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BASIC CATEGORY THEORY

At the heart of this short introduction to category theory is the idea of a universal property, important throughout mathematics. After an introductory chapter giving the basic definitions, separate chapters explain three ways of expressing universal properties: via adjoint functors, representable functors and limits. A final chapter ties all three together.

The book is suitable for use in courses or for independent study. Assuming relatively little mathematical background, it is ideal for beginning graduate students or advanced undergraduates learning category theory for the first time. For each new categorical concept, a generous supply of examples is provided, taken from different parts of mathematics. At points where the leap in abstraction is particularly great (such as the Yoneda lemma), the reader will find careful and extensive explanations. Copious exercises are included.

Tom Leinster has held postdoctoral positions at Cambridge and the Institut des Hautes Études Scientifiques (France), and held an EPSRC Advanced Research Fellowship at the University of Glasgow. He is currently a Chancellor's Fellow at the University of Edinburgh. He is also the author of *Higher Operads, Higher Categories* (Cambridge University Press, 2004), and one of the hosts of the research blog, The *n*-Category Café.

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Basic Category Theory

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Note to the reader

This is not a sophisticated text. In writing it, I have assumed no more mathematical knowledge than might be acquired from an undergraduate degree at an ordinary British university, and I have not assumed that you are used to learning mathematics by reading a book rather than attending lectures. Furthermore, the list of topics covered is deliberately short, omitting all but the most fundamental parts of category theory. A 'further reading' section points to suitable follow-on texts.

There are two things that every reader should know about this book. One concerns the examples, and the other is about the exercises.

Each new concept is illustrated with a generous supply of examples, but it is not necessary to understand them all. In courses I have taught based on earlier versions of this text, probably no student has had the background to understand every example. All that matters is to understand enough examples that you can connect the new concepts with mathematics that you already know.

As for the exercises, I join every other textbook author in exhorting you to do them; but there is a further important point. In subjects such as number theory and combinatorics, some questions are simple to state but extremely hard to answer. Basic category theory is not like that. To understand the question is very nearly to know the answer. In most of the exercises, there is only one possible way to proceed. So, if you are stuck on an exercise, a likely remedy is to go back through each term in the question and make sure that you understand it *in full*. Take your time. Understanding, rather than problem solving, is the main challenge of learning category theory.

Citations such as Mac Lane (1971) refer to the sources listed in 'Further reading'.

This book developed out of master's-level courses taught several times at the University of Glasgow and, before that, at the University of Cambridge. In turn, the Cambridge version was based on Part III courses taught for many

years by Martin Hyland and Peter Johnstone. Although this text is significantly different from any of their courses, I am conscious that certain exercises, lines of development and even turns of phrase have persisted through that long evolution. I would like to record my indebtedness to them, as well as my thanks to François Petit, my past students, the anonymous reviewers, and the staff of Cambridge University Press.