

## Contents

---

<i>Preface</i>	<i>page xi</i>
<b>PART I DERIVED FUNCTORS AND HOMOTOPY (CO)LIMITS</b>	
<b>1 All concepts are Kan extensions</b>	<b>3</b>
1.1 Kan extensions	3
1.2 A formula	6
1.3 Pointwise Kan extensions	9
1.4 All concepts	11
1.5 Adjunctions involving simplicial sets	12
<b>2 Derived functors via deformations</b>	<b>17</b>
2.1 Homotopical categories and derived functors	18
2.2 Derived functors via deformations	23
2.3 Classical derived functors between abelian categories	29
2.4 Preview of homotopy limits and colimits	30
<b>3 Basic concepts of enriched category theory</b>	<b>32</b>
3.1 A first example	33
3.2 The base for enrichment	34
3.3 Enriched categories	35
3.4 Underlying categories of enriched categories	39
3.5 Enriched functors and enriched natural transformations	44
3.6 Simplicial categories	47
3.7 Tensors and cotensors	48
3.8 Simplicial homotopy and simplicial model categories	54
<b>4 The unreasonably effective (co)bar construction</b>	<b>58</b>
4.1 Functor tensor products	59
4.2 The bar construction	60

4.3	The cobar construction	62
4.4	Simplicial replacements and colimits	64
4.5	Augmented simplicial objects and extra degeneracies	66
<b>5</b>	<b>Homotopy limits and colimits: The theory</b>	<b>69</b>
5.1	The homotopy limit and colimit functors	70
5.2	Homotopical aspects of the bar construction	72
<b>6</b>	<b>Homotopy limits and colimits: The practice</b>	<b>76</b>
6.1	Convenient categories of spaces	77
6.2	Simplicial model categories of spaces	81
6.3	Warnings and simplifications	82
6.4	Sample homotopy colimits	84
6.5	Sample homotopy limits	89
6.6	Homotopy colimits as weighted colimits	92
 <b>PART II ENRICHED HOMOTOPY THEORY</b>		
<b>7</b>	<b>Weighted limits and colimits</b>	<b>99</b>
7.1	Weighted limits in unenriched category theory	99
7.2	Weighted colimits in unenriched category theory	105
7.3	Enriched natural transformations and enriched ends	108
7.4	Weighted limits and colimits	109
7.5	Conical limits and colimits	112
7.6	Enriched completeness and cocompleteness	114
7.7	Homotopy (co)limits as weighted (co)limits	116
7.8	Balancing bar and cobar constructions	119
<b>8</b>	<b>Categorical tools for homotopy (co)limit computations</b>	<b>121</b>
8.1	Preservation of weighted limits and colimits	121
8.2	Change of base for homotopy limits and colimits	124
8.3	Final functors in unenriched category theory	126
8.4	Final functors in enriched category theory	129
8.5	Homotopy final functors	130
<b>9</b>	<b>Weighted homotopy limits and colimits</b>	<b>136</b>
9.1	The enriched bar and cobar construction	136
9.2	Weighted homotopy limits and colimits	139
<b>10</b>	<b>Derived enrichment</b>	<b>145</b>
10.1	Enrichments encoded as module structures	146
10.2	Derived structures for enrichment	149
10.3	Weighted homotopy limits and colimits, revisited	155
10.4	Homotopical structure via enrichment	158
10.5	Homotopy equivalences versus weak equivalences	162

### PART III MODEL CATEGORIES AND WEAK FACTORIZATION SYSTEMS

<b>11 Weak factorization systems in model categories</b>	167
11.1 Lifting problems and lifting properties	167
11.2 Weak factorization systems	172
11.3 Model categories and Quillen functors	174
11.4 Simplicial model categories	180
11.5 Weighted colimits as left Quillen bifunctors	182
<b>12 Algebraic perspectives on the small object argument</b>	190
12.1 Functorial factorizations	191
12.2 Quillen's small object argument	192
12.3 Benefits of cofibrant generation	195
12.4 Algebraic perspectives	198
12.5 Garner's small object argument	201
12.6 Algebraic weak factorization systems and universal properties	208
12.7 Composing algebras and coalgebras	214
12.8 Algebraic cell complexes	216
12.9 Epilogue on algebraic model categories	220
<b>13 Enriched factorizations and enriched lifting properties</b>	222
13.1 Enriched arrow categories	223
13.2 Enriched functorial factorizations	224
13.3 Enriched lifting properties	228
13.4 Enriched weak factorization systems	233
13.5 Enriched model categories	235
13.6 Enrichment as coherence	237
<b>14 A brief tour of Reedy category theory</b>	240
14.1 Latching and matching objects	241
14.2 Reedy categories and the Reedy model structures	243
14.3 Reedy cofibrant objects and homotopy (co)limits	247
14.4 Localizations and completions of spaces	252
14.5 Homotopy colimits of topological spaces	257

### PART IV QUASI-CATEGORIES

<b>15 Preliminaries on quasi-categories</b>	263
15.1 Introducing quasi-categories	265
15.2 Closure properties	266
15.3 Toward the model structure	269
15.4 Mapping spaces	273

<b>16 Simplicial categories and homotopy coherence</b>	282
16.1 Topological and simplicial categories	282
16.2 Cofibrant simplicial categories are simplicial computads	284
16.3 Homotopy coherence	286
16.4 Understanding the mapping spaces $\mathcal{C}X(x, y)$	290
16.5 A gesture toward the comparison	296
<b>17 Isomorphisms in quasi-categories</b>	298
17.1 Join and slice	299
17.2 Isomorphisms and Kan complexes	303
17.3 Inverting simplices	306
17.4 Marked simplicial sets	307
17.5 Inverting diagrams of isomorphisms	312
17.6 A context for invertibility	315
17.7 Homotopy limits of quasi-categories	316
<b>18 A sampling of 2-categorical aspects of quasi-category theory</b>	318
18.1 The 2-category of quasi-categories	319
18.2 Weak limits in the 2-category of quasi-categories	321
18.3 Arrow quasi-categories in practice	324
18.4 Homotopy pullbacks	325
18.5 Comma quasi-categories	326
18.6 Adjunctions between quasi-categories	328
18.7 Essential geometry of terminal objects	333
<i>Bibliography</i>	337
<i>Glossary of Notation</i>	343
<i>Index</i>	345