its Applications*
28 Clemens/Kollár (eds.): *Current Topics in Complex Algebraic Geometry*
29 Nowakowski (ed.): *Games of No Chance*
30 Grove/Petersen (eds.): *Comparison Geometry*
31 Levy (ed.): *Flavors of Geometry*
32 Cecil/Chern (eds.): *Tight and Taut Submanifolds*
33 Axler/McCarthy/Sarason (eds.): *Holomorphic Spaces*
34 Ball/Milman (eds.): *Convex Geometric Analysis*
35 Levy (ed.): *The Eightfold Way*
36 Gavosto/Krantz/McCallum (eds.): *Contemporary Issues in Mathematics Education*
37 Schneider/Siu (eds.): *Several Complex Variables*
38 Billera/Björner/Green/Simion/Stanley (eds.): *New Perspectives in Geometric Combinatorics*
39 Haskell/Pillay/Steinhorn (eds.): *Model Theory, Algebra, and Geometry*
40 Bleher/Its (eds.): *Random Matrix Models and Their Applications*
41 Schneps (ed.): *Galois Groups and Fundamental Groups*
42 Nowakowski (ed.): *More Games of No Chance*
43 Montgomery/Schneider (eds.): *New Directions in Hopf Algebras*
44 Buhler/Stevenhagen (eds.): *Algorithmic Number Theory: Lattices, Number Fields, Curves and Cryptography*
45 Jensen/Ledet/Yui: *Generic Polynomials: Constructive Aspects of the Inverse Galois Problem*
46 Rockmore/Healy (eds.): *Modern Signal Processing*
47 Uhlmann (ed.): *Inside Out: Inverse Problems and Applications*
48 Gross/Kotiuga: *Electromagnetic Theory and Computation: A Topological Approach*
49 Darmon/Zhang (eds.): *Heegner Points and Rankin L-Series*
50 Bao/Bryant/Chern/Shen (eds.): *A Sampler of Riemann–Finsler Geometry*
51 Avramov/Green/Huneke/Smith/Sturmfels (eds.): *Trends in Commutative Algebra*
52 Goodman/Pach/Welzl (eds.): *Combinatorial and Computational Geometry*
53 Schoenfeld (ed.): *Assessing Mathematical Proficiency*
54 Hasselblatt (ed.): *Dynamics, Ergodic Theory, and Geometry*
55 Pinsky/Birnir (eds.): *Probability, Geometry and Integrable Systems*
56 Albert/Nowakowski (eds.): *Games of No Chance 3*
57 Kirsten/Williams (eds.): *A Window into Zeta and Modular Physics*
58 Friedman/Hunsicker/Libgober/Maxim (eds.): *Topology of Stratified Spaces*
59 Caporaso/M^{c}Kernan/Mustaţă/Popa (eds.): *Current Developments in Algebraic Geometry*
60 Uhlmann (ed.): *Inverse Problems and Applications: Inside Out II*
61 Breuillard/Oh (eds.): *Thin Groups and Superstrong Approximation*

Symplectic, Poisson, and Noncommutative Geometry

Edited by

Tohru Eguchi
Rikkyo University

Yakov Eliashberg
Stanford University

Yoshiaki Maeda
Keio University

CAMBRIDGE
UNIVERSITY PRESS

Tohru Eguchi
Rikkyo University
t.eguchi@rikkyo.ac.jp

Yakov Eliashberg
Stanford University
eliash@math.stanford.edu

Yoshiaki Maeda
Keio University
maeda@math.keio.ac.jp

Silvio Levy (*Series Editor*)
Mathematical Sciences Research Institute
levy@msri.org

The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation and the *Pacific Journal of Mathematics* for the publication of this series.

CAMBRIDGE
UNIVERSITY PRESS

32 Avenue of the Americas, New York NY 10013-2473, USA

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107056411

First published 2014

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-05641-1 Hardback

Symplectic, Poisson and Noncommutative Geometry
MSRI Publications
Volume **62**, 2014

Contents

Symplectic, Poisson and Noncommutative Geometry
MSRI Publications
Volume **62**, 2014

Preface

Symplectic geometry has its origin in physics, particularly as a mathematical framework for Hamiltonian mechanics, but it has undergone a remarkable development in the past thirty years as an independent subject in mathematics. Through the work of many outstanding mathematicians, symplectic geometry and its offspring, symplectic topology, have become very active research areas. There are now many applications of symplectic methods back to mathematical physics; for example, Floer theory has contributed new insights to quantum field theory. In a related direction, noncommutative geometry has developed an alternative mathematical quantization scheme, based on a geometric approach to operator algebras. Finally, deformation quantization, a blend of symplectic methods and noncommutative geometry, approaches quantum mechanics from a more algebraic viewpoint, as it addresses quantization as a deformation of Poisson structures. Thus, symplectic geometry and noncommutative geometry offer unique but related perspectives on quantum theory, and with deformation quantization they have led to many fruitful connections between mathematics and physics.

Two research programs, the 2009–2010 MSRI (Mathematical Sciences Research Institute at Berkeley) thematic year on Symplectic and Contact Geometry and Topology and the 2010 RIMS (Research Institute of Mathematical Sciences at Kyoto University) thematic year on Perspectives in Deformation Quantization and Noncommutative Geometry, held joint workshops at MSRI in May 2010.

The conference on *Symplectic and Poisson Geometry in Interaction with Analysis, Algebra and Topology* was held at MSRI on May 4–7, 2010. It coincided with the first anniversary of Professor Alan Weinstein's retirement from UC Berkeley and was organized jointly with the UC Berkeley Mathematics Department. Professor Weinstein has been one of the most influential figures in symplectic and Poisson geometry and analysis over the past forty years. His fundamental work has inspired many others and has greatly contributed to the development of central concepts in symplectic and Poisson geometry, as well as to the establishment of symplectic geometry as an independent discipline within

mathematics. This conference served to celebrate Professor Weinstein's contributions to symplectic and Poisson geometries and analysis, and to mathematics at large.

The second MSRI conference was one of the two Hayashibara Forum conferences on *Symplectic Geometry, Noncommutative Geometry and Physics*, which were held at MSRI on May 10–14, 2010, and at RIMS on November 1–5, 2010, as part of the RIMS thematic year 2010. These conferences aimed to highlight connections among the fields in their title and to further interactions between mathematicians and physicists working in related areas. The talks at these conferences were devoted to recent developments in symplectic geometry and noncommutativity in geometric settings, as well as possible physical applications.

The present volume consists of refereed papers by the invited speakers at these workshops, and are either presentations of new results which have not previously appeared or comprehensive reviews including recent developments in these areas.

The workshops were held with the support of MSRI and the Hayashibara Foundation. The editors and workshop organizers wish to thank the Hayashibara Foundation and MSRI for their generous financial support and for their encouragement and help with organization in the planning phase.

Tohru Eguchi
Yakov Eliashberg
Yoshiaki Maeda