

## Symbol index for Volume 1

- $\mathbf{1}$  (the unit of a unital algebra), 2  
 $\mathbb{1}$  (the unit of the unital extension), 33  
 $\|\cdot\|_\pi$  (projective tensor norm), 31  
 $|\cdot|_n$  (natural  $C^*$ -norm on  $M_n(\mathbb{C})$ ), 166  
 $\|\cdot\|_n$  ( $n \in \mathbb{N}$ ), 167–175  
 $\{\cdot\cdot\}$ , 127, 130, 324, 463  
 $[a, b] = ab - ba$ , 126  
 $[a, b, c] = (ab)c - a(bc)$ , 151  
 $\odot^\pi$  (natural product on the range of the projection  $\pi$ ), 154  
 $\bigoplus_{\lambda \in \Lambda}^{\ell_1} X_\lambda$  ( $\ell_1$ -sum of the family  $\{X_\lambda\}$ ), 109  
 $\bigoplus_{i \in I}^{\ell_\infty} X_i$  ( $\ell_\infty$ -sum of the family  $\{X_i\}$ ), 271  
 $[ij]$  (for  $i, j = 1, 2$ ), 538  
  
 $a \bullet b = \frac{1}{2}(ab + ba)$ , 122  
 $a^{-1}$  (the [J]-inverse of  $a$ ), 5, 188, 453, 473  
 $a^{[n]}$  (plenary powers of  $a$ ), 566  
 $a^\diamond$  (the quasi-[J]-inverse of  $a$ ), 431, 585  
 $\text{Ann}(A)$  (annihilator of  $A$ ), 4  
 $\text{Aut}(A) = \text{Aut}(A, A)$ , 384  
 $\text{Aut}(A, B)$  (isomorphisms from  $A$  onto  $B$ ), 384  
 $\text{Aut}^+(A) = \{F \in \text{Aut}(A) : F^\bullet = F, \text{sp}(F) \subseteq \mathbb{R}_0^+\}$ , 384  
 $\text{Aut}^*(A, B)$  ( $*$ -isomorphisms from  $A$  onto  $B$ ), 387  
 $A(a)$  (subalgebra of  $A$  generated by  $a$ ), 262  
 $A(S)$  (subalgebra of  $A$  generated by  $S$ ), 9  
 $\bar{A}(S)$  (closed subalgebra of  $A$  generated by  $S$ ), 9  
 $A_{\mathbb{1}}$  ( $= A$  or  $A_{\mathbb{1}}$  depending on whether or not  $A$  is unital), 407  
 $A_{\mathbb{1}}$  (unital extension of  $A$ ), 33  
 $A_{\mathbb{C}}$  (complexification of  $A$ ), 32  
 $A_{\mathbb{R}}$  (real algebra underlying  $A$ ), 97  
 $A_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce subspaces of  $A$  relative to the idempotent  $e$ ), 178  
 $A_\Omega = \{x \in A : [J-]\text{sp}(A, x) \subseteq \Omega\}$ , 65, 486  
 $A_{\mathcal{U}}$  (ultrapower of  $A$ ), 272  
 $A^{(0)}$  (opposite algebra of  $A$ ), 13  
 $A^{(u)}$  ( $u$ -isotope of  $A$ ), 519  
  
 $A^+$  (positive part of  $A$ ), 47, 613  
 $A^{\text{ant}}$  (antisymmetrized algebra of  $A$ ), 560  
 $A^K = \{x \in A : V(A, \mathbf{1}, x) \subseteq K\}$ , 649  
 $A^{\text{sym}}$  (symmetrized algebra of  $A$ ), 122  
 $(A_i)_{\mathcal{U}}$  (ultraproduct of the family  $\{A_i\}$ ), 271  
 $\mathcal{A}(E)$  (flexible quadratic algebra of the pre- $H$ -algebra  $E$ ), 204  
 $\mathcal{A}(K)$  (associative algebra of the compact set  $K \subseteq [1, \infty[$ ), 537  
 $\mathcal{A}(U, \vartheta, \mathbb{K})$ , 257  
 $\mathcal{A}_p(U, \vartheta, \mathbb{K})$  ( $1 \leq p < \infty$ ), 257  
 $\mathcal{A}(V, \times, (\cdot, \cdot))$  (quadratic algebra of  $(V, \times, (\cdot, \cdot))$ ), 182  
 $*\mathbb{A}$  (for  $\mathbb{A} = \mathbb{C}, \mathbb{H}$ , or  $\mathbb{O}$ ), 278  
 $\mathbb{A}^*$  (for  $\mathbb{A} = \mathbb{C}, \mathbb{H}$ , or  $\mathbb{O}$ ), 278  
 $\mathbb{A}^*$  (for  $\mathbb{A} = \mathbb{H}$ , or  $\mathbb{O}$ ), 220  
 $\mathbb{A}_n$   $n \in \mathbb{N} \cup \{0\}$  (Cayley–Dickson algebras), 199  
 $\mathfrak{A}\text{-Rad}(A)$  ( $\mathfrak{A}$ -radical of  $A$ ), 580  
  
 $BL(X, Y)$  (bounded linear operators from  $X$  to  $Y$ ), 3  
 $BL(X) = BL(X, X)$ , 3  
 $B(I, X)$  (bounded functions from  $I$  to  $X$ ), 117, 307  
 $B(x, y)$  (Bergmann operator of  $(x, y)$ ), 509  
 $BC = \{xy : (x, y) \in B \times C\}$ , 2  
 $\beta_{u, K}(r) = \inf\{1 - \|u + rx\| : x \in K\mathbb{B}_X, \tau(u, x) \leq -1\}$ , 299  
 $\mathcal{B}(A)$  (Baer radical of  $A$ ), 601  
 $\mathcal{B}(K)$  (Jordan algebra of the compact set  $K \subseteq [1, \infty[$ ), 553  
 $\mathbb{B}_X$  (closed unit ball of  $X$ ), 2  
  
 $\text{co}(S)$  (convex hull of  $S$ ), 28  
 $|\text{co}|(S)$  (absolutely convex hull of  $S$ ), 99  
 $\overline{\text{co}}(S)$  (closed convex hull of  $S$ ), 99  
 $|\overline{\text{co}}|(S)$  (closed absolutely convex hull of  $S$ ), 99  
 $c_0$  (null sequences in  $\mathbb{K}$ ), 3

- $C_A$  (extended centroid of  $A$ ), 195
- $C_b(E, A)$  (bounded continuous functions from  $E$  to  $A$ ), 3
- $C_b^{\mathbb{C}}(E) = C_b(E, \mathbb{C})$ , 150
- $C_p([1, 2], \mathcal{S}_3) = \{\alpha \in C([1, 2], \mathcal{S}_3) : \alpha(1) \in \mathbb{R}p\}$ , 560
- $C_p(K, \mathcal{C}_3) = \{\alpha \in C(K, \mathcal{C}_3) : \alpha(1) \in \mathbb{C}p\}$ , 555
- $C_p(K, M_2(\mathbb{C})) = \{\alpha \in C(K, M_2(\mathbb{C})) : \alpha(1) \in \mathbb{C}p\}$ , 545
- $C_0^{\mathbb{K}}(E) = C_0^{\mathbb{K}}(E)$  when  $E$  is compact, 3
- $C_0^{\mathbb{K}}(E)$  ( $\mathbb{K}$ -valued continuous functions on  $E$  vanishing at infinity), 3
- $C_0^{\mathbb{T}}(E) = \{x \in C_0^{\mathbb{C}}(E) : x(zt) = zx(t) \forall (z, t) \in \mathbb{T} \times E\}$ , 498
- $C(\mathbb{C})$  (complex octonions), 205
- $C(\mathbb{R})$  (Cayley–Dickson doubling of  $M_2(\mathbb{R})$ ), 218
- $C(E, A)$  (continuous functions from  $E$  to  $A$ ), 3
- $C_0(E, X)$  ( $X$ -valued continuous functions on  $E$  vanishing at infinity), 330
- $\mathcal{C}\mathcal{D}(A)$  (Cayley–Dickson doubling of  $A$ ), 176
- $\mathcal{C}\mathcal{N}(\mathbf{X}, \mathbb{K})$  (free complete normed non-associative  $\mathbb{K}$ -algebra on  $\mathbf{X}$ ), 261
- $\mathcal{C}_3$  (three-dimensional spin factor), 553
- $\overset{*}{\mathbb{C}}$  (McClay algebra), 216
- $\deg(A)$  (degree of  $A$ ), 212
- $\text{dens}(E)$  (density character of  $E$ ), 257
- $\text{dom}(\cdot)$  (domain of a partially defined operator), 194, 640
- $\text{Der}^*(A)$ , 384
- $\text{Dis}(X, u)$  (dissipative elements of  $X$  relative to  $u$ ), 291
- $D(X, u)$  (states of  $X$  relative to  $u$ ), 94
- $D^Y(X, u) = D(X, u) \cap Y$ , 99
- $D^{w^*}(X, x) = D(X, x) \cap X_{w^*}$ , 285
- $D(K)$  (Banach space of  $K$ ), 645
- $d\hat{f}(a) : X \rightarrow X$  (formal differential of  $f$  at  $a$ ), 652
- $\delta_X(u, \cdot) : \mathbb{R}^+ \rightarrow \mathbb{R}$  (modulus of midpoint local convexity of  $X$  at  $u$ ), 111
- $\Delta_A$  (characters on  $A$ ), 21
- $\Delta = \Delta_A$ , 21
- $\exp(a)$  (exponential of  $a$ ), 10, 342
- $(\exp - 1)(a) = \sum_{n=1}^{\infty} \frac{a^n}{n!}$ , 609
- $\text{ext}(S)$  (extreme points of  $S$ ), 107
- $\text{Ea}(K)$  (extremal algebra of  $K$ ), 647
- $\text{Ea}^1(K)$  (derivable elements of  $\text{Ea}(K)$ ), 651
- $\text{Ea}^n(K)$   $n \in \mathbb{N}$  ( $n$ -times derivable elements of  $\text{Ea}(K)$ ), 669
- $\text{Ea}^{\infty}(K) = \bigcap_{n \in \mathbb{N}} \text{Ea}^n(K)$ , 669
- $\eta : [1, \infty[ \rightarrow M_2(\mathbb{C})$ , 537
- $\eta_K = \eta|_K$ , 537
- $\eta_{ij} : [1, \infty[ \rightarrow M_2(\mathbb{C})$ , 537
- $\eta_{ij}^K = (\eta_{ij})|_K$ , 537
- $f(a)$ , 46, 57–59, 479, 484, 648
- $\tilde{f} : A_{\Omega} \rightarrow A$ , 66, 486
- $f^*(x) = \tilde{f}(x^*)$  for  $(x, f) \in X \times X'$ , 146
- $f[ij]$  (for  $f \in C^{\mathbb{C}}(K)$  and  $i, j = 1, 2$ ), 538
- $f'$  (for  $f \in \text{Ea}^1(K)$ ), 651
- $F' : Y' \rightarrow X'$  (transpose of the operator  $F : X \rightarrow Y$ ), 29
- $F \otimes G$  (operator tensor product of  $F$  and  $G$ ), 30
- $F^* : K \rightarrow H$  (adjoint of the operator  $F : H \rightarrow K$ ), 38
- $F^{\mathbb{K}}(E)$  ( $\mathbb{K}$ -valued functions on  $E$ ), 2
- $F_i(f)(x_1, \dots, x_n)$  ( $0 \leq i \leq n$ ), 370
- $\mathfrak{F}(X, Y)$  (finite-rank operators from  $X$  to  $Y$ ), 73
- $\mathfrak{F}(X) = \mathfrak{F}(X, X)$ , 75
- $\mathcal{F}(\mathbf{X}, \mathbb{K})$  (free non-associative  $\mathbb{K}$ -algebra on  $\mathbf{X}$ ), 258
- $\mathcal{F}_p(\mathbf{X}, \mathbb{K})$  ( $1 \leq p < \infty$ ), 258
- $\mathcal{F} : \mathcal{A}(K) \rightarrow C(K, M_2(\mathbb{C}))$ , 538
- $G : A \rightarrow C^{\mathbb{C}}(\Delta)$  (Gelfand representation for complete normed unital associative and commutative complex algebras), 22
- $G : J \rightarrow C_0^{\mathbb{T}}(\Lambda)$  (Gelfand representation for complex Banach Jordan  $*$ -triples), 500
- $\Gamma$  (a contour in  $\mathbb{C}$ ), 58
- $\Gamma_A$  (centroid of  $A$ ), 4
- $\Gamma_e(A)$  (left centralizers on  $A$ ), 254
- $\mathcal{G}(X)$  (surjective linear isometries on  $X$ ), 332
- $\mathcal{G} : \mathcal{B}(K) \rightarrow C(K, \mathcal{C}_3)$ , 553
- $H(X, *)$  ( $*$ -invariant elements of  $(X, *)$ ), 39
- $H_1 \hat{\otimes} H_2$  (Hilbert tensor product of  $H_1$  and  $H_2$ ), 417
- $H_3(\mathbb{O})$  (Albert exceptional Jordan algebra), 337
- $\mathcal{H}(\Omega)$  ( $\mathbb{C}$ -valued holomorphic functions on  $\Omega$ ), 59
- $\mathbb{H}$  (algebra of Hamilton quaternions), 176
- $\text{id}(x_0) = \{e \in A : ex_0 = x_0\}$ , 437
- $\text{Ind}_{\Gamma}(z_0)$  (index of  $z_0$  with respect to  $\Gamma$ ), 58
- $\text{Inv}(A)$  (invertible elements of  $A$ ), 5
- $I_X$  (identity mapping on  $X$ ), 2
- $(I : A) = \{x \in A : xA + Ax \subseteq I\}$ , 602
- $\Im(z)$  (imaginary part of  $z$ ), 132
- $\text{J-Inv}(A)$  (J-invertible elements of  $A$ ), 453, 475
- $\text{J-Rad}(A)$  (Jacobson radical of  $A$ ), 569
- $\text{J-sp}(A, a)$  (J-spectrum of  $a$  relative to  $A$ ), 456, 476
- $J^{(e)}$  ( $e$ -homotope algebra of  $J$ ), 465
- $J_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce subspaces of  $J$  relative to the tripotent  $e$ ), 505
- $\ker(x_0) = \{a \in A : ax_0 = 0\}$ , 437
- $k(F) = \max\{k \geq 0 : k\|x\| \leq \|F(x)\| \forall x \in X\}$ , 250
- $K(X, u) = \bigcap_{f \in D(X, u)} \ker(f)$ , 351

- $\mathfrak{K}(X, Y)$  (compact operators from  $X$  to  $Y$ ), 70
- $\mathfrak{K}(X) = \mathfrak{K}(X, X)$ , 75
- $\mathbb{K} = \mathbb{R}$  or  $\mathbb{C}$ , 1
- $\mathbb{K}[\mathbf{x}]$  (polynomials over  $\mathbb{K}$  in the indeterminate  $\mathbf{x}$ ), 9
- $\mathbb{K}(\mathbf{x})$  (fractions over  $\mathbb{K}$  in the indeterminate  $\mathbf{x}$ ), 57
- $\text{lin}(S)$  (linear hull of  $S$ ), 351
- $L(X, Y)$  (linear mappings from  $X$  to  $Y$ ), 1
- $L(X) = L(X, X)$ , 1
- $L_a$  (left multiplication by  $a$ ), 13
- $L_S := \{L_x : x \in S\}$ , 433
- $L_a^X$  (left multiplication by  $a$  on the bimodule  $X$ ), 637
- $L_x^B = (L_x)|_B$ , 348
- $L(x, y)(z) = \{xyz\}$ , 465
- $L(J, J) = \{L(x, y) : x, y \in J\}$ , 468
- $\Lambda_J$  (nonzero triple homomorphisms from  $J$  to  $\mathbb{C}$ ), 499
- $\Lambda = \Lambda_J$ , 499
- $m' : Z' \times X \rightarrow Y'$ , 124
- $m'' : Y'' \times Z' \rightarrow X'$ , 124
- $m''' : X'' \times Y'' \rightarrow Z''$ , 124
- $m^l = m'''$ , 124
- $m^r(y, x) = m(x, y)$ , 126
- $m^*(x, y) = (m(x^*, y^*))^*$ , 146
- $M(A)$  (algebra of multipliers of  $A$ ), 126, 325
- $M_n(X)$  ( $n \times n$  matrices with entries in the vector space  $X$ ), 166
- $M_n(A)$  ( $n \times n$  matrices with entries in the algebra  $A$ ), 167
- $M_\infty(\mathbb{K})$  (infinite matrices over  $\mathbb{K}$  with a finite number of nonzero entries), 267
- $M_{a,b}(x) = axb$ , 601
- $\mathcal{M}(\mathbf{X})$  (free monad generated by  $\mathbf{X}$ ), 258
- $\mathcal{M}(B)^A$ , 357
- $\mathcal{M}^\sharp(A)$  (multiplication ideal of  $A$ ), 443
- $n(a)$  (algebraic norm of  $a$ ), 181
- $n(X, u)$  (numerical index of  $(X, u)$ ), 98
- $n^Y(X, u)$ , 99
- $n^{w^*}(X, u) = n^{X^*}(X, u)$ , 295
- $n_{\mathbb{R}}(X, u)$  (real numerical index of  $(X, u)$ ), 353
- $N(X)$  (spatial numerical index of  $X$ ), 105
- $\mathcal{N}(\mathbf{X}, \mathbb{K})$  (free normed non-associative  $\mathbb{K}$ -algebra on  $\mathbf{X}$ ), 258
- $\omega_K(z) = \max\{|e^{wz}| : w \in K\}$ , 645
- $\mathbb{O}$  (algebra of Cayley numbers), 176
- $p(a) = \sum_{k=0}^n \alpha_k a^k$  for  $p(\mathbf{x}) = \sum_{k=0}^n \alpha_k \mathbf{x}^k \in \mathbb{K}[\mathbf{x}]$ , 9
- $\mathbf{p}(a_1, \dots, a_n)$  (valuation of  $\mathbf{p}$  at  $(a_1, \dots, a_n)$ ), 262
- $P(X)$  (continuous products on  $X$ ), 405
- $p_A$  (product of  $A$ ), 408
- $P_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce projections relative to  $e$ ), 178, 505
- $\varphi(X, u, r) = \sup \left\{ \frac{\|u+r\mathbf{x}\| - 1}{r} - \tau(u, x) : x \in \mathbb{B}_X \right\}$ , 299
- $\pi_1(\Gamma) = \{x : (x, f) \in \Gamma \text{ for some } f\}$ , 106
- $\Pi(X) = \{(x, f) : x \in \mathbb{S}_X, f \in D(X, x)\}$ , 106
- $\Pi(Y, X) = \{(y, x') \in \mathbb{S}_Y \times \mathbb{S}_{X'} : x' \in D(X, y)\}$ , 116
- $\mathbb{P}$  (algebra of pseudo-octonions), 220
- q-Inv( $A$ ) (quasi-invertible elements of  $A$ ), 440
- $Q_x(y) = \{xyx\}$ , 506
- $Q_{x,z}(y) = \{xyz\}$ , 507
- $\mathcal{Q}_a = \left\{ \frac{p(\mathbf{x})}{q(\mathbf{x})} \in \mathbb{K}(\mathbf{x}) : q(a) \in \text{Inv}(A) \right\}$ , 57
- $\mathcal{Q}\mathcal{F}\mathcal{M}(A)$  (quasi-full multiplication algebra of  $A$ ), 578
- $\tau(a)$  (spectral radius of  $a$ ), 6, 381
- $\text{Rad}(A)$  (radical of  $A$ ), 429
- $R_a$  (right multiplication by  $a$ ), 13
- $R_S := \{R_x : x \in S\}$ , 433
- $R_a^X$  (right multiplication by  $a$  on the bimodule  $X$ ), 637
- $\Re(z)$  (real part of  $z$ ), 95
- $\mathfrak{s}(a)$  (substitute of the spectral radius of  $a$ ), 566
- $\text{sp}(A, a)$  (spectrum of  $a$  relative to  $A$ ), 12
- $\text{sp}(a) = \text{sp}(A, a)$ , 12
- s-Rad( $A$ ) (strong radical of  $A$ ), 20, 427
- $S^c$  (commutant of  $S$ ), 24
- $S^{cc} = (S^c)^c$ , 24
- $\sigma(x)$  (triple spectrum of  $x$ ), 504
- $\mathcal{S}_3$  (three-dimensional real spin factor), 560
- $\S$  (algebra of sedenions), 199
- $\mathbb{S}_X$  (unit sphere of  $X$ ), 2
- $\mathfrak{S}(\Phi)$  (separating space of  $\Phi$ ), 18
- $t(a)$  (trace of  $a$ ), 181
- $\tau(u, x) = \max \Re(V(X, u, x))$ , 291
- $\tau^l : C^c(F) \rightarrow C^c(E)$  for  $\tau : E \rightarrow F$ , 85
- $\vartheta : M_2(\mathbb{C}) \rightarrow M_2(\mathbb{C})$ , 553–554
- $\Theta : \mathcal{A}(K) \rightarrow \mathcal{A}(K)$ , 552–554
- $\mathbb{T} = \mathbb{S}_{\mathbb{C}}$ , 10
- uw-Rad( $A$ ) (ultra-weak radical of  $A$ ), 580
- $U(X, u) = \{f \in \mathbb{B}_P(X) : f(x, u) = f(u, x) = x \forall x \in X\}$ , 405
- $U_a = L_a(L_a + R_a) - L_{a^2}$ , 121
- $U_{a,b} = \frac{1}{2}[L_a(L_b + R_b) + L_b(L_a + R_a)] - L_{a \bullet b}$ , 364, 453
- $U_a^X(x) = a(ax + xa) - a^2x$  for  $x \in X$ , 637
- $U_x^B = (U_x)|_B$ , 348

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| <p><math>v(X, u, x)</math> (numerical radius of <math>x</math> relative to <math>(X, u)</math>), 98<br/> <math>v(x) = v(X, u, x)</math>, 98<br/> <math>V(X, u, x)</math> (numerical range of <math>x</math> relative to <math>(X, u)</math>), 94<br/> <math>V(x) = V(X, u, x)</math>, 94</p> <p><math>w\text{-Rad}(A)</math> (weak radical of <math>A</math>), 578<br/> <math>W(f)</math> (spatial numerical range of <math>f : \mathbb{S}_Y \rightarrow X</math>), 116, 308<br/> <math>W(T)</math> (spatial numerical range of <math>T : X \rightarrow X</math>), 107<br/> <math>W(T)</math> (spatial numerical range of <math>T : Y \rightarrow X</math>), 116<br/> <math>\mathcal{W}(A) = \{a \in A : L_a, R_a \in \text{Rad}(\mathcal{L}\mathcal{F}\mathcal{M}(A))\}</math>, 578<br/> <math>\mathfrak{W}(X, Y)</math> (weakly compact operators from <math>X</math> to <math>Y</math>), 70<br/> <math>\mathfrak{W}(X) = \mathfrak{W}(X, X)</math>, 75</p> <p><math>x^{(2n+1)} = \{x^{(2n-1)}x\}</math> (triple powers of <math>x</math>), 468<br/> <math>X'</math> ((topological) dual of <math>X</math>), 2</p> | <p><math>X''</math> (bidual of <math>X</math>), 2<br/> <math>(X, u)</math> (numerical-range space), 94<br/> <math>X \otimes_{\pi} Y</math> (projective tensor product of <math>X</math> and <math>Y</math>), 31<br/> <math>X \oplus_1 Y</math> (<math>\ell_1</math>-sum of <math>X</math> and <math>Y</math>), 109<br/> <math>X_{\mathbb{R}}</math> (real vector space underlying <math>X</math>), 95<br/> <math>X_{\mathbb{C}}</math> (complexification of <math>X</math>), 31<br/> <math>X_n</math> (continuous <math>n</math>-linear mappings from <math>X^n</math> to <math>X</math>), 370<br/> <math>X_{\mathcal{U}}</math> (ultrapower of <math>X</math>), 271<br/> <math>(X_i)_{\mathcal{U}}</math> (ultraproduct of the family <math>\{X_i\}</math>), 271<br/> <math>\mathcal{U}(U, \mathbb{K})</math> (free vector space over <math>\mathbb{K}</math> generated by <math>U</math>), 257</p> <p><math>y \otimes f : x \rightarrow f(x)y</math>, 73</p> <p><math>Z(A)</math> (centre of <math>A</math>), 192<br/> <math>\mathcal{Z}(B)</math> (centre modulo the radical of <math>B</math>), 597</p> |
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- $\|S\| = \sup\{\|s\| : s \in S\}$ , 603  
 $\|f\|_E := \sup_{x \in E} \|f(x)\|$ , 54  
*ANBP* (approximate norm-1 boundedness property), 614  
 $A(k, n) = \{\alpha = (\alpha_1, \dots, \alpha_k) \in \mathbb{N}^k : |\alpha| = n\}$ , 20  
 $A_0(k, n) = \{\alpha = (\alpha_1, \dots, \alpha_k) \in (\mathbb{N} \cup \{0\})^k : |\alpha| = n\}$ , 20  
 $A^{(a)}$  (a-homotope of  $A$ ), 589  
 $\text{Ann}(I)$  (annihilator of the ideal  $I$ ), 562  
 $\text{Ann}(X)$  (annihilator of the Jordan  $*$ -triple  $X$ ), 225  
 $\text{ad}_a(x) = [a, x]$ , 176  
 $A_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce subspaces of  $A$  relative to the idempotent  $e$ ), 375  
 $A_*^+$  (cone of  $w^*$ -continuous positive linear functionals on  $A$ ), 286  
 $A^{(\lambda)}$  ( $\lambda$ -mutation of  $A$ ), 325  
 $\text{Aut}(\Omega)$  (biholomorphic mappings from  $\Omega$  onto  $\Omega$ ), 61  
 $\text{aut}(\Omega)$  (complete holomorphic vector fields on  $\Omega$ ), 138  
 $\text{Aut}^0(\Omega) = \text{Aut}(\Omega) \cap \mathcal{P}^1(X, X)$ , 187  
 $\text{Aut}_0(\Omega)$  (connected component of  $I_\Omega$  in  $(\text{Aut}(\Omega), T_a)$ ), 183  
 $\text{aut}^0(\Omega) = \text{aut}(\Omega) \cap \mathcal{P}^1(X, X)$ , 179  
 $\text{aut}_0(\Omega) = \text{aut}(\Omega) \cap (\mathcal{P}^0(X, X) \oplus \mathcal{P}^2(X, X))$ , 179  
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 $BL_s^n(X, Y)$  (bounded symmetric  $n$ -linear operators from  $X \times \dots \times X$  to  $Y$ ), 20  
 $\beta(A)$ , 614  
 $C_b(E, A)$  (bounded continuous functions from  $E$  to  $A$ ), 54  
 $C(\mathbb{F})$  (split octonion algebra over  $\mathbb{F}$ ), 360  
 $C(E, A)$  (continuous functions from  $E$  to  $A$ ), 53  
 $c(X, A)$  (core of  $X$  in  $A$ ), 413  
 $\mathcal{C}\mathcal{D}(A)$  (Cayley–Dickson doubling of  $A$ ), 359  
 $Df(x_0)$  (derivative of  $f$  at  $x_0$ ), 22  
 $Df(x_0; x)$  (derivative of  $f$  at  $x_0$  in the direction  $x$ ), 42  
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 $F^*(a) := (F(a^*))^*$  (definition of the operator  $F^* : A \rightarrow B$  for an operator  $F : A \rightarrow B$  with  $A$  and  $B$  semi- $H^*$ -algebras), 488  
 $F^\bullet : B \rightarrow A$  (adjoint of the operator  $F : A \rightarrow B$  for  $A$  and  $B$  semi- $H^*$ -algebras), 488  
 $g^\sharp : \mathcal{H}(U, \Omega_X) \rightarrow \mathcal{H}(g(U), \Omega_Y)$  ( $g : \Omega_X \rightarrow \Omega_Y$  biholomorphic mapping), 112  
 $g^\diamond : \mathcal{H}(U, X) \rightarrow \mathcal{H}(g(U), Y)$  ( $g : \Omega_X \rightarrow \Omega_Y$  biholomorphic mapping), 112  
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 $\mathcal{H}_b(\Omega, Y)$  (bounded holomorphic mappings from  $\Omega$  to  $Y$ ), 54  
 $\mathcal{H}_b^\Omega(B, Y)$  (holomorphic mappings from  $\Omega$  to  $Y$  which are bounded on  $B$ ), 55  
 $\mathcal{H}_0(\Omega, Y)$  (holomorphic mappings from  $\Omega$  to  $Y$  which are bounded on every  $B \in \Omega$ ), 56  
 $\mathcal{H}\mathcal{S}(H)$  (Hilbert–Schmidt operators on  $H$ ), 481

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Symbol index for Volume 2

- $[I : A] = \{F \in \mathcal{M}(A) : F(A) \subseteq I\}$ , 574
- $\overline{I}^\pi = \text{Ann}(\text{Ann}(I))$ , 572
- $k(F) = \max\{k \geq 0 : k\|x\| \leq \|F(x)\| \forall x \in X\}$ , 586
- $K_\delta = K + \delta\Delta_X$ , 55
- $K \Subset \Omega$  ( $K$  lies strictly inside  $\Omega$ ), 55
- $\Lambda : \mathcal{H}(U, Y) \rightarrow \mathcal{H}(U, Y)$  (differential operator on  $\mathcal{H}(U, Y)$  associated to  $\Lambda$ ), 106
- $\Lambda^n$  ( $n$ th power of  $\Lambda$  as a differential operator), 116
- $\overline{\text{lin}}(S)$  (closed linear hull of  $S$ ), 563
- $m(X, X_*)$  (Mackey topology of a dual Banach space  $X$ ), 290
- $\text{MC}(S)$  (multiplicatively closed subset generated by  $S$ ), 606
- $\mathcal{M}(A)$  (algebra of multipliers of  $A$ ), 315
- $\text{Mult}(X)$ , 319
- $\mathcal{M}(A)$  (multiplication algebra), 367
- $\mathfrak{M}(A)$ , 406
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- $\mathcal{N}_{\text{ann}} = \{a \in A : \mathcal{N}(a) = 0\}$ , 572
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- $\mathcal{N}(A)$  (nucleus of  $A$ ), 413
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- $NSI$  (norm square inequality), 638
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- $P_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce projections relative to  $e$ ), 375
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- $Q(A)$  (symmetric Martindale algebra of quotients), 403
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- $r_b(f, x_0)$  (radius of boundedness of  $f$  at  $x_0$ ), 46
- $S(A, \tau) = \{a \in A : \tau(a) = -a\}$ , 445
- $S^{\text{ann}} = \{F \in \mathcal{M}(A) : F(S) = 0\}$ , 572
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- $\overline{S}^\varepsilon = (S^{\text{ann}})_{\text{ann}}$ , 572
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- $\tau_C(A)$  (trace class elements of  $A$ ), 526
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- $t^+(x)$  (positive lifetime of  $x$ ), 94
- $t^-(x)$  (negative lifetime of  $x$ ), 94
- $T_p(M)$  (tangent space of  $M$  at  $p$ ), 160
- $T(M) := \bigcup_{p \in M} T_p(M)$  (tangent bundle of  $M$ ), 162
- $\mathcal{TC}(H)$  (trace-class operators on  $H$ ), 524
- $\text{Trees}(m, n)$  (trees of degree  $m$  and height  $n$ ), 30
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