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The Riemann Hypothesis for Function Fields

Frobenius Flow and Shift Operators

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CAMBRIDGE
UNIVERSITY PRESS

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107047211

First published 2014

Printed in the United Kingdom by CPI Group Ltd, Croydon CR0 4YY

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-04721-1 Hardback
ISBN 978-1-107-68531-4 Paperback

To my beautiful wife Jena

Cambridge University Press
978-1-107-04721-1 - The Riemann Hypothesis for Function Fields: Frobenius Flow and Shift Operators
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Preface

This book grew out of an attempt to understand the paper [Conn1], in which Alain Connes constructs a beautiful noncommutative space with a view to proving the Riemann hypothesis. That paper is supplemented by Shai Haran's papers [Har2, Har3], which give a similar construction with more details on some of the computations. Connes' proof is explored in Chapter 6, where his method is applied with an aim of proving the Riemann hypothesis for a curve over a finite field (Weil's theorem).

Chapter 5 presents Bombieri's proof [Bom1] of the Riemann hypothesis for curves over a finite field. This chapter is not necessary for Chapter 6, and can be skipped by a reader who is only interested in understanding Connes' approach.

Chapters 1, 2, and 3 provide background. Chapter 1 is an exposition of the theory of valued fields, and in Chapters 2 and 3, we present Tate's thesis [Ta] for curves over a finite field.

There are numerous exercises throughout the book where the reader is asked to work out a detail or explore related material. The exercises that are labelled as 'problems' ask questions that may not have a definite answer.

This book is not primarily about number fields, but occasionally we discuss the connection between number fields and function fields. We have included several diagrams to help the reader create a mental picture of this connection.

The author believes that Connes' approach provides the first truly convincing heuristic argument for the Riemann hypothesis. He also believes that working out this argument for the function field case is the key to getting it to work for the integers. It is therefore not surprising that we do not reach our goal in Chapter 6. This book provides the basis for further research in this direction.

Acknowledgements The research for this book was started at the University of California in Riverside and continued over the years at Rutgers University (New Jersey), Utah Valley State College (now Utah Valley University), the Institut des Hautes Études Scientifiques (IHÉS) in Bures-sur-Yvette, France, of which I was a member during February of 2007, and the Georg-August-Universität in Göttingen, Germany, during the author's sabbatical year at the invitation of Professor Dr. Ralf Meyer in 2011.

The material and financial support of the IHÉS and the Department of Mathematics and the School of Science and Health of Utah Valley University is gratefully ackowledged. The generous support from Professor Meyer during the author's sabbatical year is also gratefully acknowledged. I also want to acknowledge the many excellent teachers whom I had at the university of Nijmegen, Riverside, the IHÉS, Rutgers University, and Göttingen. I fondly remember Serge Lang, whose way of doing mathematics has been an example ever since we first met. I miss him. I want to thank the students and professors who gave talks in my seminar in Göttingen when I was there in 2011. That seminar greatly accelerated the evolution of this book.