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978-1-107-05381-6 - Supersymmetric Field Theories: Geometric Structures and Dualities
Sergio Cecotti
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SUPERSYMMETRIC FIELD THEORIES

Adopting an elegant geometrical approach, this advanced pedagogical text describes deep and intuitive methods for understanding the subtle logic of supersymmetry, while avoiding lengthy computations.

The book describes how complex results and formulae obtained using other approaches can be significantly simplified when translated to a geometric setting. Introductory chapters describe geometric structures in field theory in the general case, while detailed later chapters address specific structures such as parallel tensor fields, G-structures, and isometry groups. The relationship between structures in supergravity and period maps of algebraic manifolds, Kodaira–Spencer theory, modularity, and the arithmetic properties of supergravity, are also addressed.

Relevant geometric concepts are introduced and described in detail, providing a self-contained toolkit of useful techniques, formulae and constructions. Covering all the material necessary for the application of supersymmetric field theories to fundamental physical questions, this is an outstanding resource for graduate students and researchers in theoretical physics.

SERGIO CECOTTI is Associate Professor at Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste, Italy.

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Preface

The focus of the present book is on the *geometric structures* underlying *all* supersymmetric field theories (classical and quantum). The language of geometric structures on smooth manifolds allows us to describe in a uniform and highly unified way all possible situations: rigid supersymmetry as well as local supergravity, in all space–time dimensions D, for all SUSY extensions $\mathcal{N}$, and all kinds of supersymmetries: superPoincaré, superconformal, and even rigid SUSY on general curved space–times.

This book evolved out of the lecture notes of a course in supergravity and supersymmetry taught at SISSA. The lectures were aimed at graduate students who already had a knowledge of supersymmetry and supergravity in the *standard* approaches (superfields, the Noether method, etc.), and the course was meant as an advanced (and perhaps deeper) topic. This explains why this book does not contain many materials that are fundamental tools for a physicist working in the field of supersymmetry but are more than adequately covered by existing books and reviews (see, e.g., the recent book *Supergravity* by D.Z. Freedman and A. van Proeyen (Cambridge University Press, 2012); our book instead focuses on the geometric aspects, with particular emphasis on the geometric structures that are *universal*, that is, that are present *mutatis mutandis* in all possible situations.

The geometric tools introduced in this book allow recovery of all the results obtained from the more classical approaches to SUSY, and typically more quickly and with less pain (however, for specific problems other viewpoints may be more efficient).

In our tale there are four main characters: (i) the Atiyah–Bott–Shapiro classification of Clifford modules; (ii) Berger's theorem on the Riemannian holonomy groups and the allied results on parallel tensor and spinor fields; (iii) Kostant theorem on the interplay of the holonomy and isometry groups, which describes the gauging of all SUSY field theories; (iv) Griffiths' theory of variations of Hodge structures, which gives a unifying view on the geometry of electromagnetic

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dualities. We pay particular attention to their arithmetic aspects, which are crucial for the quantum theory (and have never been discussed previously, to the best of our knowledge).

In particular, we give a new (simpler and more intrinsic) interpretation of rigid special Kähler geometry as a *flat* (G, H)–structure on the scalars' manifold. We also discuss, in the appropriate geometric setting, some recent major breakthroughs, such as the world–volume theory on a stack of M2–branes, including the Bagger–Lambert and ABJM models.

Some "phenomenological" topics, and some applications to other areas of theoretical physics, would deserve a more detailed discussion: perhaps it would be worthwhile to return to them in an enlarged (and corrected) second edition.

General references on supergravity include:

Van Nieuwenhuizen, P. (1981). Supergravity. *Phys. Rept.*, **68**, 189–398.
Salam, A., and Sezgin, E. (1989). *Supergravity in Diverse Dimensions*, vols. 1, 2. World Scientific.
Castellani, L., D'Auria, R., and Fré, P. (1991). *Supergravity and Superstrings: A Geometrical Perspective*, vols. 1, 2, 3. World Scientific.
Freedman, D.Z., van Proeyen, A. (2012). *Supergravity*. Cambridge University Press.
Fré, P. (2013). *Gravity, A Geometrical Course*, vol. 2. Springer.

Organization of the book

The book is divided into three parts. The purpose of Part I is to motivate the geometric structure approach by showing how differential geometric structures naturally appear in field theory in non–supersymmetric theories (Chapter 1) as well as in the supersymmetric ones (Chapter 2). Parts II and III are the body of the book, where the theory is developed in detail. In particular, Part II is the technical core of the book, where the general results are deduced and then illustrated in detail for the class of field theories having more than eight supersymmetries. In Parts II and III the geometry is discussed in full detail. Chapters in which geometry is presented in a rather rigorous way (stating explicitly Definitions, Lemmas, Theorems, etc.) are followed by physical chapters in which the geometry of the previous chapter is used to construct and understand supergravity and supersymmetric theories. As a rule, *starting from Chapter 3,* odd–*numbered chapters are purely geometric, while* even–*numbered ones contain physical applications and constructions.* Part III applies the general result to the theories having fewer than nine supersymmetries. The even chapters of this third part are somewhat sketchy, since they have a substantial overlap with existing literature to which the reader is referred. In the Appendix we present a quick review of the language of G–structures on smooth manifolds.

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Notations

The math symbols we use are defined in the text. Recurring symbols are:

- $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, and $\mathbb{O}$ denote respectively the natural, integer, rational, real, complex numbers, the Hamilton quaternions, and the Cayley octaves; $\mathbb{R}^\times$, $\mathbb{C}^\times$, ... the multiplicative group of non–zero elements in $\mathbb{R}$, $\mathbb{C}$, ...
- $V^\vee$ stands for the dual of the vector space V, $\otimes$ for the tensor product of vector spaces, $\odot$ for the *symmetric* tensor product of vector spaces, and $\wedge$ for the *antisymmetric* one. The same notation applies to vector bundles.
- The algebra of $n \times n$ matrices with entries in the algebra $\mathbb{F}$ is denoted $\mathbb{F}(n)$, the vector space of $n \times m$ matrices as $\mathbb{F}(n, m)$.
- $\mathbb{C}l(n)$ stands for the universal Clifford algebra in dimension n, $\mathbb{C}l^0(n)$ for its even subalgebra.
- If $G, H, K, L, \ldots, SU(n), SO(n), \ldots$ are Lie groups, the corresponding algebras are denoted as $\mathfrak{g}, \mathfrak{h}, \mathfrak{k}, \mathfrak{l}, \ldots, \mathfrak{su}(n), \mathfrak{so}(n), \ldots$.
- $Sp(2n)$ denotes the symplectic group with fundamental representation of dimension $2n$ corresponding to Cartan's Lie algebra C_n.
- Given a smooth manifold $\mathcal{M}$, its universal cover is denoted $\widetilde{\mathcal{M}}$, its tangent bundle $T\mathcal{M}$ and its cotangent bundle $T^*\mathcal{M}$.
- The kth Betti number of the manifold $\mathcal{M}$ is written $B_k(\mathcal{M})$, its Euler characteristic $\chi(\mathcal{M})$.
- The space of sections of the bundle/sheaf $\mathcal{E}$ over U is written $\Gamma(U, \mathcal{E})$ or simply $\mathcal{E}(U)$.
- The space of smooth k–forms over U is denoted $\Lambda^k(U)$.
- The sheaf of (germs of) holomorphic p–forms is denoted Ω^p.

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