

Index of notation

blank space, 24	$ $, 74	CRing , 19
gf , 10	$[\ , \]$, 30	D , 4
αF , 37	\otimes , 6	E , 117, 154
$F\alpha$, 37	\times , 16, 68, 109	ev , 148
α_A , 28	\prod , 68, 111	FDVect , 32
$\mathcal{A}(A, B)$, 10	$+$, 68, 127	Field , 46
$\mathcal{A}(A, -)$, 84	Σ , 68, 127	FinSet , 35
$\mathcal{A}(-, A)$, 88	\amalg , 68	Grp , 11
$\mathcal{A}(f, -)$, 88	\coprod , 127	H^A , 84
$\mathcal{A}(-, f)$, 90	\oplus , 110	H_A , 88
$\mathcal{A}(A, D)$, 146	\mathcal{A}/A , 59	H^f , 88
$D(-)(A)$, 148	A/\mathcal{A} , 60	H_f , 90
$\mathcal{B}^{\mathcal{A}}$, 30	A/\sim , 70	H^\bullet , 88
B^A , 69, 112, 164	\wedge , 111	H_\bullet , 90
$(f_i)_{i \in I}$, 111	\bigwedge , 111	Hom , 10, 90
A, B, ... (typeface), 118	\vee , 128	Hom , 23
\neg , 24	\bigvee , 128	I , 7
$\bar{}$, 42, 118, 126	$(\)^{-1}$, 12	\lim , 119
\sim , 96	\emptyset , 13	\lim_{\leftarrow} , 126
$\hat{}$, 96, 165	0, 127	\lim_{\rightarrow}
$(\)^*, (\)_\bullet$, 150	1, 1, 10, 18, 30, 112	Mon , 18
$*$, 37	1, 13	\mathbb{N} , 15
V^* , 24	2, 31, 69	\mathcal{O} , 24, 89
f^* , 23, 88	2, 117	ob , 10
f_* , 84, 90	Δ , 50, 73, 142	$(\)^{\text{op}}$, 16
\circ , 10, 18, 30	ε , 51	P , 117
$g \circ -$, 84, 90	η , 51	\mathcal{P} , 55, 69, 89
$- \circ f$, 88	π_1 , 21	P_A , 161
\forall , 3	χ , 69	Ring , 11
$\exists!$, 3	Ab , 18	S^1 , 85
\rightarrow , 10	$(\)_{\text{ab}}$, 45	Set , 11
\hookrightarrow , 6	Bilin , 86	T , 117
\twoheadrightarrow , 99	C , 23	Top , 12
\Rightarrow , 29, 59, 60	CAT , 18	Top* , 21
\dashv , 41	Cat , 77	Toph , 17
\perp , \top , 49	Cone , 142	Toph* , 85
\cong , 12, 26, 32	CptHff , 122	Vect $_k$, 12
\simeq , 34		$\mathbb{Z}[x]$, 8
\leq , 15, 74		

Index

- abelianization, 45
- adjoint functor theorems, 159–164
 - general, 162, 171–173
 - special, 163
- adjunction, 41
 - composition of adjunctions, 49
 - vs. equivalence, 55
 - fixed points of, 57
 - free–forgetful, 43–46
 - via initial objects, 60–63, 100, 101
 - limits preserved in, 158
 - naturality axiom for, 42, 50–51, 91, 101
 - nonexistence of adjoints, 159
 - uniqueness of adjoints, 43, 106
- aerial photography, 87
- algebra, 92
 - for algebraic theory, 46
 - associative, 42–43
- algebraic geometry, 21, 36, 92
- algebraic theory, 46
- algebraic topology, 20
- applied mathematics, 9
- arithmetic, 69, 112, 158, 165
 - cardinal, 163, 168
- arity, 46
- arrow, 10, *see also* map
- associative algebra, 42–43
- associativity, 10, 151
- axiom of choice, 71, 135
- bicycle inner tube, 133
- bilinear, *see* map, bilinear
- black king, 72
- Boolean algebra, 36
- C^* -algebra, 36
- canonical, 33, 39
- Cantor, Georg, 78
 - Cantor’s theorem, 74
 - Cantor–Bernstein theorem, 74
- cardinality, 74, 163, 168
- cartesian closed category, 164–167
- category, 10
 - cartesian closed, 164–167
 - category of categories, 18, 77
 - adjunctions with **Set**, 78, 167
 - comma, *see* comma category
 - complete, 159
 - coslice, 60
 - discrete, 13, 78, 87
 - functor out of, 29, 31, 32
 - drawing of, 13
 - of elements, 154, 156
 - equivalence of categories, 34
 - vs. adjunction, 55
 - essentially small, 76
 - finite, 121
 - isomorphism of categories, 26
 - large, 75
 - locally small, 75, 84
 - monoidal closed, 165
 - one-object, 14–15, *see also* monoid *and* group
 - opposite, 16
 - product of categories, 16, 26, 39
 - slice, *see* slice category
 - slimmed-down, 35
 - small, 75, 118
 - 2-category of categories, 38
 - well-powered, 168
- centre, 26
- characteristic function, 69
- chess, 72

- class, 11, 75
- closure, 55
- cocone, 126, *see also* cone
- codomain, 11
- coequalizer, 128, *see also* equalizer
- cohomology, 24
- colimit, 126, *see also* limit
 - and integration, 151
 - map out of, 147
- collection, 11
- comma category, 59
 - limits in, 172
- commutes, 11
- complete, 159
- component
 - of map into product, 111
 - of natural transformation, 28
- composition, 10
 - horizontal, 37
 - vertical, 37
- computer science, 9, 79, 80
- cone, 118
 - limit, 119
 - as natural transformation, 142
 - set of cones as limit, 146
- connectedness, 156
- contravariant, 22, 90
- coproduct, 127, *see also* sum
- coprojection, 126
- coreflective, 46
- coslice category, 60
- counit, *see* unit and counit
- covariant, 22
- creation of limits, 138–139, 172
- density, 154, 156
- determinant, 29
- diagonal, *see* functor, diagonal
- diagram, 118
 - commutative, 11
 - string, 55
- direct limit, 131
- discrete, *see* category, discrete *and* topological
 - space, discrete
- disjoint union, 68, *see also* set, category of,
 - sums in
- domain, 11
- duality, 16, 35, 132
 - algebra–geometry, 23, 35
 - Gelfand–Naimark, 36
 - Pontryagin, 36
 - principle of, 16, 49
- Stone, 36
 - terminology for, 126
 - for vector spaces, 24, 32
- duck, 104
- Eilenberg, Samuel, 9
- element
 - category of elements, 154, 156
 - as function, 67
 - generalized, 92, 105, 117, 123, 156
 - least, *see* least element
 - of presheaf, 99
 - universal, 100
- embedding, 102
- empty family, 111, 127
- epic, 133, *see also* monic
 - regular, 135
 - split, 135
- epimorphism, 133, *see also* epic
- equalizer, 112, 132
 - map into, 146
 - vs. pullback, 124
 - of sets, 70, 113
- equivalence of categories, 34
 - vs. adjunction, 55
- equivalence relation, 70, 135
 - generated by relation, 128
- equivariant, 29
- essentially small, 76
- essentially surjective on objects, 34
- evaluation, 32, 95, 148
- explicit description, 44, 163
- exponential, 164, *see also* set of functions
 - preserved by Yoneda embedding, 168
- faithful, 25, 27
- family, 68
 - empty, 111, 127
- fibred product, 115, *see also* pullback
- field, 46, 83, 159
- figure, *see* element, generalized
- fixed point, 57, 77
- forgetful, *see* functor, forgetful
- fork, 112
- foundations, 71–73, 80
- Fourier analysis, 36, 78
- free functor, 19
- Fubini's theorem, 151
- full, *see* functor, full *and* subcategory, full
- function
 - characteristic, 69
 - injective, 123
 - intuitive description of, 66

- number of functions, 67
- partial, 64
- set of functions, 47, 69, 164
- surjective, 133
- functor, 17
 - category, 30, 38, 164
 - limits in, 148–153
 - composition of functors, 18
 - contravariant, 22, 90
 - covariant, 22
 - diagonal, 50, 73, 142
 - essentially surjective on objects, 34
 - faithful, 25, 27
 - forgetful, 18
 - left adjoint to, 43, 87, 163
 - preserves limits, 158
 - is representable, 85, 87
 - free, 19
 - full, 25
 - full and faithful, 34, 103
 - identity, 18
 - limit of, 171, 173
 - image of, 25
 - product of functors, 148
 - representable, 84, 89
 - and adjoints, 86, 167
 - colimit of representables, 153–156
 - isomorphism of representables, 104–105
 - limit of representables, 152–153
 - preserves limits, 145–147
 - sum of representables, 156
 - ‘seeing’, 83, 85
 - set-valued, 84
- G -set, 22, 50, 157, *see also* monoid, action of
- general adjoint functor theorem (GAFT), 162, 171–173
- generalized element, *see* element, generalized
- generated equivalence relation, 128
- greatest common divisor, 110
- greatest lower bound, 111
- group, 6, 101, 103, *see also* monoid
 - abelian
 - coequalizer of, 130
 - finite limit of, 123
 - abelianization of, 45
 - action of, 50, 157, *see also* monoid, action of
 - category of groups, 11
 - colimits in, 137
 - epics in, 134
 - equalizers in, 114
 - is not essentially small, 77
 - isomorphisms in, 12
 - limits in, 121, 137–140
 - is locally small, 76
 - monics in, 123
 - free, 19, 44, 63, 163, 168
 - free on monoid, 45
 - fundamental, 7, 21, 85, 131
 - isomorphism of elements of, 39
 - non-homomorphisms of groups, 36
 - normal subgroup of, 135
 - as one-object category, 14
 - opposite, 26
 - order of element of, 85, 105
 - representation of, *see* representation
 - topological, 36
- holomorphic function, 153
- hom-set, 75, 90
- homology, 21
- homotopy, 17, 85, *see also* group, fundamental
- identity, 10
 - as zero-fold composite, 11
- image
 - of functor, 25
 - of homomorphism, 130
 - inverse, *see* inverse image
- inclusion, 6
- indiscrete space, 7, 47
- infimum, 111
- ∞ -category, 38
- initial, *see* object, initial *and* set, weakly initial
- injection, 123
- injective object, 140
- integers, *see* \mathbb{Z}
- interchange law, 38
- intersection, 110, 120
 - as pullback, 116, 130
- inverse, 12
 - image, 57, 89
 - as pullback, 115
 - limit, 120
 - right, 71
- isomorphism, 12
 - of categories, 26
 - and full and faithful functors, 103
 - natural, 31
 - preserved by functors, 26
- join, 128
- Kan extension, 157
- kernel, 6, 8, 114

- Kronecker, Leopold, 78
- large, 75
- least element, 128, 171, 173
 - as meet, 161
- least upper bound, 128
- Lie algebra, 42–43
- limit, 118
 - as adjoint, 144
 - vs. colimit, 132, 147, 161
 - non-commutativity with colimits, 152
 - commutativity with limits, 150, 159
 - computed pointwise, 148
 - cone, 119
 - creation of, 138–139, 172
 - direct, 131
 - finite, 121
 - in functor category, 148–153
 - functoriality of, 139
 - has limits, 121
 - of identity, 171, 173
 - informal usage, 119
 - inverse, 120
 - large, 161–162, 171, 173
 - map between limits, 143
 - map into, 147
 - non-pointwise, 150
 - preservation of, 136
 - by adjoint, 158
 - from products and equalizers, 121
 - from pullbacks and terminal object, 125
 - reflection of, 136
 - as representation of cone functor, 142
 - small, 119, 161–162, 173
 - uniqueness of, 143, 145
- locally small, 75, 84
- loop, 92
- lower bound, 111
- lowest common multiple, 128
- Mac Lane, Saunders, 9
- manifold, 133
- map, 10
 - bilinear, 4, 86, 105, 165
 - need not resemble function, 13
 - order-preserving, 22, 26
- matrix, 40
- meet, 111
- metric space, 91
- minimum, 110
- model, 46
- monic, 123
 - composition of monics, 135
 - pullback of, 125, 135
 - regular, 135
 - split, 135
- monoid, 15
 - action of, 22, 24, 29, 31, 85, *see also* group,
 - action of
 - epics between monoids, 134
 - free group on, 45
 - homomorphism of monoids, 21
 - as one-object category, 15, 29, 35, 77
 - opposite, 26
 - Yoneda lemma for monoids, 99
- monoidal closed category, 165
- monomorphism, 123, *see also* monic
- morphism, 10, *see also* map
- n -category, 38
- natural isomorphism, *see* isomorphism, natural
- natural numbers, 15, 71, 158, *see also*
 - arithmetic
- natural transformation, 28
 - composition of, 30, 36–38
 - identity, 30
- naturally, 32
- object, 10
 - initial, 48, 127
 - as adjoint, 49
 - as limit of identity, 171, 173
 - uniqueness of, 48
 - injective, 140
 - need not resemble set, 13
 - probing of, 81
 - projective, 140
 - set of category, 78, 85
 - terminal, 48, 112, *see also* object, initial
- open subset, 89
- order-preserving, 22, 26
- ordered set, 15, 31
 - adjunction between, 54, 56, 160–162
 - complete small category is, 162, 168
 - vs. preordered set, 16, 167
 - product in, 110–111
 - sum in, 128
 - totally, 39
- partial function, 64
- partially ordered set, 15, *see also* ordered set
- permutation, 39
- pointwise, 23, 148, 165
- polynomial, 21, *see also* ring, polynomial
- poset, 15, *see also* ordered set
- power, 112

series, 153
 set, 69, 89, 110, 128
 predicate, 57
 preimage, *see* inverse image
 preorder, 15, *see also* ordered set
 preservation, *see* limit, preservation of
 presheaf, 24, 50
 category of presheaves
 is cartesian closed, 166
 limits in, 152
 monics and epics in, 156
 slice of, 157
 is topos, 169
 as colimit of representables, 153–156
 element of, 99
 prime numbers, 153
 product, 108, 111
 associativity of, 151
 binary, 111
 commutativity of, 151
 empty, 111
 functoriality of, 139
 informal usage, 109
 map into, 145, 153
 as pullback, 115
 uniqueness of, 109
 projection, 108, 118
 projective object, 140
 pullback, 114
 vs. equalizer, 124
 of monic, 125, 135
 pasting of pullbacks, 124
 square, 115
 pushout, 130, *see also* pullback
 quantifiers as adjoints, 57
 quotient, 132, 134
 of set, 70, 129
 reflection (adjunction), 57
 reflection of limits, 136
 reflective, 46
 relation, 128, *see also* equivalence relation
 representable, *see* functor, representable
 representation
 of functor, 84, 89
 as universal element, 99–102
 of group or monoid
 linear, 22, 50, 157
 regular, 85, 99
 ring, 2
 category of rings, 11
 epics in, 134

is not essentially small, 77
 isomorphisms in, 12
 limits in, 121, 137–140
 is locally small, 76
 monics in, 123
 free, 87
 of functions, 22, 89
 polynomial, 8, 19, 87
 SAFT (special adjoint functor theorem), 163
 sameness, 33–34
 scheme, 21
 section, 71
 sequence, 71, 92
 set
 axiomatization of sets, 79–82
 category of sets, 11, 67
 coequalizers in, 129
 colimits in, 131
 epics in, 133
 equalizers in, 70, 113
 is not essentially small, 76
 isomorphisms in, 12
 limits in, 120
 is locally small, 75
 monics in, 123
 products in, 47, 68, 107, 109
 pushouts in, 130
 sums in, 68, 127
 as topos, 82, 167
 conflicting meaning in ZFC, 80
 definition of, 71–73
 empty, 67, 72
 finite, 35, 76
 of functions, 47, 69, 164
 history, 78–82
 intuitive description of, 66
 one-element, 1, 67, 112
 open, 89
 quotient of, 70, 129
 size of, 74–75
 structurelessness of, 66
 two-element, 69, 89, 167
 -valued functor, 84
 weakly initial, 162, 171–173
 shape
 of diagram, 118
 of generalized element, 92
 sheaf, 24, 167
 Sierpiński space, 93
 simultaneous equations, 21, 113, 122
 slice category, 59

- of presheaf category, 157
 - small, 75, 118, 119
 - special adjoint functor theorem, 163
 - sphere, 132–133
 - Stone–Čech compactification, 164
 - string diagram, 55
 - subcategory
 - full, 25, 103
 - reflective, 46
 - subobject, 125
 - classifier, 167, 168
 - subset, 69, 125
 - sum, 127, *see also* product
 - empty, 127
 - map out of, 147
 - as pushout, 131
 - supremum, 128
 - surface, 132–133
 - surjection, 133
 - tensor product, 5–6, 86, 105, 165
 - terminal, *see* object, terminal
 - thought experiment, 120, 165, 169
 - topological group, 36
 - topological space, 6, 55, *see also* homotopy
 - and group, fundamental
 - category of topological spaces, 12
 - colimits in, 137
 - epics in, 134
 - equalizers in, 113
 - is not essentially small, 77
 - isomorphisms in, 12
 - limits in, 121, 137
 - is locally small, 76
 - products in, 109
 - compact Hausdorff, 122, 164
 - discrete, 4, 47, 87
 - functions on, 22, 24, 89
 - Hausdorff, 134
 - indiscrete, 7, 47
 - open subset of, 89
 - subspace of, 113
 - as topos, 167
 - two-point, 89
 - topos, 82, 167–169
 - total order, 39
 - transpose, 42
 - triangle identities, 52, 56
 - 2-category, 38
 - type, 79–81
 - underlying, 18
 - union, 68, 128
 - as pushout, 130
 - uniqueness, 1, 3, 31, 105
 - of constructions, 10, 17, 28, 42, 94
 - unit and counit, 51
 - adjunction in terms of, 52, 53
 - injectivity of unit, 63
 - unit as initial object, 60–63, 100
 - universal
 - element, 100
 - enveloping algebra, 43
 - property, 1–7
 - determines object uniquely, 2, 5
 - upper bound, 128
 - van Kampen’s theorem, 7, 131
 - variety, 36
 - vector space, 3, 4, 40, *see also* bilinear map
 - category of vector spaces, 12
 - is not cartesian closed, 165
 - colimits in, 137
 - epics in, 134
 - equalizers in, 114
 - is not essentially small, 76
 - limits in, 121, 123, 137–140
 - is locally small, 76
 - monics in, 123
 - products in, 110
 - sums in, 127
 - direct sum of vector spaces, 110, 128
 - dual, 24, 32
 - free, 20, 43, 87
 - unit of, 51, 58, 100
 - functions on, 24
 - of linear maps, 23
 - vertex, 118, 126
 - weakly initial, 162, 171–173
 - well-powered, 168
 - word, 19
 - Yoneda embedding, 90, 102–103
 - does not preserve colimits, 153
 - preserves exponentials, 168
 - preserves limits, 152
 - Yoneda lemma, 94
 - for monoids, 99
 - \mathbb{Z} (integers)
 - as group, 39, 83, 101, 103
 - as ring, 2, 48
 - ZFC (Zermelo–Fraenkel with choice), 79–82