1 Preliminary notions

Elementary particles are at the deepest level of the structure of matter. Students have already met the upper levels, namely the molecules, the atoms and the nuclei. These structures are small and their physics is properly described by non-relativistic quantum mechanics, by the Schrödinger equation. It is not relativistic because the speeds of the electrons in a molecule or in an atom and of the protons and neutrons in a nucleus are much smaller than the speed of light.

Protons and neutrons contain quarks, which have very small masses, corresponding to rest energies much smaller than their kinetic energy, and their speed is close to that of light. The structure of the nucleons, and more generally of the hadrons that we shall discuss, is described by relativistic quantum mechanics. The relevant equation, the Dirac equation, will be recalled.

The relativity theory is important in particle physics also for a different reason: the study of elementary particles requires experiments with beams accelerated at very high energies. There are two reasons for this: (a) the creation of new particles by, for example, annihilating a particle–antiparticle pair requires an initial energy large enough to be converted in the mass–energy of the new particle; (b) to study the internal structure of an object we must probe it with adequate resolving power, which increases with the energy of the probe, as we shall discuss.

In this chapter the student will learn the basic notions that will be necessary for her/his further study.

We shall start be recalling the fundamental elements of relativity, building on what students already know. The fundamental concepts of energy, momentum and mass, the relations amongst them and their transformations between reference systems, in particular the laboratory and centre of mass frame, will be clearly discussed. The students are urged to work on several numerical problems, which may be found at the end of the chapter, together with an introduction to the methods to solve them. This is the only way to master, in particular, relativistic kinematics.

Experiments on elementary particles study their collisions and decays. This chapter continues introducing the basic concepts appearing in their description. We shall then introduce the different types of particles (hadrons, quarks and leptons) and their fundamental interactions. Here and in the following we proceed, when appropriate, by successive approximations. Indeed, this is the way in which experimental science itself makes its progress.

The basic components of a collision experiment are a beam of high-energy particles, protons, antiprotons, electrons, neutrinos, etc., and a target on which they collide. The student will find in this chapter a basic description of the sources of such particles, which are the naturally occurring cosmic rays, used in the first years of the research, and the different types of accelerators. The products of a collision or of a decay, which are also elementary particles, are detected and their properties (energy, momentum, charge) measured with suitable 'detectors'. The progress of our knowledge is fully linked to the experimental 'art' of detector design and development. Detectors are made of matter, solid, liquid or gaseous. Consequently, a fair degree of knowledge of the interactions of charged and neutral high-energy particles with matter, with its atoms and molecules, is necessary to understand how detectors work and this is introduced in this chapter. This chapter introduces the principal types of detector and the principles of their operation. In later chapters the detectors' systems as implemented in important experiments will be described. We shall see here, in particular, how to measure the energy, momentum and mass of a particle, in the different energy ranges and situations in which they are met.

1.1 Mass, energy, linear momentum

Elementary particles have generally very high speeds, close to that of light. Therefore, we recall a few simple properties of relativistic kinematics and dynamics in this section and in the next three.

Let us consider two reference frames in rectilinear uniform relative motion $S(t,x,y,z)$ and $S'(t',x',y',z')$. We choose the axes as represented in Fig. 1.1. At a certain moment, which we take as $t'=t=0$, the origins and the axes coincide. The frame S' moves relative to S with speed $\mathbf{V}$, in the direction of the x-axis.

We introduce the following two dimensionless quantities relative to the motion in S of the origin of S'

$$\boldsymbol{\beta} \equiv \frac{\mathbf{V}}{c} \tag{1.1}$$

and

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \tag{1.2}$$

called the 'Lorentz factor'. An event is defined by the four-vector of the co-ordinates $(ct,\mathbf{r})$. Its components in the two frames (t,x,y,z) and (t',x',y',z') are linked by the Lorentz transformations (Lorentz 1904, Poincaré 1905) as

$$\begin{aligned} x' &= \gamma(x-\beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma(ct-\beta x). \end{aligned} \tag{1.3}$$

The Lorentz transformations, when joined to the rotations of the axes, form a group that E. Poincaré, who first recognised this property in 1905, called the proper Lorentz group. The group contains the parameter c, a constant with the dimensions of the velocity. A physical

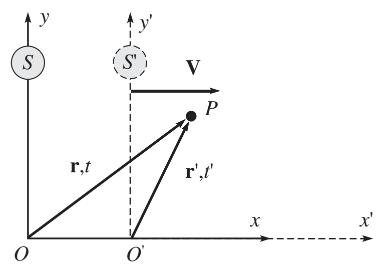

Fig. 1.1. Two reference frames in rectilinear relative motion.

entity moving at speed c in a reference frame moves with the same speed in any other. In other words, c is invariant under Lorentz transformations. It is the propagation speed of all the fundamental perturbations: light and gravitational waves (Poincaré 1905).

The same relationships are valid for any four-vector. Of special importance is the energy–momentum vector $(E/c, \mathbf{p})$ of a free particle

$$\begin{aligned} p_{x'} &= \gamma\left(p_x - \beta\frac{E}{c}\right) \\ p_{y'} &= p_y \\ p_{z'} &= p_z \\ \frac{E'}{c} &= \gamma\left(\frac{E}{c} - \beta p_x\right). \end{aligned} \tag{1.4}$$

Notice that the same 'Lorentz factor' γ appears both in the geometric transformations (1.3) and in those of dynamic quantities (1.4).

The transformations that give the components in S as functions of those in S', the inverse of (1.3) and (1.4), can be most simply obtained by changing the sign of the speed $\mathbf{V}$.

The norm of the energy–momentum is, as for all the four-vectors, an invariant; the square of the mass of the system multiplied times the invariant factor c^4

$$m^2c^4 = E^2 - p^2c^2. \tag{1.5}$$

This is a fundamental expression: it is the definition of the mass. It is, we repeat, valid only for a free body but is, on the other hand, completely general for point-like bodies, such as elementary particles, and for composite systems, such as nuclei or atoms, even in the presence of internal forces.

The most general relationship between the linear momentum (we shall call it simply momentum) $\mathbf{p}$, the energy E and the speed $\mathbf{v}$ is

$$\mathbf{p} = \frac{E}{c^2}\mathbf{v}, \tag{1.6}$$

which is valid both for bodies with zero and non-zero mass.

For massless particles, (1.5) can be written as

$$pc = E. \tag{1.7}$$

The photon mass is exactly zero. Neutrinos have non-zero but extremely small masses in comparison with the other particles. In the kinematic expressions involving neutrinos, their mass can usually be neglected.

If $m \neq 0$ the energy can be written as

$$E = m\gamma c^2, \tag{1.8}$$

and (1.6) takes the equivalent form

$$\mathbf{p} = m\gamma\mathbf{v}. \tag{1.9}$$

We call the reader's attention to the fact that one can find in the literature, and not only in that addressed to the general public, concepts that arose when the theory was not yet well understood and that are useless and misleading. One of these is the 'relativistic mass', which is the product $m\gamma$, and the dependence of mass on velocity. The mass is a Lorentz invariant, independent of the speed; the 'relativistic mass' is simply the energy divided by c^2 and as such the fourth component of a four-vector; this of course, is if $m \neq 0$, while for $m = 0$ relativistic mass has no meaning at all. Another related term to be avoided is the 'rest mass', namely the 'relativistic mass' at rest, which is simply the mass.

The concept of mass applies, to be precise, only to stationary states, i.e. to the eigenstates of the free Hamiltonian, just as only monochromatic waves have a well-defined frequency. Even the barely more complicated wave, the dichromatic wave, does not have a well-defined frequency. We shall see that there are two-state quantum systems, such as K^0 and B^0, which are naturally produced in states different from stationary states. For the former states it is not proper to speak of mass and of lifetime. As we shall see, the nucleons (as protons and neutrons are collectively called) are made up of quarks. The quarks are never free and consequently the definition of quark mass presents difficulties, which we shall discuss later.

Example 1.1 Consider a source emitting a photon with energy E_0 in the frame of the source. Take the x-axis along the direction of the photon. What is the energy E of the photon in a frame in which the source moves in the x direction at the speed $\upsilon = \beta c$? Compare this with the Doppler effect.

Call S' the frame of the source. Remembering that photon energy and momentum are proportional, we have $p'_x = p' = E_0/c$. The inverse of the last equation in (1.4) gives

$$\frac{E}{c} = \gamma\left(\frac{E_0}{c} + \beta p'_x\right) = \gamma\frac{E_0}{c}(1+\beta)$$

and we have

$$\frac{E}{E_0} = \gamma(1+\beta) = \sqrt{\frac{1+\beta}{1-\beta}}.$$

The Doppler effect theory tells us that, if a source emits a light wave of frequency ν_0, an observer who sees the source approaching at speed $\upsilon = \beta c$ measures the frequency ν, such that $\frac{\nu}{\nu_0} = \sqrt{\frac{1+\beta}{1-\beta}}$. This is no wonder; in fact quantum mechanics tells us that $E = h\nu$. □

1.2 The law of motion of a particle

The 'relativistic' law of motion of a particle was found by Planck in 1906 (See Planck 1906). As in Newtonian mechanics, a force $\mathbf{F}$ acting on a particle of mass $m \neq 0$ results in a variation in time of its momentum. Newton's law in the form $\mathbf{F} = d\mathbf{p}/dt$ (the form used by Newton himself) is also valid at high speed, provided the momentum is expressed by Eq. (1.9). The expression $\mathbf{F} = m\mathbf{a}$, used by Einstein in 1905, on the contrary, is wrong. It is convenient to write explicitly

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\gamma\mathbf{a} + m\frac{d\gamma}{dt}\mathbf{v}. \tag{1.10}$$

Taking the derivative, we obtain

$$m\frac{d\gamma}{dt}\mathbf{v} = m\frac{d\left(1-\frac{v^2}{c^2}\right)^{-1/2}}{dt}\mathbf{v} = -m\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-2\frac{v}{c^2}a_t\right)\mathbf{v} = m\gamma^3(\mathbf{a}\cdot\boldsymbol{\beta})\boldsymbol{\beta}.$$

Hence

$$\mathbf{F} = m\gamma\mathbf{a} + m\gamma^3(\mathbf{a}\cdot\boldsymbol{\beta})\boldsymbol{\beta}. \tag{1.11}$$

We see that the force is the sum of two terms, one parallel to the acceleration and one parallel to the velocity. Therefore, we cannot define any 'mass' as the ratio between acceleration and force. At high speeds, the mass is not the inertia to motion.

To solve for the acceleration we take the scalar product of the two members of Eq. (1.11) with $\boldsymbol{\beta}$. We obtain

$$\mathbf{F}\cdot\boldsymbol{\beta} = m\gamma\mathbf{a}\cdot\boldsymbol{\beta} + m\gamma^3\beta^2\mathbf{a}\cdot\boldsymbol{\beta} = m\gamma\left(1+\gamma^2\beta^2\right)\,\mathbf{a}\cdot\boldsymbol{\beta} = m\gamma^3\mathbf{a}\cdot\boldsymbol{\beta}.$$

Hence

$$\mathbf{a}\cdot\boldsymbol{\beta} = \frac{\mathbf{F}\cdot\boldsymbol{\beta}}{m\gamma^3}$$

and, by substitution into (1.11),

$$\mathbf{F} - (\mathbf{F}\cdot\boldsymbol{\beta})\boldsymbol{\beta} = m\gamma\mathbf{a}.$$

The acceleration is the sum of two terms, one parallel to the force, and one parallel to the speed.

Force and acceleration have the same direction in two cases only: (1) force and velocity are parallel: $\mathbf{F} = m\gamma^3\mathbf{a}$; (2) force and velocity are perpendicular: $\mathbf{F} = m\gamma\mathbf{a}$. Notice that the proportionality constants are different.

In order to have simpler expressions in subnuclear physics the so-called 'natural units' are used. We shall discuss them in Section 1.5, but we anticipate here one definition: without changing the unit of time of the S.I., we define the unit of length in such a way that $c = 1$. In other words, the unit length is the distance the light travels in a second in vacuum, namely 299 792 458 m, a very long distance. With this choice, in particular, mass, energy and momentum have the same physical dimensions. We shall often use as their unit the electronvolt (eV) and its multiples.

1.3 The mass of a system of particles, kinematic invariants

The mass of a system of particles is often called 'invariant mass', but the adjective is useless; the mass is always invariant.

The expression is simple only if the particles of the system do not interact amongst themselves. In this case, for n particles of energies E_i and momenta $\mathbf{p}_i$, the mass is

$$m = \sqrt{E^2 - P^2} = \sqrt{\left(\sum_{i=1}^{n} E_i\right)^2 - \left(\sum_{i=1}^{n} \mathbf{p}_i\right)^2}. \tag{1.12}$$

Consider the square of the mass, which we shall indicate by s, obviously an invariant quantity

$$s = E^2 - P^2 = \left(\sum_{i=1}^{n} E_i\right)^2 - \left(\sum_{i=1}^{n} \mathbf{p}_i\right)^2. \tag{1.13}$$

Notice that s cannot be negative

$$s \geq 0. \tag{1.14}$$

Let us see its expression in the 'centre of mass' (CM) frame, which is defined as the reference in which the total momentum is zero. We see immediately that

$$s = \left(\sum_{i=1}^{n} E_i^*\right)^2, \tag{1.15}$$

where E_i^* are the energies in the CM. In words, the mass of a system of non-interacting particles is also its energy in the CM frame.

Consider now a system made up of two non-interacting particles. It is the simplest system, and also a very important one. Figure 1.2 defines the kinematic variables.

The expression of s is

$$s = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = m_1^2 + m_2^2 + 2E_1E_2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2 \tag{1.16}$$

and, in terms of the velocity, $\boldsymbol{\beta} = \mathbf{p}/E$

$$s = m_1^2 + m_2^2 + 2E_1E_2(1 - \boldsymbol{\beta}_1 \cdot \boldsymbol{\beta}_2). \tag{1.17}$$

Clearly in this case, and as is also true in general, the mass of a system is not the sum of the masses of its constituents, even if these do not interact. It is also clear from Eq. (1.12) that energy and momentum conservation implies that the mass is a conserved quantity: in a

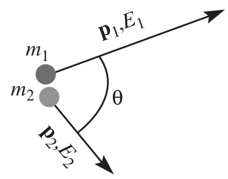

Fig. 1.2. System of two non-interacting particles.

reaction such as a collision or decay, the mass of the initial system is always equal to that of the final system. For the same reason, the sum of the masses of the bodies present in the initial state is generally different from the sum of the masses of the final bodies.

Example 1.2 We find the expressions for the mass of the system of two photons of the same energy E, if they move in equal or in different directions.

The energy and the momentum of the photon are equal, because its mass is zero, $p=E$. The total energy $E_{tot}=2E$.

If the photons have the same direction then the total momentum is $p_{tot}=2E$ and therefore the mass is $m=0$.

If the velocities of the photons are opposite, $E_{tot}=2E$, $p_{tot}=0$, and hence $m=2E$.

In general, if θ is the angle between the velocities, $p_{tot}^2=2p^2+2p^2\cos\theta=2E^2(1+\cos\theta)$ and hence $m^2=2E^2(1-\cos\theta)$. □

Notice that the system does not contain any matter, but only energy. Contrary to intuition, mass is not a measure of the quantity of matter in a body.

Now consider one of the basic processes of subnuclear physics: collisions. In the initial state two particles, a and b, are present; in the final state we may have two particles (not necessarily a and b) or more. Call these c, d, e, ... The process is

$$a+b\to c+d+e+\cdots. \tag{1.18}$$

If the final state contains only the initial particles, then the collision is said to be elastic:

$$a+b\to a+b. \tag{1.19}$$

We specify that the excited state of a particle must be considered as a different particle.

The time spent by the particles in interaction, the collision time, is extremely short and we shall think of it as instantaneous. Therefore, the particles in both the initial and final states can be considered as free.

We shall consider two reference frames, the CM frame already defined above and the laboratory frame (L). The latter is the frame in which, before the collision, one of the particles (called the target) is at rest, while the other (called the beam) moves against it. Let a be the beam particle, m_a its mass, $\mathbf{p}_a$ its momentum and E_a its energy; let b be the target mass and m_b its mass. Figure 1.3 shows the system in the initial state.

In L, s is given by

$$s=(E_a+m_b)^2-p_a^2=m_a^2+m_b^2+2m_bE_a. \tag{1.20}$$

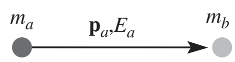

Fig. 1.3. The laboratory frame (L).

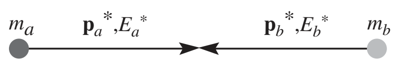

Fig. 1.4. The centre of mass reference frame (CM).

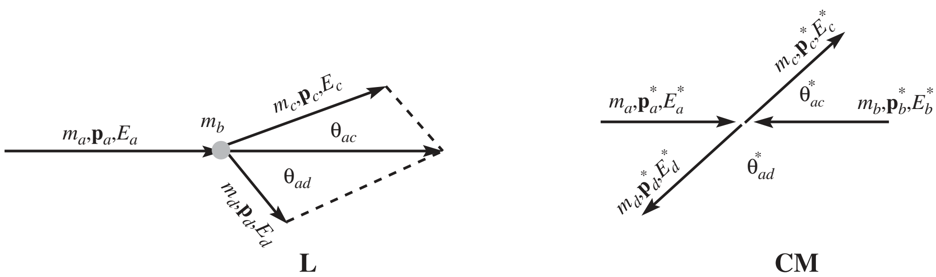

Fig. 1.5. Two-body scattering in the L and CM frames.

In practice, the energy of the projectile is often, but not always, much larger than both the projectile and the target masses. If this is the case, we can approximate Eq. (1.20) by

$$s \simeq 2m_b E_a \qquad (E_a \gg m_a, m_b). \tag{1.21}$$

We are often interested in producing new types of particles in the collision, and therefore we are also interested in the energy available for such a process. This is obviously the total energy in the CM, which, as seen in (1.21), grows proportionally to the square root of the beam energy.

Let us now consider the CM frame, in which the two momenta are equal and opposite, as in Fig. 1.4. If the energies are much larger than the masses, $E_a^* \gg m_a$ and $E_b^* \gg m_b$, the energies are approximately equal to the momenta: $E_a^* \approx p_a^*$ and $E_b^* \approx p_b^*$. Hence, they are equal to each other, and we call them simply E^*. The total energy squared is

$$s = \left(E_a^* + E_b^*\right) \approx (2E^*)^2, \tag{1.22}$$

where the approximation at the last member is valid for $E^* \gg m_a, m_b$. We see that the total centre of mass energy is proportional to the energy of the colliding particles. In the CM frame, all the energy is available for the production of new particles; in the L frame only part of it is available, because momentum must be conserved.

Now let us consider a collision with two particles in the final state: this is two-body scattering

$$a + b \to c + d. \tag{1.23}$$

Figure 1.5 shows the initial and final kinematics in the L and CM frames. Notice in particular that, in the CM frame, the final momentum is in general different from the initial momentum; they are equal in absolute value only if the scattering is elastic.

Because s is an invariant it is equal in the two frames; because it is conserved it is equal in the initial and final states. We have generically in any reference frame

$$s = (E_a + E_b)^2 - (\mathbf{p}_a + \mathbf{p}_b)^2 = (E_c + E_d)^2 - (\mathbf{p}_c + \mathbf{p}_d)^2. \tag{1.24}$$

These properties are useful to solve a number of kinematic problems, as we shall see in the 'Problems' section later in this chapter.

In two-body scattering, there are two other important kinematic variables that have the dimensions of the square of an energy: the a–c four-momentum transfer t, and the a–d four-momentum transfer u. The first is defined as

$$t \equiv (E_c - E_a)^2 - (\mathbf{p}_c - \mathbf{p}_a)^2. \tag{1.25}$$

It is easy to see that the energy and momentum conservation implies

$$t = (E_c - E_a)^2 - (\mathbf{p}_c - \mathbf{p}_a)^2 = (E_d - E_b)^2 - (\mathbf{p}_d - \mathbf{p}_b)^2. \tag{1.26}$$

In a similar way

$$u \equiv (E_d - E_a)^2 - (\mathbf{p}_d - \mathbf{p}_a)^2 = (E_c - E_b)^2 - (\mathbf{p}_c - \mathbf{p}_b)^2. \tag{1.27}$$

The three variables are not independent. It easy to show (see Problems) that

$$s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2. \tag{1.28}$$

Notice, finally, that

$$t \le 0, \quad u \le 0. \tag{1.29}$$

1.4 Systems of interacting particles

Let us now consider a system of interacting particles. We immediately stress that its total energy is not in general the sum of the energies of the single particles, $E \neq \sum_{i=1}^{n} E_i$, because the field responsible for the interaction itself contains energy. Similarly, the total momentum is not the sum of the momenta of the particles, $\mathbf{P} \neq \sum_{i=1}^{n} \mathbf{p}_i$, because the field contains momentum. In conclusion, Eq. (1.12) does not in general give the mass of the system. We shall restrict ourselves to a few important examples in which the calculation is simple.

Let us first consider a particle moving in an external, given field. This means that we can consider the field independent of the motion of the particle.

Let us start with an atomic electron of charge q_e at a distance r from a nucleus of charge Zq_e. The nucleus has a mass $M_N \gg m_e$, hence it is not disturbed by the electron motion. The electron then moves in a constant potential $\phi = -\frac{1}{4\pi\varepsilon_0}\frac{Zq_e}{r}$. The electron energy (in S.I. units) is

$$E = \sqrt{m_e^2c^4 + p^2c^2} - \frac{1}{4\pi\varepsilon_0}\frac{Zq_e^2}{r} \approx m_ec^2 + \frac{p^2}{2m_e} - \frac{1}{4\pi\varepsilon_0}\frac{Zq_e^2}{r},$$

where, in the last expression, we have taken into account that the atomic electron speeds are much smaller than c. The final expression is valid in non-relativistic situations, as the case in an atom and it is the Newtonian expression of the energy, apart from the irrelevant constant m_ec^2.

Let us now consider a system composed of an electron and a positron. The positron, as we shall see, is the antiparticle of the electron. It has the same mass and opposite charge. The difference to the hydrogen atom is that there is no longer a fixed centre of force. We must consider not only the two particles but also the electromagnetic field in which they move, which, in turn, depends on their motion. If the energies are high enough, quantum processes happen at an appreciable frequency: the electron and the positron can annihilate each other by producing photons; inversely, a photon of the field can 'materialise' in a positron–electron pair. In these circumstances, we can no longer speak of a potential.

In conclusion, the concept of potential is non-relativistic: we can use it if the speeds are small in comparison to c or, in other words, if energies are much smaller than the masses. It is correct for the electrons in the atoms, to first approximation, not for the quarks in the nucleons.

Example 1.3 Consider the fundamental level of the hydrogen atom. The energy needed to separate the electron from the proton is $\Delta E = 13.6$ eV. The mass of the atom is smaller than the sum of the masses of its constituents by this quantity: $m_H + \Delta E = m_p + m_e$. The relative mass difference is

$$-\frac{m_H - m_p - m_e}{m_H} = \frac{13.6}{9.388 \times 10^8} = 1.4 \times 10^{-8}.$$

This quantity is extremely small, justifying the non-relativistic approximation. □

Example 1.4 The processes we have mentioned above of electron–positron annihilation and pair production can take place only in the presence of another body. If not, energy and momentum cannot be conserved simultaneously. Let us now consider the following processes.

- $\gamma \to e^+ + e^-$. Let E_+ be the energy and let $\mathbf{p}_+$ be the momentum of e^+, and E_- and $\mathbf{p}_-$ those of e^-. In the initial state $s = 0$; in the final state $s = (E_+ + E_-)^2 – (\mathbf{p}_+ + \mathbf{p}_-)^2 = 2m_e{}^2 + 2(E_+E_- - p_+p_- \cos\theta) > 2m_e{}^2 > 0$. This reaction cannot occur.
- $e^+ + e^- \to \gamma$. This is just the inverse reaction, and it cannot occur either.