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David C. M. Dickson, Mary R. Hardy and Howard R. Waters

Excerpt

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1

Introduction to life insurance

1.1 Summary

Actuaries apply scientific principles and techniques from a range of other disciplines to problems involving risk, uncertainty and finance. In this chapter we set the context for the mathematics of later chapters, by describing some of the background to modern actuarial practice in life insurance, followed by a brief description of the major types of life insurance products that are sold in developed insurance markets. Because pension liabilities are similar in many ways to life insurance liabilities, we also describe some common pension benefits. We give examples of the actuarial questions arising from the risk management of these contracts. How to answer such questions, and solve the resulting problems, is the subject of the following chapters.

1.2 Background

The first actuaries were employed by life insurance companies in the early eighteenth century to provide a scientific basis for managing the companies' assets and liabilities. The liabilities depended on the number of deaths occurring amongst the insured lives each year. The modelling of mortality became a topic of both commercial and general scientific interest, and it attracted many significant scientists and mathematicians to actuarial problems, with the result that much of the early work in the field of probability was closely connected with the development of solutions to actuarial problems.

The earliest life insurance policies provided that the policyholder would pay an amount, called the **premium**, to the insurer. If the named life insured died during the year that the contract was in force, the insurer would pay a predetermined lump sum, the **sum insured**, to the policyholder or his or her estate. So, the first life insurance contracts were annual contracts. Each year the premium would increase as the probability of death increased. If the insured life became very ill at the renewal date, the insurance might not be renewed, in which case

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no benefit would be paid on the life's subsequent death. Over a large number of contracts, the premium income each year should approximately match the claims outgo. This method of matching income and outgo annually, with no attempt to smooth or balance the premiums over the years, is called **assessmentism**. This method is still used for group life insurance, where an employer purchases life insurance cover for its employees on a year-to-year basis.

The radical development in the later eighteenth century was the level premium contract. The problem with assessmentism was that the annual increases in premiums discouraged policyholders from renewing their contracts. The level premium policy offered the policyholder the option to lock-in a regular premium, payable perhaps weekly, monthly, quarterly or annually, for a number of years. This was much more popular with policyholders, as they would not be priced out of the insurance contract just when it might be most needed. For the insurer, the attraction of the longer contract was a greater likelihood of the policyholder paying premiums for a longer period. However, a problem for the insurer was that the longer contracts were more complex to model, and offered more financial risk. For these contracts actuarial techniques had to develop beyond the year-to-year modelling of mortality probabilities. In particular, it became necessary to incorporate financial considerations into the modelling of income and outgo. Over a one-year contract, the time value of money is not a critical aspect. Over, say, a 30-year contract, it becomes a very important part of the modelling and management of risk.

Another development in life insurance in the nineteenth century was the concept of **insurable interest**. This was a requirement in law that the person contracting to pay the life insurance premiums should face a financial loss on the death of the insured life – an insurance payout should not leave the beneficiary financially better off than if the insured life had not died. The insurable interest requirement ended the practice where individuals would insure persons (often public figures) with no connection to the purchaser, as a form of gambling. It also, importantly, removed the incentive for a policyholder to hasten the death of the insured life. Subsequently, insurance policies tended to be purchased by the insured life, and in the rest of this book we use the convention that the policyholder who pays the premiums is also the life insured, whose survival or death triggers the payment of the sum insured under the conditions of the contract.

The earliest studies of mortality include life tables constructed by John Graunt and Edmund Halley. A life table summarizes a survival model by specifying the proportion of lives that are expected to survive to each age. Using London mortality data from the early seventeenth century, Graunt proposed, for example, that each new life had a probability of 40% of surviving to age 16, and a probability of 1% of surviving to age 76. Edmund Halley, famous for

1.3 Life insurance and annuity contracts 3

his astronomical calculations, used mortality data from the city of Breslau in the late seventeenth century as the basis for his life table, which, like Graunt's, was constructed by proposing the average ('medium' in Halley's phrase) proportion of survivors to each age from an arbitrary number of births. Halley took the work two steps further. First, he used the table to draw inference about the conditional survival probabilities at intermediate ages. That is, given the probability that a newborn life survives to each subsequent age, it is possible to infer the probability that a life aged, say, 20, will survive to each subsequent age, using the condition that a life aged zero survives to age 20. The second major innovation was that Halley combined the mortality data with an assumption about interest rates to find the value of a whole life annuity at different ages. A whole life annuity is a contract paying a level sum at regular intervals while the named life (the annuitant) is still alive. The calculations in Halley's paper bear a remarkable similarity to some of the work still used by actuaries in pensions and life insurance.

This book continues in the tradition of combining models of mortality with models in finance to develop a framework for pricing and risk management of long-term policies in life insurance. Many of the same techniques are relevant also in pensions mathematics. However, there have been many changes since the first long-term policies of the late eighteenth century.

1.3 Life insurance and annuity contracts
 1.3.1 Introduction

The life insurance and annuity contracts that were the object of study of the early actuaries were very similar to the contracts written up to the 1980s in all the developed insurance markets. Recently, however, the design of life insurance products has radically changed, and the techniques needed to manage these more modern contracts are more complex than ever. The reasons for the changes include:

- Increased interest by the insurers in offering combined savings and insurance products. The original life insurance products offered a payment to indemnify (or offset) the hardship caused by the death of the policyholder. Many modern contracts combine the indemnity concept with an opportunity to invest.
- More powerful computational facilities allow more complex products to be modelled.
- Policyholders have become more sophisticated investors, and require more options in their contracts, allowing them to vary premiums or sums insured, for example.

- More competition has led to insurers creating increasingly complex products in order to attract more business.
- Risk management techniques in financial products have also become increasingly complex, and insurers have offered some benefits, particularly financial guarantees, that require sophisticated techniques from financial engineering to measure and manage the risk.

In the remainder of this section we describe some of the most important modern insurance contracts, which will be used as examples in later chapters. Different countries have different names and types of contracts; we have tried to cover the major contract types in North America, the United Kingdom and Australia.

The basic transaction of life insurance is an exchange; the policyholder pays premiums in return for a later payment from the insurer which is life contingent, by which we mean that it depends on the death or survival or possibly the state of health of the policyholder. We usually use the term ‘**insurance**’ when the benefit is paid as a single lump sum, either on the death of the policyholder or on survival to a predetermined **maturity date**. (In the UK it is common to use the term ‘assurance’ for insurance contracts involving lives, and insurance for contracts involving property.) An **annuity** is a benefit in the form of a regular series of payments, usually conditional on the survival of the policyholder.

1.3.2 Traditional insurance contracts

Term, whole life and endowment insurance are the traditional products, providing cash benefits on death or maturity, usually with predetermined premium and benefit amounts. We describe each in a little more detail here.

Term insurance pays a lump sum benefit on the death of the policyholder, provided death occurs before the end of a specified term. Term insurance allows a policyholder to provide a fixed sum for his or her dependents in the event of the policyholder’s death.

Level term insurance indicates a level sum insured and regular, level premiums.

Decreasing term insurance indicates that the sum insured and (usually) premiums decrease over the term of the contract. Decreasing term insurance is popular in the UK where it is used in conjunction with a home mortgage; if the policyholder dies, the remaining mortgage is paid from the term insurance proceeds.

Renewable term insurance offers the policyholder the option of renewing the policy at the end of the original term, without further evidence of the policyholder’s health status. In North America, Yearly Renewable

1.3 Life insurance and annuity contracts 5

Term (YRT) insurance is common, under which insurability is guaranteed for some fixed period, though the contract is written only for one year at a time.

Convertible term insurance offers the policyholder the option to convert to a whole life or endowment insurance at the end of the original term, without further evidence of the policyholder's health status.

Whole life insurance pays a lump sum benefit on the death of the policyholder whenever it occurs. For regular premium contracts, the premium is often payable only up to some maximum age, such as 80. This avoids the problem that older lives may be less able to pay the premiums.

Endowment insurance offers a lump sum benefit paid either on the death of the policyholder or at the end of a specified term, whichever occurs first. This is a mixture of a term insurance benefit and a savings element. If the policyholder dies, the sum insured is paid just as under term insurance; if the policyholder survives, the sum insured is treated as a maturing investment. Endowment insurance is obsolete in many jurisdictions. Traditional endowment insurance policies are not currently sold in the UK, but there are large portfolios of policies on the books of UK insurers, because until the late 1990s, endowment insurance policies were often used to repay home mortgages. The policyholder (who was the home owner) paid interest on the mortgage loan, and the principal was paid from the proceeds on the endowment insurance, either on the death of the policyholder or at the final mortgage repayment date.

Endowment insurance policies are becoming popular in developing nations, particularly for 'micro-insurance' where the amounts involved are small. It is hard for small investors to achieve good rates of return on investments, because of heavy expense charges. By pooling the death and survival benefits under the endowment contract, the policyholder gains on the investment side from the resulting economies of scale, and from the investment expertise of the insurer.

Participating insurance

Also part of the traditional design of insurance is the division of business into 'participating' or 'par' business, also known as 'with-profit', and 'non-participating', or 'non-par', also known as 'without profit'. Under participating insurance, the profits earned on the invested premiums are shared with the policyholders. In North America, the profit share often takes the form of cash dividends or reduced premiums. In the UK and in Australia the traditional approach is to use the profits to increase the sum insured, through bonuses called '**reversionary bonuses**' and '**terminal bonuses**'. Reversionary bonuses are awarded during the term of the contract; once a reversionary

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bonus is awarded it is guaranteed. Terminal bonuses are awarded when the policy matures, either through the death of the insured, or when an endowment policy reaches the end of the term. Reversionary bonuses may be expressed as a percentage of the total of the previous sum insured plus bonus, or as a percentage of the original sum insured plus a different percentage of the previously declared bonuses. Reversionary and terminal bonuses are determined by the insurer based on the investment performance of the invested premiums.

Participating insurance is used to offer policyholders a savings element with their life insurance. However, the traditional participating contract was designed primarily for the life insurance cover, with the savings aspect a secondary feature.

1.3.3 Modern insurance contracts

In recent years insurers have provided more flexible products that combine the death benefit coverage with a significant investment element, as a way of competing for policyholders' savings with other institutions, for example, banks or open-ended investment companies (e.g. mutual funds in North America, or unit trusts in the UK). Additional flexibility also allows policyholders to purchase less insurance when their finances are tight, and then increase the insurance coverage when they have more money available.

In this section we describe some examples of modern, flexible insurance contracts.

- **Universal Life insurance** combines investment and life insurance. The policyholder determines a premium and a level of life insurance cover. Premiums are flexible, as long as the accumulated value of the premiums is sufficient to pay for the designated sum insured under the term insurance part of the contract. Universal Life is a common insurance contract in North America.
- **Unitized with-profit** is a UK insurance contract; it is an evolution from the conventional with-profit policy, designed to be more transparent than the original. Premiums are used to purchase units (shares) of an investment fund, called the with-profit fund. As the fund earns investment return, the shares increase in value (or more shares are issued), increasing the benefit entitlement as reversionary bonus. The shares will not decrease in value. On death or maturity, a further terminal bonus may be payable depending on the performance of the with-profit fund.

After some poor publicity surrounding with-profit business, and, by association, unitized with-profit business, these product designs were withdrawn from the UK and Australian markets in the early 2000s. However, they will remain important for many years as many companies carry very large

1.3 Life insurance and annuity contracts 7

portfolios of with-profit (traditional and unitized) policies issued during the second half of the twentieth century.

- **Equity-linked insurance** has a benefit linked to the performance of an investment fund. There are two different forms. The first is where the policyholder's premiums are invested in an open-ended investment company style account; at maturity, the benefit is the accumulated value of the premiums. There is a guaranteed minimum death benefit payable if the policyholder dies before the contract matures. In some cases, there is also a guaranteed minimum maturity benefit payable. In the UK and most of Europe, these are called **unit-linked** policies, and they rarely carry a guaranteed maturity benefit. In Canada they are known as **segregated fund** policies and always carry a maturity guarantee. In the USA these contracts are called **variable annuity** contracts; maturity guarantees are increasingly common for these policies. (The use of the term 'annuity' for these contracts is misleading. The benefits are designed with a single lump sum payout, though there will be an option to convert the lump sum to an annuity.)

The second form of equity-linked insurance is the **Equity-Indexed Annuity (EIA)** in the USA. Under an EIA the policyholder is guaranteed a minimum return on their premium (minus an initial expense charge). At maturity, the policyholder receives a proportion of the return on a specified stock index, if that is greater than the guaranteed minimum return.

EIAs are generally rather shorter in term than unit-linked products, with seven-year policies being typical; variable annuity contracts commonly have terms of 20 years or more. EIAs are much less popular with consumers than variable annuities.

1.3.4 Distribution methods

Most people find insurance dauntingly complex. Brokers who connect individuals to an appropriate insurance product have, since the earliest times, played an important role in the market. There is an old saying amongst actuaries that 'insurance is sold, not bought', which means that the role of an intermediary in persuading potential policyholders to take out an insurance policy is crucial in maintaining an adequate volume of new business.

Brokers, or other financial advisors, are often remunerated through a **commission system**. The commission would be specified as a percentage of the premium paid. Typically, there is a higher percentage paid on the first premium than on subsequent premiums. This is referred to as a **front-end load**. Some advisors may be remunerated on a fixed-fee basis, or may be employed by one or more insurance companies on a salary basis.

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An alternative to the broker method of selling insurance is **direct marketing**. Insurers may use television advertising or other telemarketing methods to sell direct to the public. The nature of the business sold by direct marketing methods tends to differ from the broker-sold business. For example, often the sum insured is smaller. The policy may be aimed at a niche market, such as older lives concerned with insurance to cover their own funeral expenses (called **pre-need** insurance in the USA). Another mass marketed insurance contract is loan or credit insurance, where an insurer might cover loan or credit card payments in the event of the borrower's death, disability or unemployment.

1.3.5 Underwriting

It is important in modelling life insurance liabilities to consider what happens when a life insurance policy is purchased. Selling life insurance policies is a competitive business and life insurance companies (also known as life offices) are constantly considering ways in which to change their procedures so that they can improve the service to their customers and gain a commercial advantage over their competitors. The account given below of how policies are sold covers some essential points but is necessarily a simplified version of what actually happens.

For a given type of policy, say a 10-year term insurance, the life office will have a schedule of premium rates. These rates will depend on the size of the policy and some other factors known as **rating factors**. An applicant's risk level is assessed by asking them to complete a **proposal form** giving information on relevant rating factors, generally including their age, gender (where legislation permits), smoking habits, occupation, any dangerous hobbies, and personal and family health history. The life insurer may ask for permission to contact the applicant's doctor to enquire about their medical history. In some cases, particularly for very large sums insured, the life insurer may require that the applicant's health be checked by a doctor employed by the insurer.

The process of collecting and evaluating this information is called **underwriting**. The purpose of underwriting is, first, to classify potential policyholders into broadly homogeneous risk categories, and secondly to assess what additional premium would be appropriate for applicants whose risk factors indicate that standard premium rates would be too low.

On the basis of the application and supporting medical information, potential life insurance policyholders will generally be categorized into one of the following groups:

- **Preferred lives** have very low mortality risk based on the standard information. The preferred applicant would have no recent record of smoking;

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Excerpt

[More information](#)*1.3 Life insurance and annuity contracts*

9

no evidence of drug or alcohol abuse; no high-risk hobbies or occupations; no family history of disease known to have a strong genetic component; no adverse medical indicators such as high blood pressure or cholesterol level or body mass index.

The preferred life category is common in North America, but has not yet caught on elsewhere. In other areas there is no separation of preferred and normal lives.

- **Normal lives** may have some higher rated risk factors than preferred lives (where this category exists), but are still insurable at standard rates. Most applicants fall into this category.
- **Rated lives** have one or more risk factors at raised levels and so are not acceptable at standard premium rates. However, they can be insured for a higher premium. An example might be someone having a family history of heart disease. These lives might be individually assessed for the appropriate additional premium to be charged. This category would also include lives with hazardous jobs or hobbies which put them at increased risk.
- **Uninsurable lives** have such significant risk that the insurer will not enter an insurance contract at any price.

Within the first three groups, applicants would be further categorized according to the relative values of the various risk factors, with the most fundamental being age, gender and smoking status. Note, however, that gender-based premiums are no longer permitted in some jurisdictions, including the European Union countries. Most applicants (around 95% for traditional life insurance) will be accepted at preferred or standard rates for the relevant risk category. Another 2–3% may be accepted at non-standard rates because of an impairment, or a dangerous occupation, leaving around 2–3% who will be refused insurance.

The rigour of the underwriting process will depend on the type of insurance being purchased, on the sum insured and on the distribution process of the insurance company. Term insurance is generally more strictly underwritten than whole life insurance, as the risk taken by the insurer is greater. Under whole life insurance, the payment of the sum insured is certain, the uncertainty is in the timing. Under, say, 10-year term insurance, it is assumed that the majority of contracts will expire with no death benefit paid. If the underwriting is not strict there is a risk of **adverse selection** by policyholders – that is, that very high-risk individuals will buy insurance in disproportionate numbers, leading to excessive losses. Since high sum insured contracts carry more risk than low sum insured, high sums insured would generally trigger more rigorous underwriting.

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The distribution method also affects the level of underwriting. Often, direct marketed contracts are sold with relatively low benefit levels, and with the attraction that no medical evidence will be sought beyond a standard questionnaire. The insurer may assume relatively heavy mortality for these lives to compensate for potential adverse selection. By keeping the underwriting relatively light, the expenses of writing new business can be kept low, which is an attraction for high-volume, low sum insured contracts.

It is interesting to note that with no third party medical evidence the insurer is placing a lot of weight on the veracity of the policyholder. Insurers have a phrase for this – that both insurer and policyholder may assume ‘utmost good faith’ or ‘*uberrima fides*’ on the part of the other side of the contract. In practice, in the event of the death of the insured life, the insurer may investigate whether any pertinent information was withheld from the application. If it appears that the policyholder held back information, or submitted false or misleading information, the insurer may not pay the full sum insured.

1.3.6 Premiums

A life insurance policy may involve a single premium, payable at the outset of the contract, or a regular series of premiums payable provided the policyholder survives, perhaps with a fixed end date. In traditional contracts the regular premium is generally a level amount throughout the term of the contract; in more modern contracts the premium might be variable, at the policyholder’s discretion for investment products such as equity-linked insurance, or at the insurer’s discretion for certain types of term insurance.

Regular premiums may be paid annually, semi-annually, quarterly, monthly or weekly. Monthly premiums are common as it is convenient for policyholders to have their outgoings payable with approximately the same frequency as their income.

An important feature of all premiums is that they are paid at the start of each period. Suppose a policyholder contracts to pay annual premiums for a 10-year insurance contract. The premiums will be paid at the start of the contract, and then at the start of each subsequent year provided the policyholder is alive. So, if we count time in years from $t = 0$ at the start of the contract, the first premium is paid at $t = 0$, the second is paid at $t = 1$, and so on, to the tenth premium paid at $t = 9$. Similarly, if the premiums are monthly, then the first monthly instalment will be paid at $t = 0$, and the final premium will be paid at the start of the final month at $t = 9\frac{11}{12}$ years. (Throughout this book we assume that all months are equal in length, at $\frac{1}{12}$ years.)