

THE PHYSICS OF ETTORE MAJORANA

Through just a handful of papers, Ettore Majorana left an indelible mark on the fields of physics, mathematics, computer science, and even economics before his mysterious disappearance in 1938. It is only now that the importance of Majorana's work is being realized: Majorana fermions are intensely studied today, and his work on neutrino physics has provided possible explanations for the existence of dark matter.

In this unique volume, Salvatore Esposito not only explores Majorana's known papers but, even more interestingly, also unveils his unpublished works. These include powerful methods and results, ranging from the atomic two-center problem, the Thomas–Fermi model, and ferromagnetism to quasi-stationary states, n -component relativistic wave equations, and quantum scalar electrodynamics.

Featuring biographical notes and contributions from leading experts Evgeny Akhmedov and Frank Wilczek, this fascinating book will captivate graduate students and researchers interested in both frontier science and the history of science.

SALVATORE ESPOSITO is Professor of the History of Physics and Associate Professor of Theoretical Physics, and Associate Researcher at the Naples Unit of the Istituto Nazionale di Fisica Nucleare. His research interests range from neutrino physics to field theory, and he is considered to be the world expert on Majorana's work.

THE PHYSICS OF ETTORE MAJORANA

Phenomenological, Theoretical, and Mathematical

SALVATORE ESPOSITO

With contributions by E. Akhmedov and F. Wilczek



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107044029

© S. Esposito 2015

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2015

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-04402-9 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

Acknowledgments page xii

Part I	Introducing the character	1
1	Life and myth	3
1.1	Fortunes and misfortunes of a genius	3
1.2	Family and university training	6
1.3	Lone physicist in the Fermi group	9
1.4	Leipzig–Rome–Naples: the later years	12
2	The visible side	17
2.1	Ten papers depicting the future	17
2.2	Introducing the Dirac equation into atomic spectroscopy	18
2.3	Spontaneous ionization	19
2.3.1	Anomalous terms in helium	19
2.3.2	Incomplete P' triplets	21
2.3.3	Majorana–Fano–Feshbach resonances	23
2.4	Chemical bonding	24
2.4.1	Helium molecular ion	24
2.4.2	Majorana structures	26
2.5	Non-adiabatic spin-flip	27
2.5.1	Majorana sphere and a general theorem	28
2.5.2	Landau–Zener probability formula	30
2.5.3	Majorana’s holes	31
2.5.4	Majorana–Brossel effect	32
2.6	Nuclear forces	33
2.6.1	The Heisenberg model of nuclear interactions	33
2.6.2	Majorana’s exchange mechanism	35

vi	<i>Contents</i>	
	2.6.3 Thomas–Fermi formalism and Yukawa potential	37
2.7	Infinite-component equation	38
	2.7.1 A successful relativistic wave equation	39
	2.7.2 Majorana equation	40
	2.7.3 Infinite-dimensional representations of the Lorentz group	41
	2.7.4 A difficult problem for Pauli and Fierz	42
	2.7.5 Further elaborations	44
2.8	Majorana neutrino theory	47
	2.8.1 “Symmetric” Dirac equation	47
	2.8.2 Neutrino–antineutrino identity	49
	2.8.3 Racah and the neutrinoless double β -decay	50
	2.8.4 Pontecorvo and the neutrino oscillations	51
	2.8.5 Majorana fermions	52
2.9	Complex systems in physics and economics	53
	2.9.1 Genesis of paper N.10	54
	2.9.2 Statistical laws in social sciences	55
	2.9.3 A sensational success in econophysics	57
	Part II Atomic physics	61
3	Two-electron problem	63
	3.1 A long-lasting success for quantum mechanics	63
	3.2 Known solutions to the helium atom problem	64
	3.2.1 Perturbative calculations	64
	3.2.2 Variational method I	66
	3.2.3 Self-consistent field method	68
	3.2.4 Slater’s refinement	69
	3.2.5 Variational method II: Hylleraas variables	70
	3.2.6 Helium-like ions	71
	3.3 Majorana empirical relations	72
	3.4 Helium wavefunctions and broad range estimates	76
	3.5 Accurate numbers and a general theory	78
	3.5.1 A simpler alternative to Hylleraas’s method	78
	3.5.2 Majorana’s variant of the variational method	79
	3.6 Conclusions	81
4	Thomas–Fermi model	83
	4.1 Fermi universal potential	83
	4.1.1 Thomas–Fermi equation	83

<i>Contents</i>		vii
4.1.2	Numerical and approximate solutions	85
4.1.3	Mathematical properties	86
4.2	Majorana solution of the Thomas–Fermi equation	87
4.2.1	Transformation into an Abel equation	87
4.2.2	Analytic series solution	89
4.2.3	Numerical tables	92
4.3	Mathematical generalizations	93
4.3.1	Frobenius method	93
4.3.2	Scale-invariant differential equations	95
4.4	Physical applications	97
4.4.1	Modified Fermi potential for heavy atoms	98
4.4.2	Second approximation for the atomic potential	100
4.4.3	Atomic polarizability	102
4.4.4	Applications to molecules	103
4.5	Conclusions	105
Part III Nuclear and statistical physics		107
5	Quasi-stationary nuclear states	109
5.1	Probing the atomic nucleus with α particles	109
5.2	Scattering of α particles on a radioactive nucleus	111
5.2.1	Quantum-mechanical theory	111
5.2.2	Thermodynamic approach	115
5.3	Transition probabilities of quasi-stationary states	116
5.3.1	Transitions from a discrete into a continuum state	116
5.3.2	Transitions into two continuous spectra	118
5.3.3	Transitions from a continuum state	118
5.4	Nuclear disintegration by α particles	119
5.4.1	Statement of the problem	119
5.4.2	The appropriate wavefunction	121
5.4.3	Cross section	122
5.5	Conclusions	124
6	Theory of ferromagnetism	127
6.1	Towards a statistical theory of ferromagnetism	128
6.1.1	Molecular fields	128
6.1.2	Heisenberg theory	129
6.1.3	Later refinements	131
6.2	Majorana statistical model	131
6.2.1	Distribution function	134

viii	<i>Contents</i>	
6.3	Solution of the model in the continuum limit	136
6.3.1	Partition function at finite temperature	138
6.3.2	Mean magnetization	139
6.4	Applications and further results	141
6.4.1	Particular ferromagnetic geometries	141
6.4.2	Critical temperature and dimensionality	143
6.5	Conclusions	144
	Part IV Relativistic fields and group theory	147
7	Groups and their applications to quantum mechanics	149
7.1	The “Gruppenpest” in quantum mechanics	150
7.2	Unitary transformations in two dimensions	153
7.2.1	\mathcal{D}_j representation and group generators	154
7.3	Three-dimensional rotations	156
7.3.1	Group generators	157
7.4	Application: the anomalous Zeeman effect	160
7.5	Lorentz group and its representations	164
7.5.1	n -dimensional Dirac matrices	164
7.5.2	Special case: maximum allowed p for fixed n	167
7.5.3	Non-Hermitian operators	169
7.5.4	Infinite-dimensional unitary representations	170
7.6	Conclusions	173
8	Dirac equations and some alternatives	175
8.1	Searching for an equation	175
8.1.1	Massive photons and the DKP algebra	178
8.1.2	Dirac–Fierz–Pauli formalism	180
8.1.3	General equations for arbitrary spin	182
8.2	Majorana n -component spinor equations	184
8.2.1	The 16-component equation for a two-particle system	185
8.2.2	Equation for a six-component spinor	187
8.2.3	Five-component equation	189
8.3	Parallel lives (and findings)	189
8.4	Conclusions	191
	Part V Quantum field theory	193
9	Scalar electrodynamics	195
9.1	Early quantum electrodynamics	196

	<i>Contents</i>	ix
	9.1.1 Quantum field formalism	196
	9.1.2 Particles and antiparticles	198
	9.1.3 Pauli–Weisskopf theory	198
	9.2 Majorana theory of scalar electrodynamics I	201
	9.3 Majorana theory of scalar electrodynamics II	205
	9.4 Application to the nuclear structure	209
	9.5 Conclusions	212
10	Photons and electrons	214
	10.1 Photon wave mechanics	215
	10.1.1 Majorana–Oppenheimer formulation of electrodynamics	215
	10.1.2 Lorentz-invariant wave theory	217
	10.1.3 Two-component theory	219
	10.1.4 Field quantization	220
	10.2 Dynamical theory of electrons and holes	221
	10.3 Conclusions	224
	Part VI Fundamental theories and other topics	227
11	A “path integral” approach to quantum mechanics	229
	11.1 Dirac and Feynman’s mathematical approach	230
	11.2 Majorana’s physical approach	232
	11.3 Conclusions	234
12	Fundamental lengths and times	236
	12.1 Introducing elementary space-time lengths	236
	12.2 Quasi-Coulombian scattering	239
	12.3 Intrinsic time delay and retarded electromagnetic fields	242
	12.4 Conclusions	244
13	Majorana’s multifaceted life	246
	13.1 Majorana as a student	246
	13.1.1 Melting point shift due to a magnetic field	246
	13.1.2 Determination of a function from its moments	248
	13.1.3 WKB method for differential equations	251
	13.2 Majorana as a phenomenologist: spontaneous and induced ionization of a hydrogen atom	254
	13.2.1 Hydrogen atom placed in a high potential region	255
	13.2.2 Ionization of a hydrogen-like atom in an electric field	260
	13.3 Majorana as a theoretician: a unifying model for the fundamental constants	263

x	<i>Contents</i>	
13.4	Majorana as a mathematician	264
13.4.1	Improper operators	264
13.4.2	Cubic symmetry	266
13.5	Majorana as a teacher	270
	Part VII Beyond Majorana	277
14	Majorana and condensed matter physics	279
	F. WILCZEK	
14.1	Spin response and universal connection	280
14.2	Level crossing and generalized Laplace transform	282
14.3	Majorana fermions and Majorana mass: from neutrinos to electrons	285
14.3.1	Majorana’s equation	285
14.3.2	Analysis of Majorana neutrinos	287
14.3.3	Majorana mass	289
14.3.4	Majorana electrons	292
14.4	Majorinos	293
14.4.1	Kitaev chain	294
14.4.2	Junctions and the algebraic genesis of majorinos	298
14.4.3	Continuum majorinos	301
15	Majorana neutrinos and other Majorana particles: theory and experiment	303
	E. AKHMEDOV	
15.1	Weyl, Dirac, and Majorana fermions	304
15.1.1	Particle–antiparticle conjugation	306
15.1.2	Dirac dynamics and the Majorana condition	307
15.1.3	Fermion mass terms and U(1) symmetries	312
15.1.4	Feynman rules for Majorana particles	313
15.2	C, P, CP, and CPT properties of Majorana fermions	314
15.3	Mixing and oscillations of Majorana neutrinos	317
15.3.1	Neutrinos with a Majorana mass term	317
15.3.2	General case of Dirac + Majorana mass term	322
15.3.3	Dirac and pseudo-Dirac neutrino limits in the D + M case	325
15.4	Seesaw mechanism of neutrino mass generation	327
15.5	Electromagnetic properties of Majorana neutrinos	330
15.6	Majorana particles in SUSY theories	338

<i>Contents</i>	xi
15.7 Experimental searches for Majorana neutrinos and other Majorana particles	339
15.7.1 Neutrinoless double β -decay and related processes	339
15.7.2 Other lepton-number-violating processes	344
15.8 Baryogenesis through leptogenesis and Majorana neutrinos	346
15.9 Miscellaneous	350
15.10 Summary and conclusions	352
Appendix Molecular bonding in quantum mechanics	354
A.1 On the meaning of quantum state	354
A.2 Symmetry properties of a system in classical and quantum mechanics	356
A.3 Resonance forces between states that cannot be symmetrized for small perturbations and spectroscopic consequences. Theory of homopolar valence according to the method of bonding electrons. Properties of the symmetrized states that are not obtained from non-symmetrized ones with a weak perturbation	358
<i>References</i>	364
<i>Author index</i>	377
<i>Subject index</i>	379

Acknowledgments

My interest in the (unpublished) scientific work by Majorana was stimulated many years ago by Erasmo Recami. I take this opportunity to thank him warmly for his continuous encouragement and constant interest in my own historical and scientific work about the protagonist of the present book.

I have also greatly appreciated several talks and discussions with a number of my colleagues about different topics treated here; in particular, I express my gratitude to A. De Gregorio, E. Di Grezia, A. Drago, A. Naddeo, and G. Salesi.

I also thank my colleagues G. Mangano, G. Miele, and O. Pisanti for their patience, especially when I repeatedly evoked the phantom of Majorana (through his writings) in their rooms at the Department of Physics in Naples.

Last but not least, I am particularly grateful to E. Akhmedov and F. Wilczek for their enthusiasm in joining this project, as well as for their excellent contributions, which have greatly increased the value of the book.