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Excerpt
More Information

PART 1

EFFECTIVE FIELD THEORY: THE STANDARD MODEL, SUPERSYMMETRY, UNIFICATION

Cambridge University Press & Assessment
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1 Before the Standard Model

Two of the most profound scientific discoveries of the early twentieth century were special relativity and quantum mechanics. With special (and general) relativity came the notion that physics should be local. Interactions should be carried by dynamical fields in space–time. Quantum mechanics altered the questions which physicists ask about phenomena; the rules governing microscopic (and some macroscopic) phenomena were not those of classical mechanics. When these ideas were combined they took on their full force, in the form of *quantum field theory*: particles themselves are localized, finite-energy, excitations of fields. Otherwise mysterious phenomena, such as the connection of spin and statistics, were immediate consequences of this marriage. But quantum field theory posed serious challenges for its early practitioners. The Schrödinger equation seems to single out time, making a manifestly relativistic description difficult. More seriously, but closely related, in quantum field theory the number of degrees of freedom is infinite, in contrast with the quantum mechanics of atomic systems. In the 1920s and 1930s, physicists performed conventional perturbation theory calculations in the quantum theory of electrodynamics, namely quantum electrodynamics (QED), and obtained expressions which were neither Lorentz invariant nor finite. Until the late 1940s these problems stymied any quantitative progress, and there was serious doubt whether quantum field theory was a sensible framework for physics.

Despite these concerns, quantum field theory proved a valuable tool with which to consider problems of fundamental interactions. Yukawa proposed a field theory of the nuclear force in which the basic quanta were mesons. The corresponding particle was discovered shortly after the Second World War. Fermi was aware of Yukawa's theory and proposed that weak interactions arose through the exchange of some massive particle – essentially the $W^{\pm}$ bosons, which were finally discovered in the 1980s. The large mass of these particles accounted for both the short range and the strength of the weak force. Because of its very short range, one could describe it in terms of four fields interacting at a point. In the early days of the theory, these were the proton, neutron, electron and neutrino. Viewed as a theory of four-fermion interactions Fermi's theory was very successful, accounting for all experimental weak interaction results until well into the 1970s. Yet the theory raised even more severe conceptual problems than QED. At high energies the amplitudes computed in the leading approximation violated unitarity, and the higher-order terms in perturbation theory were very divergent.

The difficulties of QED were overcome in the late 1940s, by Bethe, Dyson, Feynman, Schwinger, Tomanaga and others, as experiments in atomic physics demanded high-precision QED calculations. As a result of their work, it was now possible to perform perturbative calculations in a manifestly Lorentz-invariant fashion. Exploiting covariance the infinities could be controlled and, over time, their significance came to be understood. Quantum electrodynamics achieved enormous successes, explaining the magnetic moment of the electron to extraordinary precision as well as the Lamb shift in hydrogen and other phenomena. One now, for the first time, had an example of a system of physical law that was consistent both with Einstein's principles of relativity and with quantum mechanics.

There were, however, many obstacles to extending this understanding to the strong and weak interactions, and at times it seemed that some other framework might be required. The difficulties came in various types. The infinities of Fermi's theory of weak interactions could not be controlled as in electrodynamics. Even postulating the existence of massive particles to mediate the force did not solve the problems. But the most severe difficulties came in the case of the strong interactions. The 1950s and 1960s witnessed the discovery of hundreds of hadronic resonances. It was hard to imagine that each should be described by still another fundamental field. Some theorists pronounced field theory dead and sought alternative formulations (among the outgrowths explorations was string theory, which has emerged as the most promising setting for a quantum theory of gravitation). But Gell-Mann and Zweig realized that *quarks* could serve as an organizing principle. Originally, there were only three, u, d and s, with baryon number $1/3$ and charges $2/3$, $-1/3$ and $-1/3$ (in units of the electric charge) respectively. All the known hadrons could be understood as bound states of objects with these quantum numbers. Still, there remained difficulties. First, quarks were strongly interacting and there were no successful ideas for treating strongly interacting fields. Second, those searching for quarks came up empty handed.

In the late 1960s a dramatic series of experiments at SLAC, and a set of theoretical ideas due to Feynman and Bjorken, changed the situation again. Feynman had argued that one should take seriously the idea of quarks as dynamical entities (for a variety of reasons he hesitated to call them quarks, referring to them as *partons*). He conjectured that these partons would behave as nearly free particles in situations where momentum transfers were large. He and Bjorken realized that this picture implied a scaling in deep inelastic scattering phenomena. The experiments at SLAC exhibited just this phenomenon and showed that the partons carried the electric charges of the u and d quarks.

But this situation was still puzzling. Known field theories did not behave in the fashion conjectured by Feynman and Bjorken. The interactions of particles typically became *stronger* as the energies and momentum transfers grew. This is the case, for example, in quantum electrodynamics and a simple quantum mechanical argument, based on unitarity and relativity, would seem to suggest it is true in general. But there turned out to be an important class of theories with the opposite property.

In 1954 Yang and Mills wrote down a generalization of electrodynamics where the $U(1)$ symmetry group is enlarged to a non-Abelian group, with massless gauge bosons transforming in the adjoint representation of the group. While mathematically quite beautiful, these *non-Abelian gauge theories* remained oddities for some time. First, their possible place in the scheme of things was not known (Yang and Mills themselves suggested that perhaps their vector particles were the ρ mesons). Moreover, their quantization was significantly more challenging than that of electrodynamics. It was not at all clear that these theories really made sense at the quantum level, that is, that they respected the principles of both Lorentz invariance and unitarity. The first serious effort to quantize

Yang–Mills theories was probably due to Schwinger, who chose a non-covariant but manifestly unitary gauge and carefully verified that the Poincaré algebra was satisfied. The non-covariant gauge, however, was exceptionally awkward. Real progress in formulating a covariant perturbation expansion was made by Feynman, who noted that naive Feynman rules for these theories were not unitary but that this difficulty could be removed, at least in low orders, by adding a set of fictitious fields ("ghosts"). A general formulation was provided by Faddeev and Popov, who derived Feynman's covariant rules in a path integral formulation and showed their formal equivalence to Schwinger's manifestly unitary formulation. A convincing demonstration that these theories are unitary, covariant and *renormalizable* was finally given in the early 1970s by 't Hooft and Veltman, who developed elegant and powerful techniques for performing real calculations as well as formal proofs.

In the original Yang–Mills theories the vector bosons were massless and their possible connections to known phenomena were obscure. However, Carl R. Hagen, Francois Englert, Gerald S. Guralnik, Peter W. Higgs, Robert Brout, and T. W. B. Kibble discovered a mechanism by which these particles could become massive. In 1967, Weinberg and Salam wrote down a Yang–Mills theory of weak interactions based on what has come to be referred to as the "Higgs mechanism". This finally realized Fermi's idea that weak interactions arise from the exchange of a very massive particle. To a large degree this work was ignored until 't Hooft and Veltman proved the unitarity and renormalizability of these theories. At this point the race to find precisely the correct theory and study its experimental consequences was on; Weinberg's and Salam's first guess turned out to be correct.

The possible role of Yang–Mills fields in strong interactions was, at first sight, even more obscure. To complete the story required another important fact of hadronic physics. While the quark model was very successful, it was also puzzling. The quarks were spin-$1/2$ particles, yet models of the hadrons seemed to require that the hadronic wave functions were symmetric under the interchange of quark quantum numbers. A possible resolution, suggested by Greenberg, was that the quarks carried an additional quantum number, called color, coming in three possible types. The statistics puzzle was solved if the hadron wave functions were totally antisymmetric in color. This hypothesis required that the color symmetry, unlike, say, isospin, should be exact and thus special. While seemingly contrived, it explained two other facts: the width of the π^0 meson and the value of the e^+e^- cross section to hadrons, each of which was otherwise was too large by a factor three.

To a number of researchers the exactness of this color symmetry suggested a possible role for Yang–Mills theory. So, in retrospect there was an obvious question: could it be that an $SU(3)$ Yang–Mills theory, describing the interactions of quarks, would exhibit the property required to explain Bjorken scaling, i.e. that the interactions become weak at short distances? Of course, things were not quite so obvious at the time.The requisite calculation had already been done by 't Hooft but the result seems not to have been widely known nor its significance appreciated. David Gross and his student Frank Wilczek set out to prove that no field theory had the required scaling property, while Sidney Coleman, apparently without any particular prejudice, assigned the problem to his graduate student David Politzer. All soon realized that Yang–Mills theories do have the property of asymptotic freedom: the interactions become weak at high momentum transfers or at short distances.

Experiment and theory now entered a period of remarkable convergence. Alternatives to the Weinberg–Salam theory were quickly ruled out. The predictions of quantum chromodynamics (QCD) were difficult, at first, to verify in detail. The theory predicted small violations of Bjorken scaling, depending logarithmically on energy, and it took many years to measure them convincingly. But there was another critical experimental development which clinched the picture. The existence of a heavy quark beyond the u, d and s had been predicted by Glashow, Iliopoulos and Maiani and was a crucial part of the developing Standard Model. The mass of this *charm* quark had been estimated by Gaillard and Lee. Appelquist and Politzer predicted, almost immediately after the discovery of asymptotic freedom, that heavy quarks would be bound in narrow vector resonances. In 1974 a narrow resonance was discovered in e^+e^- annihilation, the J/ψ particle, which was quickly identified as a bound state of a charm quark and its antiparticle.

Over the next 25 years, this Standard Model was subjected to more and more refined tests. One feature absent from the original Standard Model was CP(T) violation. Kobiyashi and Maskawa pointed out that if there were a third generation of quarks and leptons, then the theory could accommodate the observed CP violation in the K meson system. Two more quarks and a lepton were discovered, and their interactions and behavior were as expected within the Standard Model. Jets of particles which could be associated with *gluons* were seen in the late 1970s. The W and Z particles were produced in accelerators in the early 1980s. At CERN and SLAC, precision measurements of the Z mass and width provided stringent tests of the weak-interaction part of the theory. Detailed measurements in deep inelastic scattering and in jets provided precise confirmation of the logarithmic scaling violations predicted by QCD. The Standard Model passed every test.

At the time at which the first edition of this book went to press, the Standard Model had triumphed in almost every realm. The low-energy weak interactions were completely described by the Weinberg–Salam theory with corrections from the strong interactions, many well understood. At high energies the W and Z particles had been produced in great numbers in accelerators, and their properties – i.e. production rates and decays – compared with the theory, including the effects of QCD, at the one part per mil level. The Tevatron had performed precise studies of jet production in excellent agreement with QCD and lattice gauge theory had witnessed an enormous leap in reliability and precision, reproducing features of the hadron spectrum and yielding quantities of importance for the study of the weak decays of B mesons, for example. The only missing piece was the Higgs particle, or whatever entity was responsible for the breaking of the electroweak symmetry. In 2012, that changed. The 5σ discovery of a scalar particle was announced at CERN on July 4. By the end of the first run of the LHC at the end of the year, a good deal of circumstantial evidence had accumulated that this particle was indeed the Higgs scalar of the simplest Standard Model. 't Hooft and Veltman had received the Nobel Prize for their work on non-Abelian gauge theories in 1999. During the first 14 years of the new millennium, these successes have been recognized by several Nobel Prizes: Gross, Politzer and Wilczek for the understanding of strong interactions (2004); Nambu for his work on spontaneous symmetry breaking; Kobayashi and Maskawa for the mechanism of

CP violation in the Standard Model (2008); and Englert and Higgs for the proposal of the Higgs particle (2013). Since the publication of the first edition of this book, a Nobel Prize has been awarded for the discovery of dark energy (Perlmutter, Reiss and Schmidt, 2011).

So the question which I raised in 2006, Why write a book about Beyond the Standard Model physics?, is all the sharper now. It is still true that, for all its simplicity and success in reproducing the interactions of elementary particles, the Standard Model cannot represent a complete description of nature. In the first few chapters of this book we will review the Standard Model and its successes, including the recent discovery of the Higgs particle, which is a triumph not only for our understanding of the electroweak theory but of QCD as well. Then we will discuss some of the Standard Model's limitations. These include the *hierarchy problem*, which, at its most primitive level, represents a failure of dimensional analysis; the presence of a large number of parameters; the strong CP problem, i.e. the presence of a very small dimensionless number which violates CP. We will confront the incompatibility of quantum mechanics with Einstein's theory of general relativity, the inability of the Standard Model to account for the small but non-zero value of the cosmological constant (an even more colossal failure of dimensional analysis) and its failure to account for basic features of our universe, the excess of baryons over antibaryons, dark matter and structure. Then we will set out on an exploration of possible phenomena which might address these questions. These include: supersymmetry, technicolor and large or warped extra dimensions as possible solutions to the hierarchy problem; grand unification as a partial solution to the overabundance of parameters; and the axion for the strong CP problem. Still more ambitious is superstring theory, as a possible solution to the problem of quantizing gravity, which incorporates many features of these other proposals. We will consider the experimental constraints on new physics, which have become more severe with the first LHC run, and discuss the prospects for the future at the LHC and beyond. Finally, we will acknowledge the possibility that the resolution of some of these puzzles might involve a *landscape* or *multiverse*.

Suggested reading

A complete bibliography of the Standard Model would require a book by itself. A good deal of the history of special relativity, quantum mechanics and quantum field theory can be found in *Inward Bound*, by Abraham Pais (1986), which also includes an extensive bibliography. The development of the Standard Model is also documented in this very readable book. As a minor historical note I would add that the earliest reference in which I came across the observation that a Yang–Mills theory might underlie the strong interactions is due to Feynman, in about 1963 (Roger Dashen, personal communication, 1981), who pointed out that in an $SU(3)$ Yang–Mills theory three quarks would be bound together, as would quark–antiquark pairs.

2 The Standard Model

The interactions of the Standard Model give rise to the phenomena of our day to day experience. They explain virtually all the particles and interactions which have been observed in accelerators. Yet the underlying laws can be summarized in a few lines. In this chapter we describe the ingredients of this theory and some of its important features. Many dynamical questions will be studied in subsequent chapters. For detailed comparisons of theory and experiment there are a number of excellent texts, described in the suggested reading at the end of the chapter.

2.1 Yang–Mills theory

By the early 1950s physicists were familiar with approximate global symmetries such as isospin. Yang and Mills argued that the lesson of Einstein's general theory was that symmetries, if exact, should be local. In ordinary electrodynamics the gauge symmetry is a local Abelian symmetry. Yang and Mills explained how to generalize this to a non-Abelian symmetry group. Let's first review the case of electrodynamics. The electron field $\psi(x)$ transforms under a gauge transformation as follows:

$$\psi(x) \to e^{i\alpha(x)}\psi(x) = g_\alpha(x)\psi(x). \tag{2.1}$$

We can think of $g_\alpha(x) = e^{i\alpha(x)}$ as a group element in the group $U(1)$. The group is Abelian: $g_\alpha g_\beta = g_\beta g_\alpha = g_{\alpha+\beta}$. Quantities such as $\bar\psi\psi$ are gauge invariant, but derivative terms such as $i\bar\psi \not\partial\psi$, are not. In order to write down the derivative terms in an action or equation of motion, one needs to introduce a gauge field A_μ transforming under the symmetry transformation as

$$\begin{aligned} A_\mu &\to A_\mu + \partial_\mu\alpha \\ &= A_\mu + ig(x)\partial_\mu g^{-1}(x). \end{aligned} \tag{2.2}$$

This second form allows more immediate generalization to the non-Abelian case. Given A_μ and its transformation properties, we can define a covariant derivative,

$$D_\mu\psi = (\partial_\mu - iA_\mu)\psi. \tag{2.3}$$

This derivative has the property that it transforms like ψ itself under the gauge symmetry:

$$D_\mu\psi \to g(x)D_\mu\psi. \tag{2.4}$$

We can also form a gauge-invariant object from the gauge fields A_μ themselves. A simple way to do this is to construct the commutator of two covariant derivatives,

$$F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.5}$$

This form of the gauge transformations may be somewhat unfamiliar. Note in particular that the charge of the electron, e (the gauge coupling) does not appear in the transformation laws. Instead, the gauge coupling appears when we write down a gauge-invariant Lagrangian:

$$\mathcal{L} = i\bar{\psi}\, \not{D}\psi - m\bar{\psi}\psi - \frac{1}{4e^2}F_{\mu\nu}^2, \tag{2.6}$$

where the "slash" notation is defined by $\not{a} = a^\mu \gamma_\mu$. The more familiar formulation is obtained if we make the replacement

$$A_\mu \to eA_\mu. \tag{2.7}$$

In terms of this new field the gauge transformation law is

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha \tag{2.8}$$

and the covariant derivative is

$$D_\mu \psi = (\partial_\mu - ieA_\mu)\psi. \tag{2.9}$$

We can generalize this to a non-Abelian group, $\mathcal{G}$, by taking ψ to be a field (fermion or boson) in some representation of the group; $g(x)$ is then a matrix which describes a group transformation acting in this representation. Formally, the transformation law is the same as before,

$$\psi \to g(x)\psi(x), \tag{2.10}$$

but the group composition law is more complicated:

$$g_\alpha g_\beta \neq g_\beta g_\alpha. \tag{2.11}$$

The gauge field A_μ is now a matrix-valued field, transforming in the adjoint representation of the gauge group:

$$A_\mu \to gA_\mu g^{-1} + ig(x)\partial_\mu g^{-1}(x). \tag{2.12}$$

Formally, the covariant derivative also looks exactly as before:

$$D_\mu \psi = (\partial_\mu - iA_\mu)\psi, \quad D_\mu \psi \to g(x)D_\mu \psi. \tag{2.13}$$

Like A_μ, the field strength is a matrix-valued field:

$$F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]. \tag{2.14}$$

Note that $F_{\mu\nu}$ is not gauge *invariant* but, rather, covariant:

$$F_{\mu\nu} \to gF_{\mu\nu}g^{-1}, \tag{2.15}$$

i.e. it transforms like a field in the adjoint representation, with no inhomogeneous term.

The gauge-invariant action $\mathcal{L}$ is formally almost identical to that of the $U(1)$ theory:

$$\mathcal{L} = i\bar{\psi}\,\not{D}\psi - m\bar{\psi}\psi - \frac{1}{2g^2}\,\mathrm{Tr}\,F_{\mu\nu}^2. \tag{2.16}$$

Here we have changed the letter we use to denote the coupling constant: we will usually reserve e for the electron charge and use g for a generic gauge coupling. Note also that it is necessary to take the trace of F^2 to obtain a gauge-invariant expression.

The matrix form for the fields may be unfamiliar, but it is very powerful. One can recover expressions in terms of more conventional fields by defining

$$A_\mu = A_\mu^a T_a, \tag{2.17}$$

where T_a are the group generators in the representation appropriate to ψ. Then, for $SU(N)$, for example, if the T_as are in the fundamental representation, we have

$$\mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \quad [T^a, T^b] = if^{abc}T^c, \tag{2.18}$$

where f^{abc} are the structure constants of the group and

$$A_\mu^a = 2\,\mathrm{Tr}(T_a A^\mu), \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{abc}A_\mu^a A_\nu^b. \tag{2.19}$$

While they are formally almost identical, there are great differences between the Abelian and non-Abelian theories. Perhaps the most striking is that the equations of motion for the A_μs are non-linear in non-Abelian theories. This behavior means that, unlike the case of Abelian gauge fields, a theory of non-Abelian fields without matter is a non-trivial, interacting, theory with interesting properties. With and without matter fields, this will lead to much richer behavior even classically. For example, we will see that non-Abelian theories sometimes contain solitons, localized finite-energy solutions of the classical equations. The most interesting of these are the magnetic monopoles. At the quantum level these non-linearities lead to properties such as asymptotic freedom and confinement.

Using the form in which we have written the action, the matter fields ψ can appear in any representation of the group; one just needs to choose appropriate matrices T^a. We can also consider scalars, as well as fermions. For a scalar field ϕ, we define the covariant derivative $D_\mu\phi$ as before and add to the action a term $|D_\mu\phi|^2$ for a complex field or $(D_\mu\phi)^2/2$ for a real field.

2.2 Realizations of symmetry in quantum field theory

The most primitive exercise we can do with the Yang–Mills Lagrangian is to set $g = 0$ and examine the equations of motion for the fields A^μ. If we choose the gauge $\partial_\mu A^{\mu a} = 0$, all the gauge fields obey

$$\partial^2 A_\mu^a = 0. \tag{2.20}$$

So, like the photon, all the gauge fields A_μ^a of the Yang–Mills theory are massless. At first sight there is no obvious place for these fields in either the strong or the weak interactions. But it turns out that in non-Abelian theories the possible ways in which the symmetry may be realized are quite rich. First, the symmetry can be realized in terms of massless gauge bosons; this is known as the *Coulomb phase*. This possibility is not relevant to the Standard Model but will appear in some of our more theoretical considerations later. A second way is known as the *Higgs phase*. In this phase, the gauge bosons are massive. In the third, the *confinement phase*, there are no physical states with the quantum numbers of isolated quarks (particles in the fundamental representation), and the gauge bosons are also massive. The second phase is relevant to the weak interactions; the third, confinement, phase to the strong interactions.[1]

2.2.1 The Goldstone phenomenon

Before introducing the Higgs phase it is useful to discuss global symmetries. While we will frequently argue, like Yang and Mills, that global symmetries are less fundamental than local ones, they are important in nature. Examples are isospin, the chiral symmetries of the strong interactions and baryon number. We can represent the action of such a symmetry much as we represented the symmetry action in Yang–Mills theory:

$$\Phi \to g_\alpha \Phi, \tag{2.21}$$

where Φ is some set of fields and g is now a constant matrix, independent of spatial position. Such symmetries are typically accidents of the low-energy theory. Isospin, for example, as we will see arises because the masses of the u and d quarks are small compared with other scales of quantum chromodynamics. Then g is the matrix

$$g_{\vec{\alpha}} = e^{i\vec{\alpha}\cdot\vec{\sigma}/2} \tag{2.22}$$

acting on the u and d quark doublet. Note that $\vec{\alpha}$ is not a function of space but a continuous parameter, so we will refer to such symmetries as continuous global symmetries. In the case of isospin it is also important that the electromagnetic and weak interactions, which violate this symmetry, are small perturbations on the strong interactions.

The simplest model of a continuous global symmetry is provided by a complex field ϕ transforming under a $U(1)$ symmetry,

$$\phi \to e^{i\alpha}\phi. \tag{2.23}$$

We can take for the Lagrangian for this system

$$\mathcal{L} = |\partial_\mu\phi|^2 - m^2|\phi|^2 - \frac{1}{2}\lambda|\phi|^4. \tag{2.24}$$

If $m^2 > 0$ and λ is small, this is simply a theory of a weakly interacting, complex scalar. The states of the theory can be organized as states of definite $U(1)$ charge. This is the unbroken

[1] The differences between the confinement and Higgs phases are subtle, as was first stressed by Fradkin, Shenker and 't Hooft. But we now know that the Standard Model is well described by a weakly coupled field theory in the Higgs phase.