

Climate Change and Terrestrial Ecosystem Modeling

Climate models have evolved into Earth system models with representation of the physics, chemistry, and biology of terrestrial ecosystems. This companion book to Gordon Bonan's *Ecological Climatology: Concepts and Applications, Third Edition*, builds on the concepts introduced there and provides the mathematical foundation upon which to develop and understand ecosystem models and their relevance for these Earth system models. The book bridges the disciplinary gap among land surface models developed by atmospheric scientists; biogeochemical models, dynamic global vegetation models, and ecosystem demography models developed by ecologists; and ecohydrology models developed by hydrologists. Review questions, supplemental code, and modeling projects are provided to aid with understanding how the equations are used. The book is an invaluable guide to climate change and terrestrial ecosystem modeling for graduate students and researchers in climate change, climatology, ecology, hydrology, biogeochemistry, meteorology, environmental science, mathematical modeling, and environmental biophysics.

Gordon Bonan is Senior Scientist and Head of the Terrestrial Sciences Section at the National Center for Atmospheric Research in Boulder, Colorado. He studies the interactions of terrestrial ecosystems with climate, using models of Earth's biosphere, atmosphere, hydrosphere, and geosphere. He is the author of *Ecological Climatology: Concepts and Applications* (now in its third edition) and has published 150 peer-reviewed articles in atmospheric science, geoscience, and ecological journals on terrestrial ecosystems, climate, and their coupling. He is a fellow of the American Geophysical Union and the American Meteorological Society and has served on advisory boards for numerous national and international organizations and as an editor for several journals.

“This is a thoroughly comprehensive overview, and must be the definitive text for this field. The breadth of topics and the depth of knowledge displayed here is unparalleled. For students and expert practitioners alike, this is an extraordinarily useful resource, expertly presented using a thoughtful balance of text, figures, data tables and equations.”

Mathew Williams, University of Edinburgh

“Terrestrial ecosystem models are a fundamental basis for the science of global change impacts and biospheric climate, and an essential tool to investigate options to understand and mitigate climate change. This comprehensive overview of the history, theoretical underpinnings and application of terrestrial ecosystem models, paired with an in-depth discussion of the mathematical fundamentals of the model, and complemented with clear and helpful practical examples, is a must-read for anyone interested in using or developing models of land ecosystems, from the scale of individual ecosystems, to studies of the role of land surface dynamics in comprehensive Earth system models. I wish this book had been available at the beginning of my career.”

Sönke Zaehle, Max Planck Institute for Biogeochemistry

“Bonan’s excellent text is comprehensive, covering all the components needed to model the land surface and its interactions with the atmosphere and climate system. For each topic Bonan provides not only historical perspective and a comprehensive theoretical overview, but also the critically important discussion of how these theoretical approaches can be put into practice numerically. This book is the new definitive reference for anyone working on understanding and modeling all aspects of the land surface.”

Abigail L. S. Swann, University of Washington

“I highly recommend this groundbreaking book to everyone who wants to learn the nuts and bolts of land surface modeling and/or Earth system modeling. For many years I have been teaching *Ecological Climatology: Concepts and Applications* to geoscience and engineering students but struggling to supplement them with appropriate materials on modeling blocks and for modeling projects. Now this book arrives as a perfect addition and companion as it provides exactly the kind of materials I wanted to have.”

Zong-Liang Yang, University of Texas at Austin

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Gordon Bonan

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To Amie, again

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Preface

Writing a modeling textbook is daunting. There are many equations to introduce and explain, and writing a book that also shows how to solve the equations in a computer model is even more difficult. Why would one undertake such a task? There are several excellent topical textbooks, mostly in the area of environmental biophysics. David Gates's *Biophysical Ecology* is the classic text, but the equations, units, etc. are dated. *Principles of Environmental Physics* (John Monteith and Mike Unsworth), *An Introduction to Environmental Biophysics* (Gaylon Campbell and John Norman), *Evaporation into the Atmosphere* (Wilfried Brutsaert), and *Plants and Microclimate* (Hamlyn Jones) are still definitive introductions to plants and environmental physics. Daniel Hillel's *Environmental Soil Physics* is an essential text for soil physics. While these books provide excellent treatment of radiative transfer, turbulent fluxes, leaf biophysics, or soil physics, they do not integrate these concepts into a complete model of biosphere–atmosphere coupling. Nor do they, or other available textbooks, extend the subject matter to include ecosystems and biogeochemical cycles. This present book covers the fundamentals of environmental biophysics, integrates those principles into a complete model, and additionally broadens the scope to cover ecosystems and biogeochemical cycles. More importantly, the book explains the numerical methods needed to implement and solve the equations in a computer model. There is an enormous leap between seeing a mathematical equation in a research paper and actually using that equation in a model. In this respect, *Soil Physics with Basic* (Gaylon Campbell), *Soil Water Dynamics* (Arthur Warrick), *Numerical Methods in the Hydrological Sciences* (George Hornberger and Patricia Wiberg), and *A Theory of Forest Dynamics* (Hank Shugart) are indispensable reference books in that they also provide computer programs.

Many colleagues have contributed to this book in various ways. The roughness sublayer parameterization in Chapter 6 is an outgrowth of collaborations with Ned Patton and Ian Harman. The surface energy balance bucket model in Chapter 7 was developed in collaboration with Marysa Laguë as part of her simple land model. Martyn Clark

introduced me to the Picard iteration used to solve the Richards equation in Chapter 8 and also the probability distributed model of rainfall–runoff in Chapter 9, which is a generalization of the VIC model. Sönke Zaehle provided details of the implementation of the Kull and Kruijt (1998) photosynthesis in his O-CN terrestrial biosphere model. Collaboration with Danica Lombardozzi is seen in her work on photosynthesis and stomatal conductance, as is collaboration with Peter Franks on stomatal conductance. Mat Williams introduced me to his SPA model during a sabbatical visit to NCAR; the optimal stomatal conductance is described in Chapter 12, the plant hydraulics in Chapter 13, and the Norman radiative transfer in Chapter 14. Rosie Fisher also provided many insights to plant hydraulics. Ryan Knox shared details of ED2, its turbulence parameterization, and multilayer two-stream radiation. Ying-Ping Wang shared his CASA-CNP code, and Melannie Hartmann provided thorough documentation of the model; this model is the basis for Chapter 17. In his many visits to NCAR and his collaborations with the Community Land Model, Yiqi Luo shared details of the traceability analysis of biogeochemical models in Chapter 17. Quinn Thomas's and Will Wieder's insights to carbon–nitrogen biogeochemistry are also seen in Chapter 17. Collaborations with Melannie Hartmann and Will Wieder are evident in the DAYCENT soil organic matter model and microbial model, both described in Chapter 18. Rosie Fisher, Ryan Knox, Charlie Koven, and Jackie Shuman provided essential background on ecosystem demography models in Chapter 19.

Review questions and modeling projects are provided with each chapter. The review questions are intended to help students and readers assess whether they understand the concepts that have been introduced. These questions are structured around several essential modeling requirements: understanding units and how to convert among the different units favored in various scientific communities, deriving equations, and knowing what the equations mean and how to use them. Modeling projects are included in each chapter. The projects

aid with understanding how the equations are used, bringing various concepts together into a mathematical model, and using models to answer research questions. The projects utilize and build upon sample code provided with each chapter. It is hoped that the sample code will inspire students and instructors to devise their own projects, tailored to their specific interests. MATLAB® is used as the computational framework to illustrate code.¹ The sample code does not take advantage of the matrix capabilities of MATLAB or its full mathematical sophistication. Rather, the code is written to explicitly show in a step-by-step manner how the calculations are performed. In particular, many example codes explicitly loop over a number of

calculations, such as for soil layers, rather than using matrix algebra. The same code could be written utilizing matrix capabilities, but then the calculations would be less obvious to non-technical specialists. MATLAB purists and scientific coding experts will likely find the code offensive, but I prefer to err on the side of understandability rather than elegance.

Finally, I am indebted to Matt Lloyd at Cambridge University Press, who has supported this endeavor over the many years it took to complete, and also Zoë Pruce for her patience with this project.

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¹ MATLAB is a registered trademark of The MathWorks, Inc., Natick, Massachusetts, United States.

Mathematical Symbols

The following is a list of major symbols. Duplication of symbols is unavoidable, and the reader should always check the specific usage of a symbol within a particular chapter.

$a(z)$	leaf area density at height z ($\text{m}^2 \text{m}^{-3}$)	e	vapor pressure (Pa); <i>subscript</i> : e_a air; $e_{sat}(T_\ell)$ stomatal pore; e_s leaf surface
A	area (m^2); <i>subscript</i> : A_C crown; A_L leaf; A_S sapwood	$e_{sat}(T)$	saturation vapor pressure at temperature T (Pa); Table 3.3
A	gross photosynthesis rate ($\mu\text{mol CO}_2 \text{m}^{-2} \text{s}^{-1}$); <i>subscript</i> : A_c Rubisco-limited assimilation; A_j RuBP regeneration-limited assimilation; A_p product-limited assimilation	E	evaporation ($\text{mol H}_2\text{O m}^{-2} \text{s}^{-1}$)
A_n	net photosynthesis rate ($\mu\text{mol CO}_2 \text{m}^{-2} \text{s}^{-1}$)	f_{sun}	fractional area on a horizontal plane that is sunlit (–)
c_a, c_c, c_i, c_s	CO_2 concentration in air, chloroplast, intercellular space, and leaf surface, respectively ($\mu\text{mol mol}^{-1}$)	F_j	flux of constituent j ($\text{mol m}^{-2} \text{s}^{-1}$)
c_i	carbon mass per area of biogeochemical pool i (kg C m^{-2})	g	gravitational acceleration (m s^{-2}); Table A.4
c_j	mole fraction of constituent j (mol mol^{-1})	g_a	aerodynamic conductance ($\text{mol m}^{-2} \text{s}^{-1}$); <i>subscript</i> : g_{ac} scalars; g_{am} momentum
c_d	leaf aerodynamic drag coefficient (–)	g_b	leaf boundary layer conductance ($\text{mol m}^{-2} \text{s}^{-1}$); <i>subscript</i> : g_{bc} CO_2 ; g_{bh} heat; g_{bj} constituent j ; g_{bw} H_2O
c_{dry}	specific heat of dry biomass ($\text{J kg}^{-1} \text{K}^{-1}$)	g_b^*	canopy excess conductance between z_{om} and z_{oc} ($\text{mol m}^{-2} \text{s}^{-1}$)
c_{ice}, c_{wat}	specific heat of ice and water, respectively ($\text{J kg}^{-1} \text{K}^{-1}$); Table A.4	g_c	canopy conductance ($\text{mol m}^{-2} \text{s}^{-1}$)
c_L	leaf heat capacity ($\text{J m}^{-2} \text{K}^{-1}$)	g_h	conductance for heat ($\text{mol m}^{-2} \text{s}^{-1}$)
c_p	specific heat of moist air at constant pressure ($\text{J mol}^{-1} \text{K}^{-1}$); <i>subscript</i> : c_{pd} dry air; c_{pw} water vapor; Table A.4	g_j	conductance of constituent j ($\text{mol m}^{-2} \text{s}^{-1}$)
c_v	volumetric heat capacity ($\text{J m}^{-3} \text{K}^{-1}$); <i>subscript</i> : c_{vf} frozen soil; c_{vu} unfrozen soil; $c_{v,ice}$ ice; $c_{v,sol}$ soil solids; $c_{v,wat}$ water	g_ℓ	total leaf conductance of stomata and boundary layer ($\text{mol m}^{-2} \text{s}^{-1}$); <i>subscript</i> : $g_{\ell c}$ CO_2 ; $g_{\ell w}$ H_2O
$C(\psi)$	specific moisture capacity (m^{-1})	g_m	leaf mesophyll conductance ($\text{mol CO}_2 \text{m}^{-2} \text{s}^{-1}$)
C_h, C_m	drag coefficient (–); <i>subscript</i> : C_h heat; C_m momentum	g_r	radiative conductance ($\text{mol m}^{-2} \text{s}^{-1}$)
C_p	plant hydraulic capacitance ($\text{mol H}_2\text{O m}^{-2} \text{Pa}^{-1}$)	g_s	leaf stomatal conductance ($\text{mol m}^{-2} \text{s}^{-1}$); <i>subscript</i> : g_{sc} CO_2 ; g_{sw} H_2O
d	displacement height (m)	g_w	conductance for water vapor ($\text{mol m}^{-2} \text{s}^{-1}$)
D	stem diameter (cm)	g_0	minimum stomatal conductance ($\text{mol H}_2\text{O m}^{-2} \text{s}^{-1}$)
D	vapor pressure deficit (Pa); <i>subscript</i> : D_ℓ leaf-to-air, $e_{sat}(T_\ell) - e_a$; D_s leaf surface, $e_{sat}(T_\ell) - e_s$	g_1	slope parameter for stomatal conductance models
D_f	drag force from vegetation ($\text{mol m}^{-2} \text{s}^{-2}$)	G	soil heat flux (W m^{-2})
D_{ij}	element in the Lagrangian dispersion matrix (s m^{-1})	$G(Z)$	projection of leaf area in the direction of the solar beam (–)
D_j	molecular diffusivity of constituent j ($\text{m}^2 \text{s}^{-1}$); <i>subscript</i> : D_c CO_2 ; D_h heat; D_w H_2O ; Table A.3	Gr	Grashof number (–)
$D(\theta)$	hydraulic diffusivity ($\text{m}^2 \text{s}^{-1}$)	h	height in canopy (m); <i>subscript</i> : h_c top of canopy
		h_s	leaf surface humidity (–), $e_s/e_{sat}(T_\ell)$
		h_{s1}	relative humidity of pore space in surface soil layer (–)
		H	sensible heat flux (W m^{-2}); <i>subscript</i> : H_v virtual
		H_f	energy flux from soil freezing or thawing (W m^{-2})

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ΔH_a	activation energy (J mol ⁻¹)	K_e	Kersten number (–)
ΔH_d	deactivation energy (J mol ⁻¹)	K_L	leaf-specific plant hydraulic conductance (mol H ₂ O m ⁻² s ⁻¹ Pa ⁻¹)
i_c	infiltration rate (m s ⁻¹)	K_m	Michaelis–Menten constant (μmol mol ⁻¹)
I_c	cumulative infiltration (m)	K_n	canopy nitrogen decay coefficient (–)
I^\downarrow	solar radiation in the downward direction (μmol photon m ⁻² s ⁻¹); <i>subscript</i> : I_b^\downarrow , scattered direct beam flux; I_d^\downarrow scattered diffuse flux	K_o, K_{o25}	Michaelis–Menten constant for O ₂ and its value at 25°C (mmol mol ⁻¹)
I_{sky}^\downarrow	solar radiation incident on the top of the canopy (μmol photon m ⁻² s ⁻¹); <i>subscript</i> : $I_{sky,b}^\downarrow$ direct beam; $I_{sky,d}^\downarrow$ diffuse	K_p	leaf-specific stem hydraulic conductance (mol H ₂ O m ⁻² s ⁻¹ Pa ⁻¹)
I^\uparrow	solar radiation in the upward direction (μmol photon m ⁻² s ⁻¹); <i>subscript</i> : I_b^\uparrow scattered direct beam flux; I_d^\uparrow scattered diffuse flux	ℓ	characteristic leaf dimension (m)
\bar{I}_c	solar radiation absorbed by the canopy, per ground area (μmol photon m ⁻² s ⁻¹); <i>subscript</i> : \bar{I}_{cb} direct beam; \bar{I}_{cd} diffuse; \bar{I}_{cSun} sunlit canopy; \bar{I}_{cSha} shaded canopy	l	mixing length (m); <i>subscript</i> : l_c scalar; l_m momentum
\bar{I}_g	solar radiation absorbed by the ground (μmol photon m ⁻² s ⁻¹); <i>subscript</i> : \bar{I}_{gb} direct beam; \bar{I}_{gd} diffuse	L^\downarrow	downward longwave radiation (W m ⁻²); <i>subscript</i> : L_{sky}^\downarrow atmospheric
\bar{I}_ℓ	solar radiation absorbed by foliage, per leaf area (μmol photon m ⁻² s ⁻¹); <i>subscript</i> : $\bar{I}_{\ell b}$ direct beam; $\bar{I}_{\ell bb}$ unscattered direct beam; $\bar{I}_{\ell bs}$ scattered direct beam; $\bar{I}_{\ell d}$ diffuse	L^\uparrow	upward longwave radiation (W m ⁻²); <i>subscript</i> : L_g^\uparrow ground
$\bar{I}_{\ell sha}$, $\bar{I}_{\ell sun}$	solar radiation absorbed by shaded and sunlit leaves, per leaf area (μmol photon m ⁻² s ⁻¹)	\bar{L}	absorbed longwave radiation, per ground area (W m ⁻²); <i>subscript</i> : \bar{L}_c canopy; \bar{L}_{cSun} sunlit canopy; \bar{L}_{cSha} shaded canopy; \bar{L}_g ground
I_{PSII}	light utilized in electron transport by photosystem PS II (μmol m ⁻² s ⁻¹)	\bar{L}_ℓ	longwave radiation absorbed by foliage, per leaf area (W m ⁻²)
J	electron transport rate (μmol m ⁻² s ⁻¹)	L	leaf area index (m ² m ⁻²); <i>subscript</i> : L_{sha} shaded leaf area; L_{sun} sunlit leaf area
J_{max} , J_{max25}	maximum electron transport rate and its value at 25°C (μmol m ⁻² s ⁻¹)	ΔL	leaf area index of a canopy layer (m ² m ⁻²); <i>subscript</i> : ΔL_{sha} shaded leaf area; ΔL_{sun} sunlit leaf area
k	von Karman constant (–); Table A.4	L_c	canopy length scale (m)
k_i	loss rate, or turnover rate, of biogeochemical pool i (per unit time, e.g., d ⁻¹)	L_f	latent heat of fusion (J kg ⁻¹); Table A.4
k_p, k_{p25}	initial slope of CO ₂ response for C ₄ photosynthesis and its value at 25°C (mol m ⁻² s ⁻¹)	L_{MO}	Obukhov length scale (m)
K	eddy diffusivity (m ² s ⁻¹); <i>subscript</i> : K_c scalar; K_m momentum	L_r	root length density (m m ⁻³)
$K(\theta)$	hydraulic conductivity (m s ⁻¹); <i>subscript</i> : K_{sat} saturated conductivity	m_a	leaf mass per area (kg C m ⁻² , or kg dry mass m ⁻²)
K_b, K_d	extinction coefficient for direct beam and diffuse radiation, respectively (–)	m_i	biomass per area of biogeochemical pool i (kg m ⁻²)
K_c, K_{c25}	Michaelis–Menten constant for CO ₂ and its value at 25°C (μmol mol ⁻¹)	m_j	mass of constituent j (kg); <i>subscript</i> : m_d dry air; m_v water vapor
		M_j	molecular mass of constituent j (kg mol ⁻¹); <i>subscript</i> : M_a dry air; M_w water; Table A.4
		M_r	root biomass (kg m ⁻²)
		M_i	plant biomass per individual (kg); <i>subscript</i> : M_L leaf; M_H heartwood; M_S sapwood; M_R fine root
		n_a	leaf nitrogen per unit leaf area (kg N m ⁻²)
		n_i	nitrogen per area of biogeochemical pool i (kg N m ⁻²)
		n_j	number of moles of constituent j
		n_m	leaf nitrogen per unit leaf mass (kg N kg ⁻¹ C, or kg N kg ⁻¹ dry mass)
		Nu	Nusselt number (–)
		o_i	intercellular O ₂ concentration (mmol mol ⁻¹)

P	atmospheric pressure (Pa); <i>subscript</i> : P_d dry air	T	temperature (K); <i>subscript</i> : T_a air; T_e equilibrium; T_f freezing point of water (Table A.4); T_g ground; T_ℓ leaf; T_s surface; T_v virtual; T_A acclimation growth temperature
P	precipitation ($\text{kg H}_2\text{O m}^{-2} \text{ s}^{-1} = \text{mm H}_2\text{O s}^{-1}$); <i>subscript</i> : P_R rain; P_S snow	T_L	Lagrangian time scale (s)
P_j	partial pressure of constituent j (Pa); <i>subscript</i> : P_d dry air	T_p, T_{p25}	triose phosphate utilization rate and its value at 25°C ($\mu\text{mol m}^{-2} \text{ s}^{-1}$)
Pr	Prandtl number (–)	u	wind speed, or also specifically the velocity component in the x -directional (m s^{-1}); <i>subscript</i> : u_{ref} at some reference height
q	mole fraction (e/P ; mol mol^{-1}); <i>subscript</i> : q_a air; q_{ref} at some reference height; q_s surface; q_* characteristic scale	u_*	friction velocity (m s^{-1})
$q_{sat}(T)$	saturation water vapor mole fraction at temperature T (mol mol^{-1}), $e_{sat}(T)/P$	U	carbon input from plant production ($\text{kg C m}^{-2} \text{ d}^{-1}$, or $\text{kg C m}^{-2} \text{ y}^{-1}$)
Q	water flux (m s^{-1} , or $\text{mol m}^{-2} \text{ s}^{-1}$)	v	velocity component in the y -direction (m s^{-1})
Q_a	radiative forcing (W m^{-2})	v	general symbol for a rate; <i>subscript</i> : v_{base} base rate; v_{max} maximum rate; v_{25} rate at 25°C
Q_{10}	Q_{10} temperature parameter (–)	v_d	dry deposition velocity (m s^{-1})
r_c	leaf Nusselt number (heat) or Stanton number (scalar) (–)	V	volume (m^3)
r_j	resistance of constituent j ($\text{s m}^2 \text{ mol}^{-1}$)	V_{cmax}	maximum Rubisco carboxylation rate and its value at 25°C ($\mu\text{mol m}^{-2} \text{ s}^{-1}$)
r_l	specific root length (m kg^{-1})	V_{cmax25}	vertical velocity (m s^{-1})
r_r	fine root radius (m)	w	mole fraction of water vapor (mol mol^{-1}), e/P ; <i>subscript</i> : w_ℓ leaf-to-air, $[e_{sat}(T_\ell) - e_a]/P$; w_s leaf surface, $[e_{sat}(T_\ell) - e_s]/P$
R	runoff ($\text{kg H}_2\text{O m}^{-2} \text{ s}^{-1} = \text{mm H}_2\text{O s}^{-1}$)	w_c	Rubisco-limited carboxylation rate ($\mu\text{mol m}^{-2} \text{ s}^{-1}$)
R	respiration rate ($\text{kg C m}^{-2} \text{ s}^{-1}$); <i>subscript</i> : R_A autotrophic; R_g growth; R_m maintenance; R_H heterotrophic	w_i	light harvesting-limited carboxylation rate ($\mu\text{mol m}^{-2} \text{ s}^{-1}$)
R_A	isotope ratio of a sample A (–); <i>subscript</i> : R_d air; R_E evaporation flux; R_l liquid water; R_p photosynthate; R_v water vapor	w_j	RuBP regeneration-limited carboxylation rate ($\mu\text{mol m}^{-2} \text{ s}^{-1}$)
R_d, R_{d25}	leaf mitochondrial respiration in light, or day respiration, and its value at 25°C ($\mu\text{mol CO}_2 \text{ m}^{-2} \text{ s}^{-1}$)	w_p	product-limited carboxylation rate ($\mu\text{mol m}^{-2} \text{ s}^{-1}$)
R_n	net radiation (W m^{-2})	W	soil water ($\text{kg H}_2\text{O m}^{-2} = \text{mm H}_2\text{O}$); <i>subscript</i> : W_{ice} ice; W_{liq} liquid; W_{snow} snow
Re	Reynolds number (–)	z	vertical distance (m)
Ri_B	bulk Richardson number (–)	z_0	roughness length (m); <i>subscript</i> : z_{0c} scalars; z_{0m} momentum
\mathfrak{R}	gas constant ($\text{J K}^{-1} \text{ mol}^{-1}$); Table A.4	α_{A-B}	isotopic fractionation factor for samples A and B (–); <i>subscript</i> : α_k kinetic fractionation; α_{kb} kinetic fractionation for leaf boundary layer; α_{ks} kinetic fractionation for stomata; α_{l-v} liquid–vapor transition; α_p photosynthate
s	temperature derivative of q_{sat} at temperature T ($\text{mol mol}^{-1} \text{ K}^{-1}$), dq_{sat}/dT	α_ℓ	leaf absorptance (–)
S	storage heat flux (W m^{-2}); <i>subscript</i> : S_h sensible heat; S_q latent heat; S_v biomass	α_Λ	absorptance at a specified wavelength (–)
S^\downarrow	downward solar radiation (W m^{-2}); <i>subscript</i> : S_λ^\downarrow at specified wavelength	A	azimuth angle; <i>subscript</i> : A_ℓ leaf
S^\uparrow	upward solar radiation (W m^{-2}); <i>subscript</i> : S_λ^\uparrow at specified wavelength	β	ratio of friction velocity to wind speed at the canopy height (–); <i>subscript</i> : β_N neutral value of β
S_c	scalar source or sink flux ($\text{mol m}^{-3} \text{ s}^{-1}$)		
$S_{c/o}$	relative specificity of Rubisco (–)		
S_e	relative soil wetness (–), θ/θ_{sat} or $(\theta - \theta_{res})/(\theta_{sat} - \theta_{res})$		
Sc	Schmidt number (–)		
Sh	Sherwood number (–)		
ΔS	entropy term ($\text{J K}^{-1} \text{ mol}^{-1}$)		
t	time (s, seconds; h, hours; y, years)		

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β, β_0	two-stream upscatter parameters for diffuse and direct beam radiation, respectively (–)	$\zeta(z)$	cumulative leaf drag area (–)
β_w	soil wetness factor (–)	ρ, ρ_d, ρ_v	density of moist air ($\rho = \rho_d + \rho_v$), its dry air component, and water vapor, respectively (kg m^{-3})
B	solar elevation angle	ρ_a	density of dry air at pressure P and temperature T (kg m^{-3})
γ	psychrometric constant (Pa K^{-1})	ρ_b	soil bulk density (kg m^{-3})
γ_ℓ	incidence angle of the solar beam on a leaf	ρ_{ice}, ρ_{wat}	density of ice and water (kg m^{-3}); Table A.4
Γ	CO_2 compensation point ($\mu\text{mol mol}^{-1}$)	ρ_j	mass concentration of constituent j (kg m^{-3})
Γ_*, Γ_{*25}	CO_2 compensation point in the absence of non-photorespiratory respiration, and its value at 25°C ($\mu\text{mol mol}^{-1}$)	ρ_m	molar density (mol m^{-3})
δ_A	isotope ratio of sample A relative to a standard (–)	ρ_r	root tissue density (kg m^{-3})
Δ_{A-B}	isotopic fractionation between samples A and B (–)	ρ_s	soil particle density (kg m^{-3})
ε	emissivity (–); <i>subscript</i> : ε_g ground; ε_ℓ leaf; ε_λ at a specified wavelength	ρ_{snow}	density of snow (kg m^{-3})
ε_{A-B}	isotopic enrichment factor for samples A and B (–)	ρ_w	density of wood (kg m^{-3})
ε_j	emission factor for chemical compound j	ρ	reflectance (–); <i>subscript</i> : ρ_g ground; ρ_ℓ leaf; ρ_λ at a specified wavelength
E	quantum yield ($\text{mol CO}_2 \text{ mol}^{-1} \text{ photon}$, or $\text{mol CO}_2 \text{ J}^{-1}$)	ρ_c	canopy albedo (–); <i>subscript</i> : ρ_{cb} direct beam; ρ_{cd} diffuse
ζ	Monin–Obukhov stability parameter (–)	ρ_g	ground albedo (–); <i>subscript</i> : ρ_{gb} direct beam; ρ_{gd} diffuse
Z	solar zenith angle	σ	Stefan–Boltzmann constant ($\text{W m}^{-2} \text{ K}^{-4}$); Table A.4
θ	potential temperature (K); <i>subscript</i> : θ_{ref} air at some reference height; θ_s surface; θ_v virtual; θ_* characteristic temperature scale	σ_w	standard deviation of vertical wind velocity (m s^{-1})
θ	volumetric soil water content ($\text{m}^3 \text{ m}^{-3}$); <i>subscript</i> : θ_{ice} ice; θ_{liq} liquid; θ_{res} residual water; θ_{sat} saturation (porosity)	τ	transmittance (–); <i>subscript</i> : τ_b direct beam; τ_d diffuse radiation; τ_ℓ leaf; τ_λ at a specified wavelength
Θ	curvature factor for co-limitation (–); <i>subscript</i> : Θ_A photosynthesis; Θ_j electron transport	τ	momentum flux ($\text{mol m}^{-1} \text{ s}^{-2}$); <i>subscript</i> : τ_x zonal; τ_y meridional
Θ_ℓ	leaf inclination angle	τ_L	leaf temperature time constant (s)
ι	marginal carbon gain of water loss ($\mu\text{mol CO}_2 \text{ mol}^{-1} \text{ H}_2\text{O}$)	$T_{b,i}$	transmittance of direct beam radiation through the cumulative leaf area above canopy layer i (–)
κ	thermal conductivity ($\text{W m}^{-1} \text{ K}^{-1}$); <i>subscript</i> : κ_{air} air (Table A.4); κ_{dry} dry soil; κ_f frozen soil; κ_{ice} ice (Table A.4); κ_o other soil minerals; κ_q quartz; κ_{sat} saturated soil; κ_{snow} snow; κ_{sol} soil solids; κ_u unfrozen soil; κ_{wat} water (Table A.4)	ϕ	ratio of Rubisco oxygenation to carboxylation rates (–)
λ	latent heat of vaporization (J mol^{-1}); Table 3.6	ϕ_1, ϕ_2	terms in the Ross–Goudriaan $G(B)$ function
Λ	wavelength (m)	ϕ_c, ϕ_m	Monin–Obukhov similarity theory flux–gradient relationships for scalars and momentum, respectively (–)
μ	cosine solar zenith angle	$\hat{\phi}_c, \hat{\phi}_m$	roughness sublayer functions for ϕ_c and ϕ_m , respectively (–)
$\bar{\mu}$	average inverse of the optical depth of diffuse radiation per unit leaf area	Φ_c, Φ_m	roughness sublayer-modified flux–gradient relationships for scalars and momentum, respectively (–)
ν	molecular diffusivity for momentum, or kinematic viscosity ($\text{m}^2 \text{ s}^{-1}$); Table A.3	Φ_{PSII}	quantum yield of photosystem II (mol mol^{-1})
		χ	mass mixing ratio (kg kg^{-1}); <i>subscript</i> : χ_v water vapor
		χ_ℓ	departure of leaf angle distribution from spherical (–)

ψ	soil water potential (m or Pa); <i>subscript:</i> ψ_w at the wetting front; ψ_{sat} at saturation	$\hat{\psi}_c, \hat{\psi}_m$	integrated form of the roughness sublayer functions for scalars and momentum, respectively (–)
ψ_ℓ	leaf water potential (Pa); <i>subscript:</i> $\psi_{\ell min}$ minimum leaf water potential	ω	scattering coefficient (–); <i>subscript:</i> ω_ℓ leaf; ω_Λ at a specified wavelength
ψ_c, ψ_m	integrated form of Monin–Obukhov functions for scalars and momentum, respectively (–)	Ω	foliage clumping factor for light transmission (–)

