

## ULTRASONIC GUIDED WAVES IN SOLID MEDIA

Ultrasonic guided waves are revolutionizing the approach to nondestructive testing (NDT) and structural health monitoring (SHM). Large area inspection from a single probe position is possible, even for hidden and coated structures. Both theoretical and practical aspects of the method are presented in this book, which students and researchers can use as a textbook or reference source.

This book is intended to bring people up to speed with the latest developments in the field, especially new work in ultrasonic guided waves. It is designed for students and for researchers and managers somewhat familiar with the field in order to serve as a baseline for further work already under way. This text also includes extended problems and a corresponding solutions manual as a resource for the reader. A wave propagation animations collection will be available on the book web site. Solutions are available for instructors on the Cambridge web site. Join Dr. Joseph L. Rose on an exciting journey to explore breakthroughs in the understanding and application of ultrasonic guided waves.

Dr. Joseph L. Rose is the Paul Morrow Professor in the Engineering Science and Mechanics Department of The Pennsylvania State University. He is also chief scientist and president of FBS, Inc., a company dedicated to technology transfer, product development, and consulting on ultrasonic guided waves in NDT and SHM. Dr. Rose received his PhD from Drexel University in 1970. He is the author of 20 patents, 4 textbooks, and more than 600 articles on ultrasonics; has served as principal adviser to more than 60 PhD and 100 MS students; and is a Fellow of ASNT, ASME, IEEE, and the British Society for Nondestructive Testing. In addition, Dr. Rose has received many awards including the SPIE Lifetime Achievement Award in recognition of sustained contributions to the advancement of NDT and SHM in 2011, the Pennsylvania State University Graduate Teaching Award in 2012, and the distinction of being a finalist in the *Discover* Magazine Award for innovation in aviation and aerospace in 1995.

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# Ultrasonic Guided Waves in Solid Media

**Joseph L. Rose**  
The Pennsylvania State University



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Nomenclature

Latin script

$I, 2$	Indicates material 1 or 2
$\mathbf{A}, \mathbf{B}$	Matrices in a first-order eigensystem (Ch 9)
$A, A_1, A_2, \dots$	Amplitude constants
$A_+^{mn}$	Mode weighting functions for finite size loading of a hollow cylinder
$A_0, A_1, A_2, \dots$	Antisymmetric plate modes
$A(\theta_i)$	Discrete weighting function for element $i$ (Ch 16)
$a$	Acceleration (Ch 2); Coefficient vector for incident wave amplitude (Ch 5); Dimensionless length variable (Ch 17)
$B$	Amplitude constant; Ratio of acoustic impedances, $B = \frac{W_2}{W_1}$ (Ch 4)
$[\mathbf{C}]$	Global damping matrix (Ch 8)
$C$	Amplitude constant; Elastic constant (material stiffness) matrix; Viscous damping coefficient (Ch 2)
$C_{iklm}, C_{nm}$	Single entry in the elastic constant matrix (Ch 3)
$c, c_0$	Velocity of wave propagation
$c_E$	Velocity of energy transport
$c_f$	Fluid bulk wave velocity
$c_g$	Group velocity
$c_L$	Bulk longitudinal wave velocity
$c_p$	Phase velocity
$c_{plate}$	Plate mode wave velocity
$c_R$	Rayleigh wave velocity
$c_T, c_S$	Bulk transverse (shear) wave velocity
$\mathbf{D}(p, \omega)$	Coefficient matrix (Ch 11)
$D$	Amplitude constant; Cross-sectional area of a hollow cylinder (Ch 10)
$D_B^{(m)}$	Coefficient matrix relating to the inner boundary of layer $m$ (Ch 11)
$D_T^{(m)}$	Coefficient matrix relating to the outer boundary of layer $m$ (Ch 11)

$\{\mathbf{d}\}^{(e)}$	Nodal displacement vector (Ch 8)
$\{\dot{\mathbf{d}}\}$	Velocity vector (Ch 8)
$\{\ddot{\mathbf{d}}\}$	Acceleration vector (Ch 8)
$d$	Plate or layer thickness; Piston diameter (App A)
$dP_i$	Change in phase of wave component $i$ (Ch 2)
$ds$	Arc length (Ch 2)
$E$	Green-Lagrange strain (Ch 20)
$E$	Young's modulus; Mode excitability function (Ch 19)
$\bar{E}$	Energy density (Ch 2)
$e$	Mathematical constant, $e \approx 2.71828$
$\mathbf{F}$	External force (load) vector (Ch 8, 9); Deformation gradient (Ch 20)
$F$	Tension force (Ch 2); Excitation spectrum of transducer (Ch 19)
$f$	Frequency; Body force; Unknown coefficients that are part of the potentials equation (Ch 10)
$f^{(1,1)}$	Nonlinear forcing function associated with nonlinear terms from the primary wave field (Ch 20)
$f^{2D\text{ comb}}$	Two-dimensional comb transducer loading geometry (Ch 19)
$f^{\text{ann}}$	Annular array transducer loading geometry (Ch 19)
$f^{\text{comb}}$	Comb transducer loading geometry (Ch 19)
$f_n^{\text{surf}}$	Nonlinear surface force (Ch 20)
$f_n^{\text{vol}}$	Nonlinear volume force (Ch 20)
$fd$	Frequency-thickness product
$G$	Amplitude factor relative to transducer loading amplitude (Ch 19)
$\mathbf{G}(\omega)$	Fourier transform of $G(\theta)$ (Ch 16)
$G(\theta)$	Total angular profile of the phased array on a hollow cylinder (Ch 16)
$H$	Displacement gradient (Ch 20)
$H$	Layer thickness; Hankel function (Ch 19)
$\mathbf{H}(\omega)$	Fourier transform of $H(\theta)$ (Ch 16)
$\bar{H}$	Equivoluminal vector potential (Ch 10)
$H(\theta)$	Angular profile at a certain distance in the cylinder for element 0 in the phased array (Ch 16)
$h$	Half plate thickness; Layer thickness (Ch 12, 15); Unknown coefficients that are part of the potentials equation (Ch 10)
$\mathbf{I}$	Unit matrix (Ch 9)
$I$	Incident waveform amplitude (Ch 4); Wave intensity (Ch 4)
$I, i, J, j$	Index values
$i$	Imaginary number, $i = \sqrt{-1}$ ; Mode number
$J$	Bessel function of the first kind
$[\mathbf{K}]$	Global stiffness matrix (Ch 8)
$\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_3$	Stiffness matrices (Ch 9)
$K$	Elastic spring constant (Ch 2)
$k$	Wave number; Circular wave number (Ch 11); Index for transmitter-receiver pair (Ch 21)
$\bar{k}$	Complex wave number (Ch 2); Wave number vector (Ch 3)

Nomenclature

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$k, l, m, n$	Index values
$k_{\text{Im}}$	Imaginary component of wave number
$k_r, k_{\text{Re}}$	Real component of wave number
$\mathbf{L}_x, \mathbf{L}_y, \mathbf{L}_z$	Matrices in the strain-displacement equation (Ch 9)
$L$	Half length of loading in the axial direction on a hollow cylinder (Ch 10, 16); Length of plate (Ch 18); Element transverse length (Ch 19)
$L(m,n)$	Longitudinal mode of order (m,n) in a hollow cylinder (Ch 10)
$l_z$	Ratio of $z$ -direction and $x$ -direction wavenumbers, $l_z = \frac{k_z}{k_x}$ (Ch 6)
$[\mathbf{M}], \mathbf{M}$	Global mass matrix (Ch 8, 9)
$M$	Coefficient matrix for wave amplitudes (Ch 5); Mode number (Ch 6)
$M, m$	Mass (Ch 2); Circumferential order of a wave mode (Ch 10, 16)
$\mathbf{N}$	Normal vector of the loading surface; Shape function matrix (Ch 9)
$N$	Number array elements; Shape functions (Ch 8, 9); Number of nodes through plate thickness in SAFE analysis (Ch 18); Number of data points (Ch 21)
$\mathbf{n}$	Outward normal to a surface (Ch 20)
$n$	Mode number; Unit normal (Ch 6); Mode group index (Ch 16); Element number in an array (Ch 21)
$n_k, n_l$	Direction cosines of the normal to the wave front (e.g., $k_k = kn_k$ )
$\{\mathbf{P}\}$	Body force in volume $V$ (Ch 8)
$\mathbf{P}, P$	Acoustic Poynting vector, also called power flow or flux
$p$	Variable comparing the phase and bulk longitudinal wavenumbers, $p = \sqrt{\frac{\omega^2}{c_L^2} - k^2}$ (Ch 6); Angular wave number, $p = kR$ (Ch 11); Loading distribution (Ch 19); Pressure field amplitude (App A)
$p_1(\theta)$	Circumferential loading distribution function (Ch 10, 16)
$p_2(z)$	Axial loading distribution function (Ch 10, 16)
$\mathbf{Q}$	Nodal displacement vector (Ch 9)
$q$	Variable comparing the phase and bulk torsional (shear) wavenumbers, $q = \sqrt{\frac{\omega^2}{c_T^2} - k^2}$ (Ch 6); Body force or external loading per unit length (Ch 2)
$R$	Oblique incidence reflection factor (Ch 6); Array radius (Ch 21)
$R_{n\alpha}^M(r)$	Distribution of the particle displacement produced by mode $(M, n)$ in the $\alpha$ direction (Ch 16)
$r$	Radial coordinate direction (cylindrical coordinate system); Variable comparing the phase and bulk longitudinal wavenumbers, $r = \sqrt{k^2 - k_L^2}$ (Ch 6)
$r_m$	Inner radius of the $m$ th layer of a hollow cylinder (Ch 11)

$S$	Symmetric Lamb mode term in oblique incidence reflection factor equation; Surface over which calculation is to take place (Ch 8)
$S0, S1, S2, \dots$	Symmetric plate modes
$s$	Variable comparing the bulk longitudinal and torsional (shear) wavenumbers, $s = \sqrt{k_L^2 - k_T^2}$ ; Array element spacing (pitch) (Ch 18, 19); Data set (Ch 21)
$T$	Superscript indicating transpose (Ch 8)
$\mathbf{T}$	Unitary transformation matrix (Ch 9); Particle stress tensor (Ch 16)
$\mathbf{T}^{(e)}$	Nodal external tractions (Ch 9)
$\tilde{T}$	Second Piola-Kirchhoff stress (Ch 20)
$T$	Stress transmission coefficient (Ch 4)
$T_o$	First Piola-Kirchhoff stress (Ch 20)
$\bar{T}$	Stress field (Ch 10)
$T(m,n)$	Torsional mode of order (m,n) in a hollow cylinder (Ch 10)
$T_m$	Modal stress (Ch 20)
$t$	Time
$t, \bar{t}$	Surface traction (Ch 6, 19)
$t_i$	Physical time delay applied to element $i$ (Ch 16)
$\bar{U}, \bar{u}, \mathbf{u}$	Displacement vector
$U_{\alpha\beta}$ ( $\alpha = x, y, z$ ; $\beta = 1, 2, 3$ )	Nodal displacement of the node $\beta$ in the $\alpha$ direction (Ch 9)
$u$	Displacement
$\dot{u}$	Velocity, the first derivative of displacement with respect to time
$\ddot{u}$	Acceleration, the second derivative of displacement with respect to time
$u'$	First derivative of displacement with respect to coordinate direction $x$ (Ch 2)
$u_{,xx}$	Second derivative of displacement with respect to coordinate direction $x$ (Ch 2)
$u_R(\theta_i, t)$	Signal received from the $i$ th transducer segment (Ch 16)
$u_s(\theta, z)$	Synthesized pipe image (Ch 16)
$\mathbf{V}$	Particle velocity vector (Ch 5)
$V$	Volume of an element (Ch 9)
$\vec{v}$	Particle velocity field (Ch 10)
$\bar{\bar{v}}_v$	Normalized surface velocity of guided wave mode $v$ (Ch 19)
$v$	Displacement along coordinate direction 2
$W$	Acoustic impedance, $W = \rho c$ (Ch 4)
$W_m$	Bessel function of $m$ th order (Ch 10)
$w$	Displacement along coordinate direction 3; Array element width (Ch 18, 19)
$x$	Cartesian coordinate direction 1; Distance; An unknown
$Y$	Fluid influence term in oblique incidence reflection factor equation (Ch 6); Bessel function
$y$	Cartesian coordinate direction 2



Nomenclature

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$Z_m$	Bessel function of $m$ th order (Ch 10)
$z$	Cartesian coordinate direction 3; Axial coordinate direction (cylindrical coordinate system)

Greek script

$\alpha$	Wedge angle (Ch 1); Imaginary component of wavenumber (Ch 2, 6); Reflected wave angle (Ch 5); Half length of loading in the angular direction on a hollow cylinder (Ch 10); Variable comparing the phase and bulk longitudinal wavenumbers, $\alpha = \sqrt{\frac{\omega^2}{c_L^2} - k^2}$ (Ch 10); Ratio of wavenumber in the $x_3$ direction to the wavenumber in $x_1$ direction (Ch 12, 15); Attenuation factor (Ch 17); Complex amplitude coefficient (Ch 18)
$\alpha_g$	Angular group velocity (Ch 11)
$\alpha_g(\phi)$	Angular dependence of the guided wave amplitude (Ch 21)
$\alpha_i$	Component of the eigenvector from the solution to the Christoffel equation
$\alpha_n$	Complex wavenumbers which are poles to the integrands in Equation (17.24b) (Ch 17)
$\alpha_p$	Angular phase velocity (Ch 11)
$\beta$	Refracted wave angle (Ch 5); Stiffness matrix coefficient for Rayleigh damping (Ch 8); Variable comparing the phase and bulk torsional (shear) wavenumbers, $\beta = \sqrt{\frac{\omega^2}{c_T^2} - k^2}$ (Ch 10); Transformation matrix (Ch 15); Nonlinearity parameter (Ch 20); Scaling parameter for RAPID (Ch 21)
$\beta_n$	Complex wavenumbers which are poles to the integrands in Equation (17.24b) (Ch 17)
$\Gamma$	Surface of an element (Ch 9)
$\Gamma_{im}$	Christoffel acoustic tensor
$\gamma$	Reflection coefficient (Ch 21)
$\Delta$	Length of a 1-D element (Ch 8); Area of a 2-D element (Ch 8); Dilatation, $\Delta = \nabla \cdot u$ (Ch 11)
$\delta$	Signal magnification coefficient (Ch 21)
$\delta_{im}$	Kronecker delta, which is 0 for $i \neq m$ and 1 for $i = m$
$\delta \mathbf{u}$	Virtual displacement (Ch 9)
$\delta \epsilon$	Virtual strain (Ch 9)
$\epsilon, \{\epsilon\}$	Strain; Strain vector (Ch 8)
$\zeta$	Dimensionless length variable (Ch 17)
$\eta$	Coefficient of viscosity (Ch 17); Surface domain for applied traction (Ch 19)
$\Theta_\xi^M(M\theta)$	Angular distribution function ( $\cos(n\theta)$ or $\sin(n\theta)$ ) of the particle displacement produced by mode $(n, M)$ in the $\alpha$ direction (Ch 16)

$\theta$	Wave angle; Angular coordinate direction (cylindrical coordinate system) (Ch 10)
$\theta_{cr}$	Critical angle
$\Lambda$	Wavelength (Ch 12); Coefficient matrix (Ch 18)
$\lambda$	Lamé constant; Wavelength; Eigenvalue (Ch 18)
$\lambda_{im}$	Christoffel acoustic tensor
$\lambda_R$	Rayleigh surface wave wavelength
$\mu$	Shear modulus; Guided wave mode number (Ch 18)
$\nu$	Poisson’s ratio
$\xi, \xi$	Finite element shape function variable (Ch 9); Dimensionless length variable(Ch 17)
$\Pi_a$	Rectangle function (Ch 19)
$\rho$	Density; Mass density per length (Ch 2); Transducer traction density (Ch 19); Correlation coefficient (Ch 21)
$\rho_f$	Fluid density (Ch 6)
$\{\sigma\}, \bar{\sigma}$	Stress vector (Ch 8); Transverse stress field (Ch 18)
$\sigma$	Stress; Cauchy stress (Ch 20); Standard deviation (Ch 21)
$\tau$	Surface traction (Ch 14); Time delay (Ch 19)
$\{\Phi\}$	Surface traction (Ch 8)
$\Phi$	Dilatational scalar potential for Helmholtz decomposition, associated with longitudinal waves; Skew angle (Ch 15)
$\Phi_g(\phi)$	Angular dependence of phase variations (Ch 21)
$\phi$	Skew angle (Ch 3); Phase angle for sinusoidal wave (Ch 2)
$\phi_0$	Beam steering angle (Ch 21)
$\phi_i$	Phase of function $A(\theta_i)$ (Ch 16)
$\phi_n$	Phase delay applied to element $n$ (Ch 19)
$\psi$	Eigenvectors from the SAFEM eigenvalue problem (Ch 9)
$\psi_n$	Angular locations of array elements (Ch 21)
$\psi, \bar{\psi}$	Vector potential for Helmholtz decomposition, associated with torsional (shear) waves
$\omega$	Rotation vector, $\omega = \frac{1}{2}\nabla\bullet u$ (Ch 11)
$\omega$	Circular (angular) frequency

**Other symbols and notations**

*	Complex conjugate
$\langle \rangle$	Time average of variable inside bracket
$\otimes$	Convolution operator (Ch 16)
$\otimes^{-1}$	Deconvolution operator (Ch 16)

## Preface

This book builds on my 1999 book, *Ultrasonic Waves in Solid Media*. Like its predecessor, this book is intended to bring people up to speed with the latest developments in the field, especially new work in ultrasonic guided waves. It is designed for students and for researchers and managers familiar with the field in order to serve as a baseline for further work already under way. I hope to journey with you to provide more breakthroughs in the understanding and application of ultrasonic guided waves. The goal is to improve the health of individuals, industries, and national infrastructures through improved methods of Non-destructive Evaluation (NDE). The purpose of this book is to expand on many of the topics that were introduced in my first book. Several chapters are almost the same, but there are many new fundamental topic chapters with a total emphasis in this book being directed toward the basic principles of ultrasonic guided waves. The field of ultrasonic guided waves itself is treated as a new and separate field compared to ultrasonics and other inspection disciplines as indicated in some of the efforts put forward in inspection certification by the American Society for Non-destructive Testing (ASNT) and also in code requirements in such groups as the American Society for Mechanical Engineers (ASME) and the Department of Transportation (DOT).

The book begins with an overview and background materials in Chapters 1 through 7 and then continues on to more advanced topics in Chapters 8 through 21.

I have had the good fortune to witness the growth of ultrasonic guided waves in Non-destructive Testing (NDT) and Structural Health Monitoring (SHM) since 1985. I have been deeply interested in safety and improved diagnostics utilizing wave propagation concepts. Wave phenomena can be used to evaluate material properties nondestructively as well as to locate and measure defects in critical structures. This work has led to devices that have become valuable quality control tools and/or in-service inspection procedures for structures such as critical aircraft, pipeline, bridge, and nuclear power components whose integrity is vital to public safety.

My first exposure (1970 to 1985) to ultrasonic NDE – beyond basic pulse-echo and through-transmission testing – focused on signal processing and pattern recognition. New tomographic ultrasonic imaging procedures were developed that employed special features to assist in defect classification; these procedures supplemented or replaced the standard more localized ultrasonic test methods. In

the late 1970s, ultrasonic research was extended to medical applications. I explored linear phased array transducer systems used in real-time medical imaging. Of special interest to me at the time was tissue classification, in which we worked on differentiating malignant from benign tissue growth.

Around 1985, a newer version of ultrasonics in waveguides was conceived for faster and more sensitive ultrasonic examination. Some pioneering work on oblique incidence of the more localized ultrasonic method onto a bonded structure was carried out that could easily place longitudinal and shear energy into the bondline. The process was tedious and difficult to carry out. It was found that ultrasonic guided waves, however, could easily impinge both longitudinal and shear energy into the structure. Hence, guided wave activity was further developed for such adhesively bonded structures. Further research also revealed that guided waves – waves that travel along a surface or along a rod, tube, or platelike structure – could not only produce the same kind of two-dimensional particle velocity as that in oblique incidence but could also be much more efficient than the traditional technique of point-by-point examination. These guided wave research and application efforts continue today.

Guided wave concepts have been applied to examine the tubing in power plants and pipelines in chemical processing facilities and, importantly, to ensure the safety of large petroleum and gas pipelines. Because of their unique capabilities, guided wave techniques can be used to find tiny defects – over large distances, under adverse conditions, in structures with insulation and coatings, and in harsh environments.

Engineers, technicians, and students involved in ultrasonic NDE will appreciate the usefulness of this textbook. Even though the mathematics is sometimes detailed and sophisticated, the treatment can also be read by managers without detailed understanding of the concepts. They may find this book useful as it is designed to be read from a “black box” point of view so they can develop an understanding of what engineers, technicians, and students are talking about.

Overall, the material presented here in wave mechanics – and, in particular, guided wave mechanics – establishes a framework for the creative data collection and signal processing needed to solve many problems using ultrasonic NDE and SHM. I therefore hope that this book will be used as a reference in ultrasonic NDE by individuals at any level and as a textbook for seniors and graduate students. It is also hoped that this book will expand and promote the use of guided wave technology on both national and international levels.

## Acknowledgments

Thanks are given to many individuals for their work efforts, discussions, and contributions in wave mechanics over the past twenty years. A special tribute is made to Dr. Aleksander Pilarski, who passed away on January 6, 1994. “Olek” worked with me as a visiting professor at Drexel University and at The Pennsylvania State University from 1986 to 1988 and from 1992 to 1994. His energetic and enthusiastic style, as well as his technological contributions, had a strong influence on many of us. He was a dear friend whose memory will remain forever.

Thanks are given to all of my PhD students and many MS students for their work efforts and valuable discussions. In particular, special thanks for assistance in the preparation of this text are given to the following very talented individuals, with a brief description of their backgrounds.

Dr. Michael Avioli has worked with me for more than twenty-five years providing signal processing and pattern recognition support in guided wave analysis. He made special contributions in transform methods.

Cody Borigo is currently an engineer at FBS, Inc., and is conducting his PhD thesis research with me at The Pennsylvania State University. His research experience includes guided wave NDE in composites, guided wave tomography, ultrasonic vibrations, phased annular array transducers, and ultrasonic ice sensing and deicing for helicopters and fixed-wing aircraft.

Dr. Jason Philtron received his PhD in Acoustics with me from The Pennsylvania State University in 2013. He is currently a postdoc in my ultrasonic research group. Dr. Philtron’s research has focused on ultrasonic guided wave bond evaluation in thick structures, the use of phased arrays for optimal guided wave mode and frequency selection, guided wave tomography, and ultrasonic ice sensing.

Huidong Gao was born in Nantong, China, in 1978. He received his BS and MS degrees from Nanjing University, China, and his PhD degree with me from The Pennsylvania State University in 2007. Dr. Gao is now a principal research engineer at Innerspec Technologies, Inc. His primary research interest is advanced ultrasonic NDT techniques including guided waves, electromagnetic acoustic transducers (EMATs), and high-power UT applications. Dr. Gao is the 2011 Young NDT Professional Award recipient and the author of *Ultrasonic Testing*, a two-volume series book for NDT personnel training published by the ASNT.

Cliff Lissenden is a professor of engineering science and mechanics at The Pennsylvania State University. He came to The Pennsylvania State University in 1995 with expertise in mechanical behavior of materials. Dr. Lissenden now specializes in the use of ultrasonic guided waves for SHM and NDE. His current research investigates monitoring adhesively bonded or mechanically fastened joints in platelike structures and the generation of wave modes at higher harmonics to characterize precursors to macroscale damage.

Yang Liu is currently a research assistant on nonlinear methods in the Guided Wave NDE Lab, The Pennsylvania State University with Dr. Lissenden and myself.

Vamshi Chillara is a PhD candidate in the Engineering Science and Mechanics Department at The Pennsylvania State University.

Dr. Jing Mu, a scientist at FBS, Inc., obtained her PhD degree with me from The Pennsylvania State University in August 2008. Her research experience includes guided wave mechanics analysis and Finite Element Method (FEM) simulations. Dr. Mu specializes in ultrasonic guided wave inspection techniques of pipe structures including active phased array focusing, synthetic focusing, and advanced signal processing for pipe imaging.

Dr. Jason K. Van Velsor received his PhD in engineering science and mechanics with me from The Pennsylvania State University in 2009. He is currently an employee of Structural Integrity Associates. Dr. Van Velsor is an authority in the field application of guided wave technology for the long-range inspection of piping and holds multiple domestic and international certifications in this area. His practical experience includes the application of guided wave methods in nuclear and fossil power generation, oil and gas (on-shore and off-shore), gas transmission, water and wastewater, and pulp and paper industries.

Dr. Fei Yan is a scientist at FBS, Inc. He obtained his PhD degree with me in engineering mechanics from The Pennsylvania State University in 2008. Dr. Yan's research focuses on ultrasonic guided wave NDE and SHM applications including a variety of structures and composite materials. In particular, he has been involved in the development of guided wave phased arrays for isotropic and anisotropic composite plate structures, phased comb and annular array transducers, guided wave tomography SHM systems, and an ultrasonic vibration method.

Dr. Li Zhang is a scientist for FBS, Inc., and has focused on theoretical calculations and numerical simulations of guided wave behavior in various structures, phased array focusing and synthetic focusing in pipelines, and numerical simulations of ultrasonic sensor characteristics. She also obtained a PhD with me at The Pennsylvania State University.

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