

SYNCHRONIZATION

A modern introduction to synchronization phenomena, this text presents recent discoveries and the current state of research in the field, from low-dimensional systems to complex networks. The book describes some of the main mechanisms of collective behavior in dynamical systems, including simple coupled systems, chaotic systems, and systems of infinite dimension. After introducing the reader to the basic concepts of nonlinear dynamics, the book explores the main synchronized states of coupled systems and describes the influence of noise and the occurrence of synchronous motion in multistable and spatially extended systems. Finally, the authors discuss the underlying principles of collective dynamics on complex networks, providing an understanding of how networked systems are able to function as a whole in order to process information, perform coordinated tasks, and respond collectively to external perturbations. The demonstrations, numerous illustrations, and application examples will help advanced graduate students and researchers gain an organic and complete understanding of the subject.

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SYNCHRONIZATION

From Coupled Systems to Complex Networks

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Preface

Understanding, predicting, and controlling the way complex systems coordinate their dynamics in a cooperative manner have been, and still remain, among the fundamental challenges in modern scientific research. Their overwhelming difficulty stems from two major issues: extracting the proper dynamics of a solitary system, and capturing the complex way through which different systems (or units) interact to function together and in coordination with one another.

This book describes some of the most important mechanisms through which collective behavior of dynamical systems emerges, starting from the case of simple coupled systems, up to chaotic systems, infinite-dimensional systems, space-extended systems, and complex networks.

We focus on synchronization (from the Greek $\sigmaύν$ = together and $\chiρόνος$ = time), which literally means “happening at the same time.” Synchronization is actually a process where, due to their interactions or to an external driving force, dynamical systems adjust some properties of their trajectories so that they eventually operate in a macroscopically coherent way.

The word *synchronization* appeared first in 1620, at a time when determining longitudes was a challenge for transoceanic voyages. To find a solution to this problem, Christiaan Huygens invented the pendulum clock in 1657. For practical purposes, two clocks were required in general, in case one of the two stopped working properly. So, Huygens studied the behavior of two simultaneously operating maritime clocks, and noticed that they evolved in a synchronized manner and oscillated in the same plane when they were close to one another. Since then, synchronization has been investigated in numerous fields, such as mechanics, chemistry, neuroscience, biology, ecology, and social interactions, to quote just a few examples.

Synchronization is ubiquitous in natural phenomena: the organization of the world, outside and inside us, mostly depends on how different parts, units, and components are able to synchronize. The study of synchronization, as behavior of

coupled systems correlated in time, is therefore a fast-developing research topic with applications in almost all areas of science and engineering, ranging from chaotic communication to complex biological and social networks.

After introducing the reader to the basic concepts of nonlinear dynamics, the book explains the main synchronization states, such as complete synchronization, phase synchronization, lag and anticipated synchronization, generalized synchronization, and intermittent synchronization, which happen among coupled systems. Then, we move toward describing the influence of noise on synchronization, and the occurrence of synchronous motion in multistable systems and spatially extended systems, with a description of related effects, including amplitude and oscillation death.

Finally, the book discusses underlying principles of collective dynamics on complex networks, to provide an understanding of how networked systems are able to function as a whole in order to process information, perform coordinated tasks, give rise to parallel computing operations, and respond collectively to external perturbations or noise. We also review recent progress towards establishing a realistic and comprehensive approach capable of explaining and predicting some of the collective activity of real-world networks.

Furthermore, some of the most important applications of synchronization in different areas of science and engineering are given attention throughout this book: electronic circuits, lasers, chaotic communication, and neural networks.

The basic aim is to provide a first approach to synchronization, for readers who are interested in understanding its fundamental concepts and applications in several fields, from students and technicians to scientists and engineers conducting interdisciplinary research, both theoretical and experimental.