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978-1-107-04407-4 - Actuarial Mathematics for Life Contingent Risks: Second Edition

David C. M. Dickson, Mary R. Hardy and Howard R. Waters

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Actuarial Mathematics for Life Contingent Risks

How can actuaries best equip themselves for the products and risk structures of the future? In this ground-breaking textbook, three leaders in actuarial science give a modern perspective on life contingencies.

The book begins with actuarial models and theory, emphasizing practical applications using computational techniques. The authors then develop a more contemporary outlook, introducing multiple state models, emerging cash flows and embedded options. This expanded edition contains more examples and exercises designed to help with exam preparation as well as developing up-to-date expertise. There are brand new sections and chapters on discrete time Markov processes, on models involving joint lives and on Universal Life insurance and participating traditional insurance.

Balancing rigour with intuition, and emphasizing applications, this textbook is ideal for university courses, for qualified actuaries wishing to renew and update their skills and for individuals preparing for the professional actuarial examinations of the Society of Actuaries or Institute and Faculty of Actuaries. The book covers the entire SOA MLC syllabus and will be especially valuable for students preparing for the new, long answer exam questions.

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SECOND EDITION

DAVID C. M. DICKSON

University of Melbourne

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University of Waterloo, Ontario

HOWARD R. WATERS

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*To
Carolann,
Vivien
and Phelim*

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Preface to the second edition

Life insurance has undergone enormous change in the last two to three decades. New and innovative products have been developed at the same time as we have seen vast increases in computational power. In addition, the field of finance has experienced a revolution in the development of a mathematical theory of options and financial guarantees, first pioneered in the work of Black, Scholes and Merton, and actuaries have come to realize the importance of that work to risk management in actuarial contexts.

In this book we have adapted the traditional approach to the mathematics of life contingent risk to be better adapted to the products, science and technology that are relevant to current and future actuaries, taking into consideration both demographic and financial uncertainty. The material is presented with a certain level of mathematical rigour; we intend for readers to understand the principles involved, rather than to memorize methods or formulae. The reason is that a rigorous approach will prove more useful in the long run than a short-term utilitarian outlook, as theory can be adapted to changing products and technology in ways that techniques, without scientific support, cannot. However, this is a very practical text. The models and techniques presented are versions, a little simplified in parts, of the models and techniques in use by actuaries in the forefront of modern actuarial management.

The first seven chapters set the context for the material, and cover traditional actuarial models and theory of life contingencies, with modern computational techniques integrated throughout, and with an emphasis on the practical context for the survival models and valuation methods presented. Through the focus on realistic contracts and assumptions, we aim to foster a general business awareness in the life insurance context, at the same time as we develop the mathematical tools for risk management in that context.

From Chapter 8, we move into more modern theory and methods.

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In Chapter 8 we introduce multiple state models, which generalize the life–death contingency structure of previous chapters. Using multiple state models allows a single framework for a wide range of insurance, including income replacement insurance where benefits and premiums depend on the health status of the policyholder; critical illness insurance, which pays a benefit on diagnosis of certain serious medical disorders, and some insurance policies which pay additional benefits in the case of accidental death.

In Chapter 9 we apply the models and results from multiple state models to insurance involving two lives, typically domestic partners. It is common for partners to purchase life insurance cover or annuity income products where the benefits depend on both lives, not on a single insured life.

In Chapter 10 we apply the theory developed in the earlier chapters to problems involving pension benefits. Pension mathematics has some specialized concepts, particularly in funding principles, but in general this chapter is an application of the theory in the preceding chapters.

In Chapter 11 we move to a more sophisticated view of interest rate models and interest rate risk. In this chapter we explore the crucially important difference between diversifiable and non-diversifiable risk.

In Chapter 12 we introduce a general algorithm for projecting the emerging surplus of insurance policies, by considering the year-to-year net cash flows. One of the liberating aspects of the computer revolution for actuaries is that we are no longer required to summarize complex benefits in a single actuarial value; we can go much further in projecting the cash flows to see how and when surplus will emerge. This is much richer information that the actuary can use to assess profitability and to better manage portfolio assets and liabilities. In life insurance contexts, the emerging cash flow projection is often called ‘profit testing’.

In Chapter 13 we follow up on the cash flow projections of Chapter 12 to show how profit testing can be used to design and assess products for which policyholders share profits with the insurer. The first type of policy examined is a traditional with-profits policy, where profits are distributed as cash dividends, or as additional life insurance benefit. The second type is the Universal Life policy, which is very popular in North America.

In Chapter 14 we use the emerging cash flow approach to assess equity-linked contracts, where a financial guarantee is commonly part of the contingent benefit. The real risks for such products can only be assessed taking the random variation in potential outcomes into consideration, and we demonstrate this with Monte Carlo simulation of the emerging cash flows.

The products that are explored in Chapter 14 contain financial guarantees embedded in the life contingent benefits. Option theory is the mathematics

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of valuation and risk management of financial guarantees. In Chapter 15 we introduce the fundamental assumptions and results of option theory.

In Chapter 16 we apply option theory to the embedded options of financial guarantees in insurance products. The theory can be used for pricing and for determining appropriate reserves, as well as for assessing profitability.

The material in this book is designed for undergraduate and graduate programmes in actuarial science, for those self-studying for professional actuarial exams, and for practitioners interested in updating their skill set. The content has been designed primarily to prepare readers for practical actuarial work in life insurance and pension funding and valuation. The text covers all the most recent syllabus requirements for the MLC exam of the Society of Actuaries and for the CT5 exam of the UK Institute and Faculty of Actuaries. Some of the topics in this book are not currently covered by those professional exams, and many of the topics that are in the exams are covered in significantly more depth in this book, particularly where we believe the content will be valuable beyond the exams.

Students and other readers should have sufficient background in probability to be able to calculate moments of functions of one or two random variables, and to handle conditional expectations and variances. We assume familiarity with the binomial, uniform, exponential, normal and lognormal distributions. Some of the more important results are reviewed in Appendix A. We also assume that readers have completed an introductory level course in the mathematics of finance, and are aware of the actuarial notation for interest, discount and annuities-certain.

Throughout, we have opted to use examples that liberally call on spreadsheet-style software. Spreadsheets are ubiquitous tools in actuarial practice, and it is natural to use them throughout, allowing us to use more realistic examples, rather than having to simplify for the sake of mathematical tractability. Other software could be used equally effectively, but spreadsheets represent a fairly universal language that is easily accessible. To keep the computation requirements reasonable, we have ensured that every example and exercise can be completed in Microsoft Excel, without needing any VBA code or macros. Readers who have sufficient familiarity to write their own code may find more efficient solutions than those that we have presented, but our principle was that no reader should need to know more than the basic Excel functions and applications. It will be very useful for anyone working through the material of this book to construct their own spreadsheet tables as they work through the first seven chapters, to generate mortality and actuarial functions for a range of mortality models and interest rates. In the worked examples in the text, we have worked with greater accuracy than we record,

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so there will be some differences from rounding when working with intermediate figures.

One of the advantages of spreadsheets is the ease of implementation of numerical integration algorithms. We assume that students are aware of the principles of numerical integration, and we give some of the most useful algorithms in Appendix B.

The material in this book is appropriate for two one-semester courses. The first seven chapters form a fairly traditional basis, and would reasonably constitute a first course. Chapters 8–16 introduce more contemporary material. Chapter 15 may be omitted by readers who have studied an introductory course covering pricing and delta hedging in a Black–Scholes–Merton model. Chapter 10, on pension mathematics, is not required for subsequent chapters, and could be omitted if a single focus on life insurance is preferred.

Changes from the first edition

The major changes are listed here.

- The material on joint life models has been substantially expanded, and placed in a separate chapter. In the first edition, the joint life material was incorporated in Chapter 8.
- The material on profit sharing and Universal Life, in Chapter 13, is new. Some of this has been adapted from the monograph ‘Supplementary Notes for Actuarial Mathematics for Life Contingent Risks’, previously available as a free supplement to the first edition.
- Additional content in Chapter 7 (policy values) covers modified premium valuation and its relationship to deferred acquisition costs and net premium valuation. This content is relevant for any readers who need to understand US valuation methods, and may be omitted by those who do not.
- More short, examination-style questions, which do not require spreadsheets, have been added to the exercises in many of the chapters. The questions are designed to help students prepare for exams as well as develop understanding. To support these questions, we have included some exam-style tables in Appendix D.
- Other, smaller changes include new sections on mortality reduction factors, discrete time Markov chains, and construction of multiple decrement models.

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We are grateful to the Society of Actuaries for permission to reproduce questions from their MLC exams, for which they own copyright. The relevant questions are noted in the text.

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