

---

## Index

---

- $C_0$ -semigroup of linear operators, 407
- $Q$ -Wiener process, 81
- $Q$ -Wiener process with respect to filtration, 95
- $\alpha$ -Hölder continuous process, 66
- $\pi$ -system, 17
- $\sigma$ -field, 15
- $m$ th-Wick power, 7
  
- absolute continuity, 282
- absolutely continuous measure, 51, 285, 295
- adapted process, 71
- additive noise, xvii, 121, 200, 308
- analytic case, 176
- analytic semigroup, 134, 413
- approximately controllable system, 428
- associated control system, 350, 364
- averaging, 404
  
- backward stochastic differential equations, 393
- beam equations, 389
- Birkhoff theorem, 303
- Bismut, Elworthy and Li formula, 266
- Bochner integrable, 19
- Bochner theorem, 61
- border case, 139
- Borel  $\sigma$ -field, 15
- Brownian bridge process, 8
- Brownian sheet, 88, 94, 102
- Burgers equations, 375
  
- cable equation, 9
- Cahn–Hilliard equations, 383
- Cameron–Martin formula, 59
- Cauchy problem, 406
- Cauchy problem, parabolic, 413
- Chapman–Kolmogorov equation, 249
- characteristic function, 35, 36
- closed operator, 408
- compact case, 332
- compact family of measures, 31
- complete probability space, 18
- conditional expectation, 26
  
- continuous dissipative mappings, 444
- continuous with probability 1, 66
- contraction  $C_0$ -semigroup, 409
- control theory, xv
- correlation operator, 25
- countable systems, 368
- covariance kernel, 91
- covariance of a measure, 47
- covariance operator, 10, 25
- cross quadratic variation, 75, 76
- cylindrical set, 35
- cylindrical Wiener, 128
- cylindrical Wiener process, 84, 336
  
- delay equations, 2, 369
- delay system, 126
- dependence on initial data, 235
- deterministic Cauchy problem, 122
- differentiability, 238, 244
- Dirichlet boundary conditions, 421
- Dirichlet realization, 423
- dissipative mapping, 442
- dissipative nonlinearities, 204, 207, 210, 328
- dissipativity condition, 327
- distribution of a random variable, 18
- dual semigroup  $P_t^*$ , 301
  
- elementary process, 95
- equation of population genetics, 9
- equation of stochastic quantization, 6
- equation of the free field, 6
- equations on Banach spaces, 200
- equations with fractional Brownian motion, 398
- equations with cylindrical noise, 196
- equations with Lévy noise, 398
- equations with Lipschitz nonlinearities, 186
- equations with multiplicative noise, 317
- equations with random boundary conditions, 372
- equivalent measures, 51
- ergodicity, 305
- estimates, 114
- Euler approximations, 207

Euler equations, 380  
 evolutionary Lyapunov equation, 318  
 existence and uniqueness results, xiv  
 existence of a continuous modification, 192  
 existence of a martingale solution, 221  
 existence of strong solutions, 156  
 exit place, 358  
 exit problem, xvii  
 exit rate, 353  
 exit set, 358  
 exponential estimates of Freidlin–Wentzell, 343  
 exponentially stable generator, 428  
 exponentially stable semigroup, 348

factorization formula, 129, 166, 193, 228  
 factorization method, xvi, 129, 135, 221  
 factorization procedure, 221  
 Feldman–Hajek theorem, 50, 55, 283  
 Feller property, 247  
 Fernique theorem, xv, 36, 37  
 Feynman–Kac formula, 247  
 filtering equations, 375  
 filtration, 71  
 first order equations, 369  
 first minimum energy problem, 364  
 Fokker–Planck equations, 401  
 fractional power and interpolation space, 414  
 fractional powers, 414

Garsia, Rademich and Rumsay lemma, 68  
 Gaussian measure, 36  
 Gaussian measures in Banach spaces, 36  
 Gaussian measures on Hilbert spaces, 46  
 Gaussian processes, 8, 77  
 generalization of the stochastic integral, 117  
 generalized solution, 211, 212  
 generalized Wiener processes, 100, 196  
 generated  $\sigma$ -field, 17  
 geometric SDES, 382  
 Girsanov theorem, xvi, 291  
 gradient flows, 402  
 growth of surface equation, 381

Hida's white noise, 395

interpolation spaces, 415  
 invariant measure, 301, 306, 308, 329, 332, 403  
 Itô formula, 106

Kardar, Parisi and Zhang equation, 376  
 Kolmogorov test, 67, 145  
 Kolmogorov's backward equation, 254  
 Korteweg–de Vries equation, 386  
 Krylov–Bogoliubov existence theorem, 304  
 Kuramoto–Sivashinsky equation, 382

large deviation principle, 340  
 large deviations, 404  
 law of random variable, 18  
 Lévy's theorem, 75, 292  
 Liapunov equation, 317, 429

lifts of diffusion, 1  
 Lipschitz nonlinearities, 200  
 local inversion theorem, 238  
 localization procedure, 99  
 lower exit rate, 353

Malliavin calculus, 400  
 manifold valued processes, 372  
 Markov processes, 77, 247  
 Markov property, xiv, 252  
 martingale property of the stochastic integral, 100  
 martingale solutions, xvi, 220, 296  
 martingales, 73  
 maximal inequalities for real valued  
   submartingales, 73  
 maximal regularity, 168, 418  
 mean of a measure, 47  
 mean square continuous process, 66  
 mean square stability, xvi, 317  
 measurable decomposition, 24  
 measurable mapping, 15  
 measurable process, 65  
 measurable selectors, 25  
 measurable spaces, 15  
 mild solution, 161, 176, 187  
 mild solution of Kolmogorov's equation, 271  
 mild solution of the Liapunov equation, 318  
 minimal energy, 353, 434  
 mixed conditions, 421  
 mixing, 307  
 modification of a process, 65  
 motion of a string, 4  
 multiplicative noise, xvii, 159, 215  
 Musiela's equation, 10

Navier–Stokes equations, 377  
 Nemytskii operator, 202, 331, 443  
 Neumann problem, 202  
 Neumann realization, 423  
 nonlinear equations on Banach spaces, 215  
 nonlinear heat equation, 199, 217, 232  
 nonlinear Schrödinger equations, 390  
 normal filtration, 71  
 nuclear operator, 26, 436  
 null controllable system, 260, 428  
 numerical methods, 399

operator valued random variables, 23  
 Ornstein–Uhlenbeck process, xiii

parabolic type equations, xiii  
 plate equation, 424, 426  
 porous media equations, 384  
 positive definite, 60, 87, 91  
 predictable  $\sigma$ -field, 72, 97  
 predictable process, 72  
 predictable sets, 72  
 predictable version, 161  
 probability measure, 18  
 probability measures in Hilbert spaces, 46  
 probability space, 18

- process, integrable, 73  
 product  $\sigma$ -field, 20  
 progressively measurable, 71  
 Prokhorov theorem, 31  
 pseudo-contraction  $C_0$ -semigroup, 409
- quadratic variation, 75  
 quasi-geostrophic equation, 381  
 quasi-potential, 364
- random boundary conditions, 372  
 random variables, 15  
 rate function, 9, 340, 341  
 rate of convergence, 340  
 rate of divergence, 353  
 reaction-diffusion equation, 8, 370  
 regular measure, 30  
 regular semigroup, 307  
 regularity of weak solutions, 143  
 relatively compact family of measures, 31  
 renormalization, 8  
 representation theorem, 222  
 reproducing kernel space, 39, 40, 344  
 resolvent operator, 408  
 resolvent set, 408  
 rough paths, 396
- Schrödinger equation, 390  
 second minimum energy problem, 364  
 self-adjoint case, 172  
 semigroup theory, xiv  
 semigroup, exponentially stable, 428  
 semigroup, strongly Feller at a moment  $r > 0$ , 315  
 semilinear equations, xv, 9, 350  
 short time asymptotic, 405  
 simple random variable, 15  
 simple stopping time, 77  
 singular measures, 51  
 skew-symmetric generator, 149  
 Skorohod embedding theorem, 32, 221  
 Skorohod equations, 371  
 Sobolev embedding theorem, 422  
 spatial correlation, 87  
 spatial regularity, 135  
 spatially homogeneous process, 90, 150, 370, 388  
 spectrum, 408  
 square integrable martingales, 74  
 state reachable, 428  
 stationary, 78  
 stationary measure, 301  
 stochastic calculus, xiv  
 stochastic characteristics, 181  
 stochastic climate models, 380  
 stochastic conservation laws, 386  
 stochastic convolution, 123, 148, 161, 166, 168  
 stochastic convolutions in  $L^p$  spaces, 173  
 stochastic delay equations, xiii, 2  
 stochastic evolution equations, xiii  
 stochastic flows, xiii, 2, 246, 247
- stochastic Fubini theorem, xv, 110, 193  
 stochastic integral, 95  
 stochastic magneto-hydrodynamics equations, 379  
 stochastic PDEs in Banach spaces, 392  
 stochastic processes, 65  
 stochastic quantization equation, 6, 7, 373  
 stochastic wave equation, 165  
 stochastically continuous process, 65  
 stochastically integrable process, 99, 103  
 stopping times, 77  
 strict solutions, 256  
 strict solutions of Kolmogorov's equation, 254, 267  
 strong Bochner integral, 24  
 strong Markov processes, 251  
 strong solution, 122, 156, 160, 181, 187, 218  
 strongly damped wave equation, 179, 425  
 strongly Feller, 307, 308, 314, 315  
 strongly measurable, 24  
 subdifferential, 201, 440  
 submartingale, 73  
 supermartingale, 73  
 symmetric and Gaussian measure, 124  
 symmetric Gaussian distribution, 36  
 system, approximately controllable, 432  
 system, mean square stable, 325  
 system, null controllable, 432  
 system, null controllable in time  $r$ , 429  
 system, stabilizable, 428
- temporal regularity, 135  
 tight family, 30, 226  
 trace of  $I$ , 436  
 trajectories, 65  
 transition function, 247  
 transition semigroup, xv, 259, 260  
 type of the semigroup  $S(\cdot)$ , 409
- ultraboundedness, 402  
 uniformly attracted set, 351  
 upper exit rate, 353
- variation of constants formula, 410  
 variational case, 171, 180, 322  
 variational generators, 413  
 variational operator, 413  
 version of a process, 65
- wave equation, 127, 165, 387, 424  
 weak solution, 122, 125, 129, 134, 161, 187, 221, 296, 409  
 white noise, 9, 42  
 Wiener chaos expansions, 395  
 Wiener processes, xiii, xvii, 90, 150, 370, 388
- Yosida approximations, 131, 408, 444
- Zakai equation, 3, 184