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Gregory V. Vereshchagin , Alexey G. Aksenov
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Introduction

This book presents the subject of relativistic kinetic theory (KT). It starts from fundamental concepts and ideas and arrives at a vast spectrum of applications through the bridge of various numerical methods. It is not by chance that we adopted such an approach. KT of gases is perhaps the most fundamental theory in the classical (nonquantum) domain. It has been developed with the goal to derive the properties of matter at the macroscopic level, which is accessible to direct experiments and observations, based on the study of properties of microscopic particles and their mutual interactions, to which one has no direct access. The atomic picture of the world has emerged in this way. Indeed, KT was born in the nineteenth century, the golden age of classical physics. Based on the atomic picture [1], such properties as heat and electrical conductivity as well as viscosity and diffusion found natural explanations. The term originates from the Greek, where κινησις means "motion." In fact, all the properties of the medium may be understood from the analysis of its microscopical structure and motions.

Nowadays KT has to be considered in a wider context of statistical mechanics, which appeared at the end of the nineteenth century, essentially in the works of Maxwell, Boltzmann, and Gibbs. It should be emphasized that the ideas and principles of KT influenced the development of many other branches of science, including mathematics (probability theory, ergodic theory), biology (evolutionary biology, population genetics), and economics (financial markets, econophysics). Within physics, KT is closely related to statistical physics, thermodynamics, and hydro- and gasdynamics. Today one can say that the main task of kinetic theory is the explanation of various macroscopic properties of a medium based on known microscopic properties and interactions. In a general context, KT is a theory of nonequilibrium systems. Indeed, all the above-mentioned fields of physics assume that the medium is in its most probable microphysical state, called *equilibrium*. Clearly, any macroscopic manifestation of deviations from this microscopic equilibrium should be considered within KT. Because basic phenomena in

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the microworld are described in a quantum language, KT uses extensively quantum theory. In fact, the basic principles and equations of KT may be derived from quantum field theory [2].

The first classical applications of KT concerned gases. A successful description of ideal and nonideal gases has been reached within the framework of Newtonian mechanics. At the same time, the progress in stellar dynamics [3] led to the formulation of the collisionless Boltzmann equation with the mean gravitational potential, satisfying the Poisson equation. The latter was later rediscovered in the context of plasma physics.

With the appearance of special relativity, KT had to be reconciled with the existence of the limiting speed, the speed of light. In particular, equilibrium distributions, i.e., Maxwell-Boltzmann distribution of velocities, had to be modified. These developments resulted in the work of Jüttner on relativistic equilibrium distribution function already in 1911 [4].

It soon became clear that there is another natural arena for the application of KT, which is plasma physics [5, 6]. The major difference between plasma and gas is the long-range nature of electromagnetic interaction, which has been accommodated by introduction of the mean field description [7]. While classical KT theory of plasmas developed rapidly in the 1930s, it was essentially nonrelativistic. Even the Landau damping phenomenon was discussed within the nonrelativistic framework, despite the fact that its analysis requires the use of the Vlasov equation, which is Lorentz invariant.

The formulation of KT within special relativity was completed in the 1960s, and it is presented in several monographs (see, e.g., [2, 8, 9]). Relativistic astrophysics emerged in the same period. The main triggers were the discoveries of the cosmic background radiation (CMB), pulsars, and quasars. Observation of the CMB confirmed the hot model of the universe. It urged the development of models for matter at extreme densities and temperatures, characteristic of the early universe. In this way the kinetics of thermonuclear reactions was analyzed, leading to the big bang nucleosynthesis (BBN) theory. Similarly, the discovery of pulsars and their interpretation as rapidly rotating magnetized neutron stars urged the formulation of models of matter under extreme densities and in strong gravitational and electromagnetic fields.

Hence, it is natural that most applications of relativistic KT are in the fields of astrophysics and cosmology. Both fields are somewhat special in physics. They lack the very essential feature of traditional physics: the possibility to set up and control physical experiments. In both fields the available experimental data originate essentially from *observations*, and the observer has no power whatsoever to influence or change the conditions under which the observed phenomenon takes place. For this reason, numerical simulations appear to be the unique tool for development

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of theoretical models in astrophysics and cosmology, which are eventually tested against observations.

Nevertheless, relativistic KT is becoming more accessible to direct tests due to the recent progress in two fields: inertial fusion and ultra-intense lasers. Operation of ultra-intense lasers is approaching such intensities that the creation of electron-positron plasma in the laboratory is becoming technologically feasible [10]. Hence, the application of relativistic KT to electron-positron plasmas, discussed in the book, becomes of great importance.

The focus of the book is mainly on KT within special relativity. A general relativistic kinetic equation is formulated in Part I, while general relativistic effects are discussed only in Part III in relation to the gravitational instability phenomenon as well as gravitational collapse.

The formulation of KT is presented in Part I in Lorentz-invariant fashion. In Chapter 1 the evolution of basic concepts of KT, such as phase space and distribution functions, from nonrelativistic to special and general relativistic frameworks is outlined. The relation between mechanical and kinetic pictures is presented. The physical meaning of the one-particle distribution function is given and its Lorentz invariance is demonstrated. Then the most useful macroscopic quantities, such as four-current, entropy four-flux, energy-momentum tensor, and hydrodynamic velocity, are obtained. These concepts are essential to proceed with the formulation of kinetic equations and to understand the relation between KT and hydrodynamics, discussed in the following chapters. In Chapter 2, an axiomatic approach to derive kinetic equations for the one-particle distribution function is adopted, and special attention is given to the advection part. First, the kinetic equation in special relativity is presented by considering particle world lines. Then the Boltzmann equation in general relativity is derived using the Klimontovich random function. The particularly important case of scattering of two particles is considered, for which a collision integral is derived. Quantum corrections to the collision integral are also considered. The relation between KT and the radiative transfer theory is outlined. The connection between collision integrals and cross section is presented. Finally, the notion of relaxation time is introduced.

The role of averaging as one of the fundamental instruments of KT is discussed in Chapter 3. While in nonrelativistic physics, averaging appears to be straightforward, it does not prove to be so in relativistic generalization, where time and space averaging, considered separately, are not Lorentz invariant, because space and time are no longer absolute. Within general relativity, averaging is an even more complicated issue, with a fully covariant formulation of KT still missing.

In Chapter 4, equations of relativistic hydrodynamics are derived from the Boltzmann equation. It is shown that microscopic conservation of energy and momentum at each interaction between particles implies the existence of conservation laws for macroscopic quantities such as four-current and energy-momentum tensor. Then $\mathcal{H}$-theorem is proved and conditions for local thermodynamic equilibrium are formulated. The one-particle distribution function as well as some useful macroscopic quantities in equilibrium, such as density, pressure, and entropy, are obtained. The generalized continuity equation for nonequilibrium systems is also derived.

In Chapter 5 the derivation of the Bogolyubov-Born-Green-Kirkwood-Yvon hierarchy for relativistic plasma is presented. The basic idea in this approach is that any many-body system can be characterized by the set of equations of motion under the given interaction. Applying averaging to Klimontovich distribution functions, one can derive the chain of equations for many particle distribution functions. In order to obtain tractable kinetic equations, this hierarchy can then be truncated at a certain level, using expansion in small parameters or other physical considerations. In this way Maxwell-Vlasov and Belyaev-Budker equations are derived. In the last chapter of Part I, kinetic properties of dilute gas and plasma are considered. In the relativistic domain, many qualitatively new phenomena, such as particle-antiparticle production, occur in plasma. To understand these phenomena, as well as to provide the physical foundations for the derivation of the Boltzmann and Vlasov equations, it is very useful to discuss the characteristic quantities in both gases and plasmas. In particular, the plasma parameter, Coulomb logarithm, Debye length, degeneracy parameter, and Knudsen number are introduced.

Physics is an empirical science, and all its concepts are verified in experiments. By analogy, computer simulation of a physical process can be considered as numerical experimentation with all the necessary methodology, setup, and data analysis. Owing to physical limitations for both the computer memory and CPU or GPU speed, such simulations have limited space and time resolution for the simulated problems, very much like traditional physical instruments in experiments. In comparison with the physical setup the computer and the numerical method can be considered as a universal tools. During the last several decades the power of computers increased exponentially with the doubling of computing power every 18 months, following Moore's law. This provides conditions for the unprecedented development of numerical techniques and their application to various physical problems.

Numerical methods applied in relativistic KT and in hydrodynamics are discussed in Part II. In Chapter 7 an informal introduction to computational physics is presented. Although an analytic solution completely describes the problem, it is not available for most nonlinear problems. New results in modern physics are often obtained in numerical simulations. The chapter describes standard types of equations of classical mathematical physics and existing methods of their solution, focusing mainly on finite difference techniques. Systems of ordinary differential equations and problems of linear algebra are considered as well. Stability and accuracy of numerical schemes are addressed, providing the convergence of the numerical solution to the exact solution of the underlying differential equation.

In Chapter 8, numerical integration of Boltzmann equations is discussed. The approach is illustrated by the finite difference scheme on a fixed grid in the 4D phase space, and it is based on the method of lines. This method reduces the integration of partial differential equations to the solution of the system of ordinary differential equations. The latter are solved by the implicit Gear's method. The method is suitable for both optically thick and optically thin regions and is especially useful for describing neutrino transport in gravitational collapse. The Monte Carlo approach for solution of the Boltzmann equation is discussed as well. This approach is universally applied when the optical depth is small, especially in multidimensional problems.

Finally, Chapter 9 describes classical shock-capturing hydrodynamic transport in multidimensional space. The modern high-order Godunov-type methods are described. For multicomponent systems, kinetic Boltzmann equations in 7D phase space are replaced by hydrodynamics with diffusion and flux limiters in 5D phase space. The interpolation of fluxes of spectral energy density in the intermediate case between the transparent (free flow) and the nontransparent (diffusion or heat conduction) cases is introduced. A special relativistic Riemann solver is also discussed. The last section of the chapter briefly describes smooth particle hydrodynamics (SPH) and particle-in-cell (PIC) methods. Such particle-based simulations are especially useful in describing advection of a smooth flow. The common idea in this chapter is the multidimensional hydrodynamics and explicit methods for advection.

In Part III, applications of relativistic KT in astrophysics and cosmology are considered. In Chapter 10, one of the most important domains of application of relativistic KT, the theory of waves in relativistic plasma, formulated in a gauge-invariant fashion, is presented. After a brief introduction to this theory, several important applications, such as Landau damping and relativistic plasma instabilities, are considered. Collisionless shocks and their relevance to astrophysics are discussed. In Chapter 11, relaxation of nonequilibrium optically thick relativistic plasma is discussed. Collision integrals are represented as integrals over matrix elements, provided by quantum electrodynamics, describing various two-particle interactions between photons, electrons, positrons, and protons. Collision integrals for three-particle interactions are also introduced. Then a theory of thermalization, including the concepts of kinetic and thermal equilibria, is presented. Time scales for relaxation toward thermal equilibrium as functions of the total energy density and baryonic loading are reported. At the end of the chapter, a dynamical kinetic description for a mildly relativistic plasma ball is presented, including its radiation properties.

Chapter 12 is dedicated to discussion of kinetic effects related to pair creation out of a vacuum in strong electromagnetic fields. Nonlinear effects relevant for ultra-intense lasers are briefly discussed. Then the entire dynamics of energy conversion from an initial strong electric field, ending up with thermalized optically thick electron-positron-photon plasma, is studied. It is crucial that pair creation involves back reaction of pairs onto an external field. Accounting for such back reaction is imperative in this problem. As an application, emission of an electron-positron pair wind from a hot bare quark star is considered.

In Chapter 13, some essential aspects of Compton scattering are discussed and various processes in which this scattering plays an important role are illustrated. In particular, one of the most important astrophysical implications, the Sunyaev-Zeldovich effect, is addressed. The Kompaneets equation is derived. The theories of comptonization in static and relativistically moving media are reviewed. Photospheric emission from relativistic outflows is also considered.

In Chapter 14, kinetic properties of self-gravitating systems are discussed and contrasted with kinetic properties of gases and plasmas discussed in previous chapters. Here the Lorentz-invariant formulation is abandoned in favor of clarity and simplicity of presentation. First, Boltzmann equations are derived out of the Bogolyubov-Born-Green-Kirkwood-Yvon hierarchy under different approximations. Then relativistic theory of gravitational instability on the kinetic level is briefly reviewed. Collisionless relaxation and quasi-stationary states are also discussed. Finally, self-gravitating systems in equilibrium and their instability are addressed. In the last chapter, Chapter 15, an example of accurate neutrino treatment in a spherically symmetric collapse is given. The role of multidimensional effects is discussed. These results are of interest for the multidimensional models with large-scale convection as well as for the ongoing experimental search for neutrinos from supernovae.

In the Landau and Lifshitz course of theoretical physics, volume 10, "*Physical Kinetics*," is the last. Students are indeed expected to master all branches of physics before proceeding to this subject. Similarly, it is expected that graduate students in physics and astrophysics who wish to get acquainted with relativistic KT have already learned both special and general relativity and cosmology as well as quantum electrodynamics. Only with this broad and solid background will it be possible for students to make their way, employing numerous techniques and methods, to the applications of relativistic KT and find novel specific problems to be addressed and, eventually, solved. We offer in this book our vision of the foundations, numerical methods, and vast series of applications of the modern relativistic kinetic theory.

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Part I

Theoretical Foundations

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1
Basic Concepts

In this chapter the evolution of basic concepts of KT, such as phase space and distribution functions, from nonrelativistic to special and general relativistic frameworks is outlined. The relation between mechanical and kinetic pictures is presented. The physical meaning of the one-particle distribution function is given, and its Lorentz invariance is demonstrated. Then the most useful macroscopic quantities, such as four-current, entropy four-flux, energy-momentum tensor, and hydrodynamic velocity, are obtained. These concepts are essential to proceed with the formulation of kinetic equations as well as to understand the relation between kinetic theory and hydrodynamics discussed in following chapters.

1.1 Nonrelativistic Kinetic Theory

In classical (nonrelativistic) mechanics a complete description of a system composed of N interacting particles is given by their N equations of motion. In nonrelativistic KT one deals with a space of positions and velocities of these particles, called *configuration space* or the space of canonical variables: positions and momenta of particles, called the *phase space* $\mathcal{M}$. Often this mechanical description can be formulated in the language of Hamilton equations, and then an equivalent description of the system is given by a function $F(\Gamma, t)$ of time and $6N$ independent variables, defined on $\mathcal{M}$. An equation can be formulated for this function, called the *Liouville equation*, that can be written in an apparently very simple form:

$$\frac{dF(\Gamma)}{dt} = 0, \tag{1.1}$$

where the derivative is over time. Its complexity, however, is equivalent to the complexity of the original N-body problem, and in the majority of cases, it cannot be addressed directly.

A tremendous simplification occurs for such systems, where N is very large. One may define the s-particle distribution function (DF) of states depending on $6s$ variables with $1 \leq s \leq N$ and time by integrating out the remaining $6(N - s)$ degrees of freedom in $F(\Gamma, t)$. The hierarchy of integro-differential equations, which connects the s-particle DFs with the $s + 1$-particle ones, called the *Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy*, is obtained in this way. Among these s-particle DFs, the one-particle DF plays a central role in KT, as it describes the probability of finding the particle in a state with momentum in the range $(\mathbf{p}, \mathbf{p} + d^3p)$ and position in the range $(\mathbf{x}, \mathbf{x} + d^3x)$ at the moment t. The s-particle DFs describe joint probabilities, i.e., particle correlations.

Formally, this hierarchy can be truncated at a given level (usually at $s = 1, 2$) by specifying the functional form of the $s + 1$-particle DF. This is the way kinetic equations for such systems as gases or plasmas were derived out of this hierarchy, and it is called the *Bogolyubov method*, after his monograph [11]. The power of Bogolyubov's method is in its observation that the truncation of the hierarchy may be justified considering the expansion of the DF either in powers of density (for short-range interactions) or in powers of interaction energy (particularly for Coulomb interactions). Remarkably, these kinetic equations coincided with the ones derived previously on a phenomenological basis by Boltzmann and Landau, respectively. Hence the BBGKY hierarchy allows establishing kinetic equations out of the first principles.

1.2 Special Relativistic Kinetic Theory

At first glance, special relativity brings few modifications to kinetic theory. Indeed, the usual distribution function appears to be Lorentz invariant, as does the Boltzmann equation. Deep analysis shows, however, that conceptual changes are required.

First, the theory must be consistent with the existence of the limiting speed, the speed of light c. The first attempt to adopt special relativistic treatment within KT is due to Jüttner back in 1911 [4], who established the equilibrium DF in the form consistent with special relativity.

Second, the whole theory must be proven Lorentz invariant. It took some time to formulate the problem and to prove the Lorentz invariance of the one-particle DF. For the final settlement of the question, see the monograph [2] and more recent paper [12].

Third, the meaning of initial data and dynamics has to be reconsidered, following the revision of the concepts of space and time in special relativity. As one of the consequences of the modifications mentioned earlier, the Liouville equation (1.1) must be reformulated [13].