

RELATIVISTIC KINETIC THEORY

With Applications in Astrophysics and Cosmology

Relativistic kinetic theory has widespread application in astrophysics and cosmology. The interest has grown in recent years, as experimentalists are now able to make reliable measurements on physical systems where relativistic effects are no longer negligible.

This ambitious monograph is divided into three parts. Part I presents the basic ideas and concepts of this theory; equations and methods, including derivation of kinetic equations from the relativistic BBGKY hierarchy; and discussion of the relation between kinetic and hydrodynamic levels of description. Part II introduces elements of computational physics, with special emphasis on numerical integration of Boltzmann equations and related approaches as well as multicomponent hydrodynamics. Part III presents an overview of applications ranging from covariant theory of plasma response, thermalization of relativistic plasma, and comptonization in static and moving media to kinetics of self-gravitating systems, cosmological structure formation, and neutrino emission during the gravitational collapse.

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Preface

The endeavor of writing this book started from a series of lectures given by the first author for students of the International Relativistic Astrophysics PhD program (IRAP PhD) supported by the Erasmus Mundus program of the European Commission. For this book the material has been expanded and more topics incorporated. It soon became clear that an updated and systematic presentation of relativistic kinetic theory and its numerous applications in astrophysics and cosmology is lacking in the literature. Some existing monographs, presenting fundamental aspects of kinetic theory, are focused on selected applications. Others, which contain applications of kinetic theory in relativistic astrophysics and cosmology, lack the presentation of fundamental concepts of relativistic kinetic theory. Moreover, none of the existing monographs discussed in depth various numerical methods developed and successfully applied in kinetic theory in the recent decades. This last observation urged us to bridge this gap in the literature. This effort eventually resulted in the current monograph, divided in three parts. Parts I and III, with the sole exception of the last chapter, were written by the first author. Part II and the last chapter of Part III were written by the second author.

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The first author dedicates this work to his father, Victor Vereshchagin, who attracted his attention to the brilliant stars in the marvelous night sky of Petrunino when he was five years old. The encouragement, patience, and devotion of his wife, Alina, are invaluable. Without her support the book would not have been written.

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Acronyms and Definitions

AGN	active galactic nuclei
BBGKY	Bogoliubov-Born-Green-Kirkwood-Yvon (hierarchy)
CM	center of momentum
CMB	cosmic microwave background
DF	distribution function
GRB	gamma-ray burst
KT	kinetic theory
LHS	left-hand side
LP	large particle
MC	Monte Carlo (method)
ODE	ordinary differential equation
PDE	partial differential equation
PIC	particle-in-cell
QED	quantum electrodynamics
qSS	quasi-stationary state
RHS	right-hand side
SN	supernova
SPH	smoothed-particle hydrodynamics
SW	shock wave
SZ	Sunyaev-Zeldovich (effect)
$x^\mu = (ct, \mathbf{x})$	coordinate four-vector
$j^\mu = (cn, \mathbf{j})$	four-current
$p^\mu = (p^0, \mathbf{p})$	momentum four-vector
$p^0 = \sqrt{\mathbf{p}^2 + m^2 c^2}$	relativistic energy-momentum relation
$\epsilon = cp^0 = \gamma mc^2 = \varepsilon mc^2$	particle energy
$p^2 \equiv p^\mu p_\mu = m^2 c^2$	on shell condition
$g^{\mu\nu} = \text{diag} (1, -1, -1, -1)$	Minkowski metric tensor

$ds = (g_{\mu\nu}dx^\mu dx^\nu)^{1/2} = cd\tau$	interval
$d\tau = mc/[p^0(t)]dt = dt/\gamma$	proper time
$\gamma \equiv [1 - (\mathbf{v}/c)^2]^{-1/2} = p^0(p^\mu p_\mu)^{-1/2}$	Lorentz factor
$u^\mu \equiv dx^\mu/d\tau = \text{diag}(\gamma c, \gamma \mathbf{v}) = p^\mu/m$	particle four-velocity
$U^\mu U_\mu = c^2$	velocity normalization condition
$\mathbf{v} = c\mathbf{p}/p^0 = \mathbf{p}/(\gamma m)$	three-velocity vector
$\partial_\mu = (c^{-1}\partial/\partial t, \nabla)$	four-gradient
$\rho = c^{-2}T^{\mu\nu}U_\mu U_\nu$	energy density
$P = -\frac{1}{3}T^{\mu\nu}\Delta_{\mu\nu}$	pressure
$\Delta^{\mu\nu} = g^{\mu\nu} - c^{-2}U^\mu U^\nu$	projection operator
$k^\mu = (\omega/c, \mathbf{k})$	four-wave vector
$d\eta = dt/a$	conformal time
$H \equiv d\ln a/dt = a^{-2}da/d\eta$	Hubble parameter