
Index of notation

dA	area Lebesgue measure, 1
$A(\mathbb{D})$	disk algebra, 3
$A(\mathbb{D}_e)$	disk algebra on the exterior of the unit disk, 170
$A^n(\mathbb{D})$	functions such that $f^{(n)}$ belongs to the disk algebra, 64
\mathbb{A}	annulus $\{z : 1 < z < 2\}$, 29
$A \circ B$	Hadamard product of A and B , 190
$\text{Aut}(\mathbb{D})$	group of automorphisms of \mathbb{D} , 8
\mathcal{B}	Bergman space, 98
\mathcal{B}_e	Bergman space on the exterior of the unit disk, 167
$B(x, r)$	closed ball of center x and radius r , 21
$c(\cdot)$	logarithmic capacity, 24
$c^*(\cdot)$	outer logarithmic capacity, 24
$c_\alpha(\cdot)$	Riesz capacity of degree α , 27
$c_K(\cdot)$	capacity with respect to kernel K , 16
$c_K^*(\cdot)$	outer capacity with respect to kernel K , 18
Cg	Cauchy transform of g , 29
$\widetilde{C}g$	maximal Cauchy transform of g , 31
$C\mu$	Cauchy transform of measure μ , 183
\mathbb{C}	complex plane, 1
\mathbb{C}_∞	Riemann sphere, 167
C_ϕ	operator of composition with ϕ , 96
\mathbb{D}	open unit disk, 1
\mathbb{D}_e	exterior of the closed unit disk, 167
diam	diameter, 16
dist	distance, 56
\mathcal{D}	Dirichlet space, 1
$\mathcal{D}(f)$	Dirichlet integral of f , 1
$\langle \cdot, \cdot \rangle_{\mathcal{D}}$	Dirichlet inner product of f, g , 2
$\ f\ _{\mathcal{D}}$	Dirichlet norm of f , 2
$[f]_{\mathcal{D}}$	closed invariant subspace of \mathcal{D} generated by f , 146
$[f]_{\mathcal{D}}^\perp$	see Definition 9.5.5, 168
\mathcal{D}_E	the set of $f \in \mathcal{D}$ such that $f^* = 0$ q.e. on E ., 152
\mathcal{D}_w	weighted Dirichlet space, 11

\mathcal{D}_α	weighted Dirichlet space with power weight, 13
\mathcal{D}_μ	harmonically weighted Dirichlet space, 109
$\mathcal{D}_\mu(f)$	harmonically weighted Dirichlet integral of f , 109
$[f]_{\mathcal{D}_\mu}$	closed invariant subspace of \mathcal{D}_μ generated by f , 146
\mathcal{D}_ζ	local Dirichlet space at ζ , 109
$\mathcal{D}_\zeta(f)$	local Dirichlet integral of f at ζ , 109
$ E $	arclength measure of E , 25
E_t	the set of points at a distance at most t from E , 42
$f^*(\zeta)$	radial limit of f at ζ , 182
$f_i f_o$	inner-outer factorization of f , 184
f_r	r -dilation of f , 115
$f_{w,E}$	distance function corresponding to w, E , 160
$f \vee g$	outer function determined by $\max\{ f^* , g^* \}$, 122
$f \wedge g$	outer function determined by $\min\{ f^* , g^* \}$, 122
$G(x_1, \dots, x_n)$	determinant of the Gram matrix of x_1, \dots, x_n , 56
$\mathcal{H} \ominus \mathcal{K}$	orthogonal complement of \mathcal{K} in \mathcal{H} , 87
$\text{Hol}(\mathbb{D})$	holomorphic functions on \mathbb{D} , 1
$\text{Hol}_{\mathcal{B}_e, \mathcal{N}^+}(\mathbb{C}_\infty \setminus E)$	see Definition 9.6.1, 171
H^p	Hardy space, 181
H^2	the Hardy space, 182
H^∞	bounded holomorphic functions on the unit disk, 181
$I_K(\mu)$	energy of μ with respect to kernel K , 16
$K\mu$	potential of μ with respect to kernel K , 15
k_w	reproducing kernel for \mathcal{D} at w , 5
ℓ^2	square summable sequences, 2
$L^2(\mathbb{A})$	square-integrable functions on the annulus \mathbb{A} , 29
$\text{Lat}(T, \mathcal{H})$	T -invariant closed subspaces of \mathcal{H} , 140
$\log^+(x)$	maximum of $\log x$ and zero, 24
$\mathcal{M}(\mathcal{D})$	multiplier algebra of \mathcal{D} , 71
Mh	Hardy–Littlewood maximal function of h , 187
M_h	operator of multiplication by h , 72
M_z	shift operator, 132
$\widehat{\mu}(k)$	Fourier coefficients of μ , 25
\mathcal{N}	Nevanlinna class, 181
\mathcal{N}^+	Smirnov class, 185
N_F	covering number of F , 21
n_ϕ	counting function of ϕ , 96
$\mathcal{P}(F)$	Borel probability measures on F , 16
$P\mu$	Poisson integral of μ , 108
q.e.	quasi-everywhere, 31
$\rho_-(B)$	lower density of B , 194
$S(I)$	Carleson box corresponding to I , 78
$\text{supp } \mu$	closed support of μ , 19
\mathbb{T}	unit circle, 8
\widetilde{u}	increasing regularization of u , 193
W^+	analytic Wiener algebra, 3

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Index

[More information](#)

Index

- Adams, D. R., 48
 Agler, J., 14, 91, 92, 145
 Aleman, A., 130, 131, 145
 algebra, 6, 7, 12–14, 71, 110, 142, 148, 181
 Banach algebra, 6, 12, 71–73, 168, 170
 disk algebra, 3, **62**, 168, 179
 on exterior of disk, 170
 multiplier algebra, **71**, 84
 quotient algebra, 169
 subalgebra, 185
 Wiener algebra, 3
 analytic operator, **135**, 136, 137, 140, 141
 approach region, 49
 non-tangential, 9, 182
 oricyclic, 9, 45, 111
 tangential, 9, 11, 45
 exponentially, 29, **45–48**
 widest possible, 48, 49, **60**
 Arazy, J., 106
 Arcozzi, N., 14, 91
 automorphism of unit disk, **8**, 93

 Baire category theorem, 177
 Banach space, 181
 Banach–Steinhaus theorem, 5, 117
 Bergman space, **98**, 107, 167, 178
 Bergman–Smirnov exceptional set, 171–179
 Bessel’s inequality, 30
 Beurling’s theorem
 on boundary limits, 29, **31–35**, 48
 converse to, 39
 on invariant subspaces, 132, 143, **184**
 Beurling, A., 14, 48, 70, 132, 179
 Bishop, C. J., 92
 Blaschke condition, 50, **50**, 51–54, 89

 Blaschke product, 50–54, 67, 89, 90, 127,
 129, 130, 134, 151, **184**, 185
 Blaschke sequence, **50**, 61, 67, 69
 Bøe, B., 92
 Bogdan, K., 70
 Borichev, A., 48
 Bourdon, P. S., 145
 Brown, L., 49, 69, 91, 154, 179
 Brown–Shields conjecture, **154–159**, 178
 partial solution to, 154, **156**

 Cantor set, **21–23**, 63, 83, 153, 157, 159, 166
 Cantor–Lebesgue measure, 83
 capacitable set, 18
 capacity, 15, **16**, 34, 157
 capacity of Cantor set, 23, 26–28
 capacity of countable set, 19
 estimate in terms of diameter, 16
 estimate in terms of Lebesgue measure, 25
 inner capacity, 18
 is upper semicontinuous, 17
 logarithmic capacity, *see* logarithmic
 capacity
 outer capacity, 18
 Riesz capacity, *see* Riesz capacity
 strong-type inequality for, *see* strong-type
 inequality for capacity
 weak-type inequality for, *see* weak-type
 inequality for capacity
 Carleson box, **78**, 98
 Carleson measure, 37, 76, 78, 91, 131
 for Bergman space, **98**, 107
 for Dirichlet space, **76–84**, 92
 characterization of, **80**, 91
 dual formulation of, 76

- necessary condition for, 79
- sufficient condition for, 77, 79
- Carleson set, 61, **64**, 67, 68, 70, 153, 155, 158, 159, 165
- Carleson's formula, **118**, **129**, 155, 160
- Carleson, L., 49, 69, 70, 76, 91, 117, 125, 131, 179
- Carlsson, M., 145
- Cauchy transform, **29–31**, 37, 48
 - extension to unit circle, 31
 - maximal, 31
 - of a measure, 170, 172, 178, **183**
- Cauchy's integral formula, 177
- Cauchy's theorem, 158
- Cauchy–Schwarz inequality, 4, 10, 13, 22, 29, 34, 46, 63, 77, 143, 168, 174
- Caughran, J. G., 69, 70
- cellular-indecomposable operator, 145
- Chang, S.-Y. A., 48
- change-of-variable formula, 97
- character, 169
- Choquet's theorem, 18
- closed graph theorem, 14, 72, 76, 96
- Cohn, W., 49, 69, 179
- composition operator, 93, 96–107
 - boundedness of, 100
 - compactness of, **102**, 107
 - Hilbert–Schmidt, 104, 107
 - in Schatten class, 107
- concave function, 24, 91, 160, 164
- contraction, 88, 136, 137, 170
- convergence
 - in norm, 5, 43, 94, 103, 117, 141
 - locally uniform, 41, 42, 44, 60, 90, 111, 114
 - pointwise, 141, 147, 149, 156, 163
 - uniform, 117
 - weak, 94, 119, 136, 137, 140, 141, 147
 - weak*, 17–19
- convex function, 24, 45, 122
- corona problem, 76, 91
- countable set
 - as Bergman–Smirnov exceptional set, 176, 179
 - as boundary zero set, 61
 - as set of capacity zero, 19
- counting function, **96**, 107
- covering number, **21**, 166
- Cowen, C. C., 107
- Cowen, M. J., 145
- cyclic
 - function, **146**, 146–171, 176, 185
 - operator, **135**, 137
 - subspace, 143, 146, 151
 - vector, 137, 139
- de Branges–Rovnyak space, 131
- dilation, 115
- Dirichlet integral, **1**, 108, 109
 - Carleson's formula for, **118**, **129**, 155, 160
 - conformal invariance of, 7
 - Douglas' formula for, **8–11**, 113, 117
 - formula in terms of Taylor coefficients, 1
 - interpretation as area, 7
 - local, *see* local Dirichlet integral
 - of distance function, 160
 - of outer function, 118
 - weighted, **11**, 108
 - harmonically weighted, 109
- Dirichlet space, **1–3**
 - characterization via Möbius invariance, 93
 - inner product, 2
 - local, **109**, 131
 - norm, 2
 - representation formula, 29, **30**, 45, 48
 - reproducing kernel, *see* reproducing kernel
 - weighted, **11–14**, 35, 39, 91
 - harmonically weighted, *see* harmonically weighted Dirichlet space
- disk algebra, 3, **62**, 168, 179
 - on exterior of disk, 170
- distance function, 118, **159–166**, 179
 - Dirichlet integral of, 160
- dominated convergence theorem, 25, 33, 63, 119, 120, 157, 158
- Douglas' formula, **8–11**, 113, 117
 - local Douglas formula, 110, **113**, 118, 131
- Douglas, J., 14
- Douglas, R. G., 145
- dual, 67, 76, 166, 168
- Dyn'kin, E. M., 48
- energy, **16**, 28, 62, 172
 - formula in terms of Fourier coefficients, 25
- equilibrium measure, **19–21**, 43, 81
 - potential of, 19
 - uniqueness of, **26**, 27
- factorization
 - canonical, 184, 185
 - inner-outer, 125, 126, 129, 141, 143, 147–149, 151, **184**

- Fatou's lemma, 12, 20, 25, 32, 40, 60, 116, 120, 128
- Fatou's theorem, 182
- Fejér–Riesz theorem, 138
- Fields medal, 14
- Fisher, S. D., 106
- Frostman's theorem, 19
- Frostman, O., 27
- Fubini's theorem, 22, 34, 100, 110
- fusion lemma, 155
- Gallardo-Gutiérrez, E. A., 107
- Gelfand theory, 169
- González, M. J., 107
- Gram matrix, 56
- Green's function, 28
- Hadamard product, 57, 88, **190**
- Hadamard's inequality, 192
- Hahn–Banach theorem, 166, 168
- Hansson, K., 48
- Hardy space, 2, 3, 8, 13, 29, 50, 71, 73, 76, 78, 89, 132, 148, 168, **181–186**
- Hardy's inequality, 3, **182**
- Hardy–Littlewood maximal function, 46, **187–188**
- harmonic function, 13, 14, 41, 44, 108, 116, 183
- subharmonic function, 158
- superharmonic function, 91
- harmonically weighted Dirichlet integral, 109
- harmonically weighted Dirichlet space, **108–110**, 132, 146
- polynomials dense in, **116**, 137
- Hastings, W. W., 107
- Hausdorff–Young inequality, 3
- Havin, V. P., 70
- Hedberg, L. I., 48
- Hedenmalm, H., 178, 179
- Hilbert space, 2–4, 11, 14, 29, 43, 55–57, 72, 85, 88, 93, 94, 104, 109, 111, 135, 139, 140, 145, 166, 167, 182
- Hilbert–Schmidt operator, 104
- Hölder's inequality, 3
- Hruščev, S. V., 69
- ideal, 168
- increasing regularization, **193–195**, 196
- inner factor, 125, 151, 154
- inner function, 125–130, 132, 134, 143, 147, 170, **183–185**, 185
- local Dirichlet integral of, 127–128
- singular inner function, 127, **184**, 185
- inner product, 2, 5, 30, 56, 166
- semi-, 2, 138
- inner-outer factorization, 125, 126, 129, 141, 143, 147–149, 151, **184**
- interpolating sequence, 92
- invariant subspace, 108, 132, **140–144**, 146, 148–150, 152, 184, 185
- generated by a set, 146
- invertible, 150
- isometry, 2, 112, 132, 138–140
- 2-isometry, 130, 132, **135**, 136–138, 140, 141, 145
- Jensen's inequality, 122
- Julia–Carathéodory theorem, 131
- Kahane, J.-P., 180
- Kaluza, T., 69
- kernel, 15
- convex, 24
- Koebe function, 183
- Koosis, P., 69
- Korenblum, B. I., 179
- lattice operations, 122
- applied to outer functions, 122
- Lebesgue measure, 1, 25, 26, 29, 36, 61, 64, 81, 109, 110, 113, 114, 118, 128, 129, 144, 146, 151, 155, 159, 160, 182, 184
- Cantor–Lebesgue measure, 83
- Lefèvre, P., 107
- Li, D., 107
- linear functional, 138, 168
- continuous, 4, 14, 72, 94, 170
- multiplicative, 169
- positive, 138
- local Dirichlet integral, **109**, 131
- of inner function, 127–128
- Douglas formula for, 110, **113**, 118, 131
- formula in terms of Taylor coefficients, **113**, 115
- of outer function, 118
- Richter–Sundberg formula for, **118**, 121, **129**
- local Dirichlet space, **109**, 111, 131
- local Douglas formula, 110, **113**, 118, 131
- logarithmic capacity, **24–27**, 28, 29, 31, 33, 34, 40, 61, 62, 69, 80, 83, 105, 106, 146, 152, 159, 166, 171, 178, 179

- lower density, **194**, 195
Luecking, D. H., 107
- MacCluer, B. D., 107
Malliavin, P., 70
Marshall, D. E., 48, 92
maximum principle, 12, 158, 159, 173, 174
 for potentials, 24
 Smirnov's, 10, 113, 121, 175, **186**
 two-sided, 172
Maz'ya, V. G., 48, 70
McCarthy, J. E., 14, 92
Minkowski's inequality, 7
Möbius invariance, 8, 51, 93–96
 characterization of Dirichlet space via, 93
Möbius transformation, 8, 12, 174
monotone convergence theorem, 17
Monterie, M. A., 28
Morera's theorem, 158
multiplication operator, **72**, 84, 101
multiplier, 7, **71–75**, 133, 143, 152
 boundedness of, 73
 characterization of, 74, 76
 zero set of, 89–91
multiplier algebra, **71**, 84
- Nagel, A., 49, 69
Nevanlinna class, **181**, 185
non-tangential limit, 9, 29, 31, 35, **182**
norm, 2, 3, 6, 11, 12, 14, 42, 71, 88, 94, 109, 137, 168, 181
 attainment of, 85, 86
 algebra norm, 7
 multiplier norm, **72**, 89
 operator norm, 96
 semi-norm, 2
 sup-norm, 170
normal family, 60, 90
- Ohtsuka, M., 28
Olin, R. F., 145
Oppenheim's inequality, 57, **191**, 192
orthogonal basis, 111, 113
orthogonal projection, 55, 59, 85, 87, 136
orthonormal basis, 104, 105
outer factor, 151, 156, 158, 186
outer function, 10, 65, 113, 117–127, 143, 144, 147–150, 152–160, 166, 170, 171, 178, **183–185**, 185
 Dirichlet integral of, 118
 lattice operations applied to, 122
- local Dirichlet integral of, 118
- parallelogram identity, 116
Parrott's lemma, 85
Parrott, S., 91
Parseval's formula, 2, 9, 113, 167
Peller, V. V., 69
Pick interpolation, 5, **84–89**, 89, 131
Pick property, **88**, 89
 complete Pick property, 91
Plateau problem, 14
Poisson integral, 8, 13, 41, 44, **108**, 116, 120, 122, 182, 186
positive definite matrix, 55–57, **189–192**
 positive semi-definite, 55–59, 84, 85, 87, 88, **189–192**
potential, **15**, 20, 28
 maximum principle for, 24
 of equilibrium measure, 19
- quasi-everywhere, **31**, 34, 105, 152
Queffelec, H., 107
- radial limit, 8–10, 48, 105, 110, 117, 118, 125, 132, 174, **182**, 185
regular point, **127**, 128, 130
regularization lemma, 162, 163, **195**
representation formula, 29, **30**, 45, 48
reproducing kernel, **4–5**, 14, 55, 57, 72, 76, 79, 84, 88, 89, 92, 101
Richter, S., 69, 117, 125, 130, 131, 145, 179
Richter–Sundberg formula, **118**, 121, **129**
 complement to, 121
Riemann mapping, 61, 62
Riemann sphere, 167
Riesz capacity, 15, **27**, 35, 39, 70, 106
Riesz representation theorem
 for continuous functions, 138
 for Hilbert space, 4
rising-sun lemma, 162, **193**
Rochberg, R., 14, 91
Rodríguez-Piazza, L., 107
Ross, W. T., 14, 69
Rudin, W., 49, 69
- Salem, R., 180
Sarason, D., 131
Sawyer, E. T., 14, 91
Schur product theorem, 88, **190**
Schur's inequality, 192

Cambridge University Press

978-1-107-04752-5 - A Primer on the Dirichlet Space

Omar El-Fallah, Karim Kellay, Javad Mashreghi and Thomas Ransford

Index

[More information](#)

Index

211

- Schwarz inequality, *see* Cauchy–Schwarz inequality
- Seip, K., 92
- sesquilinear form, 137, 139
- Shapiro, H. S., 69
- Shapiro, J. H., 49, 69, 70, 107, 179
- Shields, A. L., 69, 91, 145, 154, 178, 179
- shift operator, **132–135**, 145, 184
characterization of, **135–140**, 145
- Shimorin, S. M., 91
- singular inner function, 127, **184**, 185
- Smirnov class, 130, 170, **185**
characterization of, 185
- Smirnov’s maximum principle, 10, 113, 121, 175, **186**
- spectral theorem, 189
- Stegenga, D. A., 91
- Stone–Weierstrass theorem, 17
- strong-type inequality for capacity, 29, **35–39**, 43, 48, 82
in weighted Dirichlet space, 39
sharpness of, 42
- subharmonic function, 158
- Sundberg, C., 69, 92, 117, 125, 131, 145, 179
- superharmonic function, 91
- Sylvester’s criterion, **189**, 191
- Taylor, B. A., 180
- Taylor, G. D., 91
- Thomson, J. E., 145
- transfinite diameter, 28
- trigonometric polynomial, 138
- uniqueness set, **50–54**, 67, 69
on boundary, **62**, 70
- unitary matrix, 189
- unitary operator, 139, 140, 142, 143
- univalent function, 154, 182
- wandering subspace theorem, 136
- weak-type inequality
for capacity, **35–39**, 48, 105, 152
in weighted Dirichlet space, 39, 106
for Hardy–Littlewood maximal function, 46, **187**
- weight
harmonic weight, 13, 108
power weight, 13
superharmonic weight, 130
- Wick, B. D., 14
- Wiener algebra, 3
- Williams, D. L., 180
- Wynn, A., 91
- zero set, 5, 50, **50–54**, 89, 90, 131, 144, 153, 182
arguments of, 67–70
moduli of, 54–61
of multiplier, 89–91
on boundary, 50, **61–67**, 70, 146, 152–155