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978-1-107-04752-5 - A Primer on the Dirichlet Space

Omar El-Fallah, Karim Kellay, Javad Mashregi and Thomas Ransford

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