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978-1-107-04306-0- Encyclopedia of Mathematics and its Applications: Non-Associative Normed Algebras: Volume 1: The Vidav–Palmer and Gelfand–Naimark Theorems

Miguel Cabrera García and Ángel Rodríguez Palacios

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