

## Symbol index

- 1** (the unit of a unital algebra), 2  
 $\mathbb{1}$  (the unit of the unital extension), 33  
 $\|\cdot\|_\pi$  (projective tensor norm), 31  
 $|\cdot|_n$  (natural  $C^*$ -norm on  $M_n(\mathbb{C})$ ), 166  
 $\|\cdot\|_n$  ( $n \in \mathbb{N}$ ), 167–175  
 $\{\cdot\cdot\cdot\}$ , 127, 130, 324, 463  
 $[a, b] = ab - ba$ , 126  
 $[a, b, c] = (ab)c - a(bc)$ , 151  
 $\odot^\pi$  (natural product on the range of the projection  $\pi$ ), 154  
 $\oplus_{\lambda \in \Lambda}^{\ell_1} X_\lambda$  ( $\ell_1$ -sum of the family  $\{X_\lambda\}$ ), 109  
 $\oplus_{i \in I}^{\ell_\infty} X_i$  ( $\ell_\infty$ -sum of the family  $\{X_i\}$ ), 271  
 $[ij]$  (for  $i, j = 1, 2$ ), 538  
 $a \bullet b = \frac{1}{2}(ab + ba)$ , 122  
 $a^{-1}$  (the [J-]inverse of  $a$ ), 5, 188, 453, 473  
 $a^{[n]}$  (plenary powers of  $a$ ), 566  
 $a^\circ$  (the quasi-[J-]inverse of  $a$ ), 431, 585  
 $\text{Ann}(A)$  (annihilator of  $A$ ), 4  
 $\text{Aut}(A) = \text{Aut}(A, A)$ , 384  
 $\text{Aut}(A, B)$  (isomorphisms from  $A$  onto  $B$ ), 384  
 $\text{Aut}^+(A) = \{F \in \text{Aut}(A) : F^\bullet = F, \text{sp}(F) \subseteq \mathbb{R}_0^+\}$ , 384  
 $\text{Aut}^*(A, B)$  ( $*$ -isomorphisms from  $A$  onto  $B$ ), 387  
 $A(a)$  (subalgebra of  $A$  generated by  $a$ ), 262  
 $A(S)$  (subalgebra of  $A$  generated by  $S$ ), 9  
 $\bar{A}(S)$  (closed subalgebra of  $A$  generated by  $S$ ), 9  
 $A_1$  ( $= A$  or  $A_1$  depending on whether or not  $A$  is unital), 407  
 $A_1$  (unital extension of  $A$ ), 33  
 $A_{\mathbb{C}}$  (complexification of  $A$ ), 32  
 $A_{\mathbb{R}}$  (real algebra underlying  $A$ ), 97  
 $A_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce subspaces of  $A$  relative to the idempotent  $e$ ), 178  
 $A_\Omega = \{x \in A : [J-]\text{sp}(A, x) \subseteq \Omega\}$ , 65, 486  
 $A_{\mathcal{U}}$  (ultrapower of  $A$ ), 272  
 $A^{(0)}$  (opposite algebra of  $A$ ), 13  
 $A^{(u)}$  ( $u$ -isotope of  $A$ ), 519  
 $A^+$  (positive part of  $A$ ), 47, 613  
 $A^{\text{ant}}$  (antisymmetrized algebra of  $A$ ), 560  
 $A^K = \{x \in A : V(A, \mathbf{1}, x) \subseteq K\}$ , 649  
 $A^{\text{sym}}$  (symmetrized algebra of  $A$ ), 122  
 $(A_i)_{\mathcal{U}}$  (ultraproduct of the family  $\{A_i\}$ ), 271  
 $\mathcal{A}(E)$  (flexible quadratic algebra of the pre- $H$ -algebra  $E$ ), 204  
 $\mathcal{A}(K)$  (associative algebra of the compact set  $K \subseteq [1, \infty]$ ), 537  
 $\mathcal{A}(U, \vartheta, \mathbb{K})$ , 257  
 $\mathcal{A}_p(U, \vartheta, \mathbb{K})$  ( $1 \leq p < \infty$ ), 257  
 $\mathcal{A}(V, \times, (\cdot, \cdot))$  (quadratic algebra of  $(V, \times, (\cdot, \cdot))$ ), 182  
 ${}^*A$  (for  $A = \mathbb{C}, \mathbb{H}$ , or  $\mathbb{O}$ ), 278  
 $\mathbb{A}^*$  (for  $A = \mathbb{C}, \mathbb{H}$ , or  $\mathbb{O}$ ), 278  
 $\mathbb{A}$  (for  $A = \mathbb{H}$ , or  $\mathbb{O}$ ), 220  
 $\mathbb{A}_n$   $n \in \mathbb{N} \cup \{0\}$  (Cayley–Dickson algebras), 199  
 $\mathfrak{A}\text{-Rad}(A)$  ( $\mathfrak{A}$ -radical of  $A$ ), 580  
 $BL(X, Y)$  (bounded linear operators from  $X$  to  $Y$ ), 3  
 $BL(X) = BL(X, X)$ , 3  
 $B(I, X)$  (bounded functions from  $I$  to  $X$ ), 117, 307  
 $B(x, y)$  (Bergmann operator of  $(x, y)$ ), 509  
 $BC = \{xy : (x, y) \in B \times C\}$ , 2  
 $\beta_{u, K}(r) = \inf\{1 - \|u + rx\| : x \in K\mathbb{B}_X, \tau(u, x) \leq -1\}$ , 299  
 $\mathcal{B}(A)$  (Baer radical of  $A$ ), 601  
 $\mathcal{B}(K)$  (Jordan algebra of the compact set  $K \subseteq [1, \infty]$ ), 553  
 $\mathbb{B}_X$  (closed unit ball of  $X$ ), 2  
 $\text{co}(S)$  (convex hull of  $S$ ), 28  
 $|\text{co}|(S)$  (absolutely convex hull of  $S$ ), 99  
 $\overline{\text{co}}(S)$  (closed convex hull of  $S$ ), 99  
 $|\overline{\text{co}}|(S)$  (closed absolutely convex hull of  $S$ ), 99  
 $c_0$  (null sequences in  $\mathbb{K}$ ), 3  
 $C_A$  (extended centroid of  $A$ ), 195  
 $C_b(E, A)$  (bounded continuous functions from  $E$  to  $A$ ), 3  
 $C_b^{\mathbb{C}}(E) = C_b(E, \mathbb{C})$ , 150  
 $C_p([1, 2], \mathcal{S}) = \{\alpha \in C([1, 2], \mathcal{S}) : \alpha(1) \in \mathbb{R}p\}$ , 560  
 $C_p(K, \mathcal{E}_3) = \{\alpha \in C(K, \mathcal{E}_3) : \alpha(1) \in \mathbb{C}p\}$ , 555  
 $C_p(K, M_2(\mathbb{C})) = \{\alpha \in C(K, M_2(\mathbb{C})) : \alpha(1) \in \mathbb{C}p\}$ , 545  
 $C^{\mathbb{K}}(E) = C_0^{\mathbb{K}}(E)$  when  $E$  is compact, 3  
 $C_0^{\mathbb{K}}(E)$  ( $\mathbb{K}$ -valued continuous functions on  $E$  vanishing at infinity), 3

- $C_0^{\mathbb{T}}(E) = \{x \in C_0^{\mathbb{C}}(E) : x(\tau t) = \tau x(t) \forall (z, t) \in \mathbb{T} \times E\}$ , 498  
 $C(\mathbb{C})$  (complex octonions), 205  
 $C(\mathbb{R})$  (Cayley–Dickson doubling of  $M_2(\mathbb{R})$ ), 218  
 $C(E, A)$  (continuous functions from  $E$  to  $A$ ), 3  
 $C_0(E, X)$  ( $X$ -valued continuous functions on  $E$  vanishing at infinity), 330  
 $\mathcal{C}\mathcal{D}(A)$  (Cayley–Dickson doubling of  $A$ ), 176  
 $\mathcal{C}\mathcal{N}(\mathbf{X}, \mathbb{K})$  (free complete normed non-associative  $\mathbb{K}$ -algebra on  $\mathbf{X}$ ), 261  
 $\mathcal{C}_3$  (three-dimensional spin factor), 553  
 $\overset{*}{\mathbb{C}}$  (McClay algebra), 216  
 $\deg(A)$  (degree of  $A$ ), 212  
 $\text{dens}(E)$  (density character of  $E$ ), 257  
 $\text{dom}(\cdot)$  (domain of a partially defined operator), 194, 640  
 $\text{Der}^*(A)$ , 384  
 $\text{Dis}(X, u)$  (dissipative elements of  $X$  relative to  $u$ ), 291  
 $D(X, u)$  (states of  $X$  relative to  $u$ ), 94  
 $D^Y(X, u) = D(X, u) \cap Y$ , 99  
 $D^{w^*}(X, x) = D(X, x) \cap X_w$ , 285  
 $D(K)$  (Banach space of  $K$ ), 645  
 $d\hat{f}(a) : X \rightarrow X$  (formal differential of  $f$  at  $a$ ), 652  
 $\delta_X(u, \cdot) : \mathbb{R}^+ \rightarrow \mathbb{R}$  (modulus of midpoint local convexity of  $X$  at  $u$ ), 111  
 $\Delta_A$  (characters on  $A$ ), 21  
 $\Delta = \Delta_A$ , 21  
 $\exp(a)$  (exponential of  $a$ ), 10, 342  
 $(\exp - 1)(a) = \sum_{n=1}^{\infty} \frac{a^n}{n!}$ , 609  
 $\text{ext}(S)$  (extreme points of  $S$ ), 107  
 $\text{Ea}(K)$  (extremal algebra of  $K$ ), 647  
 $\text{Ea}^1(K)$  (derivable elements of  $\text{Ea}(K)$ ), 651  
 $\text{Ea}^n(K)$   $n \in \mathbb{N}$  ( $n$ -times derivable elements of  $\text{Ea}(K)$ ), 669  
 $\text{Ea}^{\infty}(K) = \bigcap_{n \in \mathbb{N}} \text{Ea}^n(K)$ , 669  
 $\eta : [1, \infty[ \rightarrow M_2(\mathbb{C})$ , 537  
 $\eta_K = \eta|_K$ , 537  
 $\eta_{ij} : [1, \infty[ \rightarrow M_2(\mathbb{C})$ , 537  
 $\eta_{ij}^K = (\eta_{ij})|_K$ , 537  
 $f(a)$ , 46, 57–59, 479, 484, 648  
 $\tilde{f} : A_{\Omega} \rightarrow A$ , 66, 486  
 $f^*(x) = \tilde{f}(x^*)$  for  $(x, f) \in X \times X'$ , 146  
 $f[ij]$  (for  $f \in C^{\mathbb{C}}(K)$  and  $i, j = 1, 2$ ), 538  
 $f'$  (for  $f \in \text{Ea}^1(K)$ ), 651  
 $F' : Y' \rightarrow X'$  (transpose of the operator  $F : X \rightarrow Y$ ), 29  
 $F \otimes G$  (operator tensor product of  $F$  and  $G$ ), 30  
 $F^* : K \rightarrow H$  (adjoint of the operator  $F : H \rightarrow K$ ), 38  
 $F^{\mathbb{K}}(E)$  ( $\mathbb{K}$ -valued functions on  $E$ ), 2  
 $F_i(f)(x_1, \dots, x_n)$  ( $0 \leq i \leq n$ ), 370  
 $\mathfrak{F}(X, Y)$  (finite-rank operators from  $X$  to  $Y$ ), 73  
 $\mathfrak{F}(X) = \mathfrak{F}(X, X)$ , 75  
 $\mathcal{F}(\mathbf{X}, \mathbb{K})$  (free non-associative  $\mathbb{K}$ -algebra on  $\mathbf{X}$ ), 258  
 $\mathcal{F}_p(\mathbf{X}, \mathbb{K})$  ( $1 \leq p < \infty$ ), 258  
 $\mathcal{F} : \mathcal{A}(K) \rightarrow C(K, M_2(\mathbb{C}))$ , 538  
 $G : A \rightarrow C^{\mathbb{C}}(\Delta)$  (Gelfand representation for complete normed unital associative and commutative complex algebras), 22  
 $G : J \rightarrow C_0^{\mathbb{T}}(\Lambda)$  (Gelfand representation for complex Banach Jordan  $*$ -triples), 500  
 $\Gamma$  (a contour in  $\mathbb{C}$ ), 58  
 $\Gamma_A$  (centroid of  $A$ ), 4  
 $\Gamma_{\ell}(A)$  (left centralizers on  $A$ ), 254  
 $\mathcal{G}(X)$  (surjective linear isometries on  $X$ ), 332  
 $\mathcal{G} : \mathcal{B}(K) \rightarrow C(K, \mathcal{C}_3)$ , 553  
 $H(X, *)$  ( $*$ -invariant elements of  $(X, *)$ ), 39  
 $H_1 \otimes H_2$  (Hilbert tensor product of  $H_1$  and  $H_2$ ), 417  
 $H_3(\mathbb{O})$  (Albert exceptional Jordan algebra), 337  
 $\mathcal{H}(\Omega)$  ( $\mathbb{C}$ -valued holomorphic functions on  $\Omega$ ), 59  
 $\mathbb{H}$  (algebra of Hamilton quaternions), 176  
 $\text{id}(x_0) = \{e \in A : ex_0 = x_0\}$ , 437  
 $\text{Ind}_{\Gamma}(z_0)$  (index of  $z_0$  with respect to  $\Gamma$ ), 58  
 $\text{Inv}(A)$  (invertible elements of  $A$ ), 5  
 $I_X$  (identity mapping on  $X$ ), 2  
 $(I : A) = \{x \in A : xA + Ax \subseteq I\}$ , 602  
 $\Im(z)$  (imaginary part of  $z$ ), 132  
 $J\text{-Inv}(A)$  ( $J$ -invertible elements of  $A$ ), 453, 475  
 $J\text{-Rad}(A)$  (Jacobson radical of  $A$ ), 569  
 $J\text{-sp}(A, a)$  ( $J$ -spectrum of  $a$  relative to  $A$ ), 456, 476  
 $J^{(e)}$  ( $e$ -homotope algebra of  $J$ ), 465  
 $J_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce subspaces of  $J$  relative to the tripotent  $e$ ), 505  
 $\ker(x_0) = \{a \in A : ax_0 = 0\}$ , 437  
 $k(F) = \max\{k \geq 0 : k\|x\| \leq \|F(x)\| \forall x \in X\}$ , 250  
 $K(X, u) = \bigcap_{f \in D(X, u)} \ker(f)$ , 351  
 $\mathfrak{K}(X, Y)$  (compact operators from  $X$  to  $Y$ ), 70  
 $\mathfrak{K}(X) = \mathfrak{K}(X, X)$ , 75  
 $\mathbb{K} = \mathbb{R}$  or  $\mathbb{C}$ , 1  
 $\mathbb{K}[\mathbf{x}]$  (polynomials over  $\mathbb{K}$  in the indeterminate  $\mathbf{x}$ ), 9  
 $\mathbb{K}(\mathbf{x})$  (fractions over  $\mathbb{K}$  in the indeterminate  $\mathbf{x}$ ), 57  
 $\text{lin}(S)$  (linear hull of  $S$ ), 351  
 $L(X, Y)$  (linear mappings from  $X$  to  $Y$ ), 1  
 $L(X) = L(X, X)$ , 1  
 $L_a$  (left multiplication by  $a$ ), 13  
 $L_S := \{L_x : x \in S\}$ , 433  
 $L_a^X$  (left multiplication by  $a$  on the bimodule  $X$ ), 637  
 $L_x^B = (L_x)_{|B}$ , 348  
 $L(x, y)(z) = \{xyz\}$ , 465  
 $L(J, J) = \{L(x, y) : x, y \in J\}$ , 468  
 $\Lambda_J$  (nonzero triple homomorphisms from  $J$  to  $\mathbb{C}$ ), 499  
 $\Lambda = \Lambda_J$ , 499  
 $m' : Z' \times X \rightarrow Y'$ , 124  
 $m'' : Y'' \times Z' \rightarrow X'$ , 124  
 $m''' : X'' \times Y'' \rightarrow Z''$ , 124  
 $m^t = m''''$ , 124  
 $m^r(y, x) = m(x, y)$ , 126

$$m^*(x, y) = (m(x^*, y^*))^*, 146$$

$M(A)$  (algebra of multipliers of  $A$ ), 126, 325

$M_n(X)$  ( $n \times n$  matrices with entries in the vector space  $X$ ), 166

$M_n(A)$  ( $n \times n$  matrices with entries in the algebra  $A$ ), 167

$M_\infty(\mathbb{K})$  (infinite matrices over  $\mathbb{K}$  with a finite number of nonzero entries), 267

$$M_{a,b}(x) = axb, 601$$

$\mathcal{M}(\mathbf{X})$  (free monad generated by  $\mathbf{X}$ ), 258

$$\mathcal{M}(B)^A, 357$$

$\mathcal{M}^2(A)$  (multiplication ideal of  $A$ ), 443

$n(a)$  (algebraic norm of  $a$ ), 181

$n(X, u)$  (numerical index of  $(X, u)$ ), 98

$$n^y(X, u), 99$$

$$n^{w^*}(X, u) = n^{X^*}(X, u), 295$$

$n_{\mathbb{R}}(X, u)$  (real numerical index of  $(X, u)$ ), 353

$N(X)$  (spatial numerical index of  $X$ ), 105

$\mathcal{N}(\mathbf{X}, \mathbb{K})$  (free normed non-associative  $\mathbb{K}$ -algebra on  $\mathbf{X}$ ), 258

$$\omega_K(z) = \max\{|e^{wz}| : w \in K\}, 645$$

$\mathbb{O}$  (algebra of Cayley numbers), 176

$$p(a) = \sum_{k=0}^n \alpha_k a^k \text{ for } p(\mathbf{x}) = \sum_{k=0}^n \alpha_k \mathbf{x}^k \in \mathbb{K}[\mathbf{x}], 9$$

$\mathbf{p}(a_1, \dots, a_n)$  (valuation of  $\mathbf{p}$  at  $(a_1, \dots, a_n)$ ), 262

$P(X)$  (continuous products on  $X$ ), 405

$p_A$  (product of  $A$ ), 408

$P_k(e)$   $k = 1, \frac{1}{2}, 0$  (Peirce projections relative to  $e$ ), 178, 505

$$\varphi(X, u, r) = \sup \left\{ \frac{\|u+r\| - 1}{r} - \tau(u, x) : x \in \mathbb{B}_X \right\}, 299$$

$\pi_1(\Gamma) = \{x : (x, f) \in \Gamma \text{ for some } f\}$ , 106

$\Pi(X) = \{(x, f) : x \in \mathbb{S}_X, f \in D(X, x)\}$ , 106

$\Pi(Y, X) = \{(y, x') \in \mathbb{S}_Y \times \mathbb{S}_{X'} : x' \in D(X, y)\}$ , 116

$\mathbb{P}$  (algebra of pseudo-octonions), 220

$q\text{-Inv}(A)$  (quasi-invertible elements of  $A$ ), 440

$$Q_x(y) = \{xyx\}, 506$$

$$Q_{x,z}(y) = \{xyz\}, 507$$

$$\mathcal{Q}_a = \left\{ \frac{p(\mathbf{x})}{q(\mathbf{x})} \in \mathbb{K}(\mathbf{x}) : q(a) \in \text{Inv}(A) \right\}, 57$$

$\mathcal{Q}\mathcal{M}(A)$  (quasi-full multiplication algebra of  $A$ ), 578

$\tau(a)$  (spectral radius of  $a$ ), 6, 381

$\text{Rad}(A)$  (radical of  $A$ ), 429

$R_a$  (right multiplication by  $a$ ), 13

$R_S := \{R_x : x \in S\}$ , 433

$R_a^X$  (right multiplication by  $a$  on the bimodule  $X$ ), 637

$\Re(z)$  (real part of  $z$ ), 95

$s(a)$  (succedaneous of the spectral radius of  $a$ ), 566

$\text{sp}(A, a)$  (spectrum of  $a$  relative to  $A$ ), 12

$\text{sp}(a) = \text{sp}(A, a)$ , 12

$s\text{-Rad}(A)$  (strong radical of  $A$ ), 20, 427

$S^c$  (commutant of  $S$ ), 24

$$S^{cc} = (S^c)^c, 24$$

$\sigma(x)$  (triple spectrum of  $x$ ), 504

$\mathcal{S}_3$  (three-dimensional real spin factor), 560

$\mathbb{S}$  (algebra of sedenions), 199

$\mathbb{S}_X$  (unit sphere of  $X$ ), 2

$\mathfrak{S}(\Phi)$  (separating space of  $\Phi$ ), 18

$t(a)$  (trace of  $a$ ), 181

$$\tau(u, x) = \max \Re(V(X, u, x)), 291$$

$\tau^t : C^c(F) \rightarrow C^c(E)$  for  $\tau : E \rightarrow F$ , 45

$\vartheta : M_2(\mathbb{C}) \rightarrow M_2(\mathbb{C})$ , 553–554

$\Theta : \mathcal{A}(K) \rightarrow \mathcal{A}(K)$ , 552–554

$\mathbb{T} = \mathbb{S}_{\mathbb{C}}$ , 10

$\text{uw-Rad}(A)$  (ultra-weak radical of  $A$ ), 580

$U(X, u) =$

$$\{f \in \mathbb{B}_{P(X)} : f(x, u) = f(u, x) = x \forall x \in X\}, 405$$

$$U_a = L_a(L_a + R_a) - L_{a^2}, 121$$

$$U_{a,b} = \frac{1}{2}[L_a(L_b + R_b) + L_b(L_a + R_a)] - L_{a \bullet b}, 364, 453$$

$$U_a^X(x) = a(ax + xa) - a^2x \text{ for } x \in X, 637$$

$$U_x^B = (U_x)_B, 348$$

$v(X, u, x)$  (numerical radius of  $x$  relative to  $(X, u)$ ), 98

$v(x) = v(X, u, x)$ , 98

$V(X, u, x)$  (numerical range of  $x$  relative to  $(X, u)$ ), 94

$V(x) = V(X, u, x)$ , 94

$w\text{-Rad}(A)$  (weak radical of  $A$ ), 578

$W(f)$  (spatial numerical range of  $f : \mathbb{S}_Y \rightarrow X$ ), 116, 308

$W(T)$  (spatial numerical range of  $T : X \rightarrow X$ ), 107

$W(T)$  (spatial numerical range of  $T : Y \rightarrow X$ ), 116

$\mathcal{W}(A) = \{a \in A : L_a, R_a \in \text{Rad}(\mathcal{Q}\mathcal{M}(A))\}$ , 578

$\mathfrak{W}(X, Y)$  (weakly compact operators from  $X$  to  $Y$ ), 70

$\mathfrak{W}(X) = \mathfrak{W}(X, X)$ , 75

$$x^{(2n+1)} = \{xx^{(2n-1)}x\} \text{ (triple powers of } x), 468$$

$X'$  ((topological) dual of  $X$ ), 2

$X''$  (bidual of  $X$ ), 2

$(X, u)$  (numerical-range space), 94

$X \otimes_\pi Y$  (projective tensor product of  $X$  and  $Y$ ), 31

$X \oplus_1 Y$  ( $\ell_1$ -sum of  $X$  and  $Y$ ), 109

$X_{\mathbb{R}}$  (real vector space underlying  $X$ ), 95

$X_{\mathbb{C}}$  (complexification of  $X$ ), 31

$X_n$  (continuous  $n$ -linear mappings from  $X^n$  to  $X$ ), 370

$X_{\mathcal{U}}$  (ultrapower of  $X$ ), 271

$(X_i)_{\mathcal{U}}$  (ultraproduct of the family  $\{X_i\}$ ), 271

$\mathcal{X}(U, \mathbb{K})$  (free vector space over  $\mathbb{K}$  generated by  $U$ ), 257

$$y \otimes f : x \rightarrow f(x)y, 73$$

$Z(A)$  (centre of  $A$ ), 192

$\mathcal{Z}(B)$  (centre modulo the radical of  $B$ ), 597

Cambridge University Press

978-1-107-04306-0- Encyclopedia of Mathematics and its Applications: Non-Associative Normed Algebras: Volume 1: The Vidav–Palmer and Gelfand–Naimark Theorems

Miguel Cabrera García and Ángel Rodríguez Palacios

Index

[More information](#)


---

## Subject index

- abelian Jordan  $*$ -triple, 468
- $A$ -bimodule, 636
- $A$ -bimodule relative to  $\mathcal{V}$ , 643
- absolute value, 176
- absolute-valued algebra, 176
- absolute-valued  $C^*$ -algebra, 416
- absolute-valued left semi- $H^*$ -algebra, 253
- adjoint operation (bilinear), 124
- adjoint operator, 39
- Albert isotopic (absolute-valued algebras), 211
- Albert radical, 599
- algebra, 1
- algebra admitting power-associativity, 493
- algebra antihomomorphism, 13
- algebra homomorphism, 2
- algebra involution, 39
- algebra isomorphism, 2
- algebra norm, 2
- algebra with hermitian multiplication, 581
- algebraic algebra, 180
- algebraic algebra of bounded degree, 212
- algebraic element, 180
- algebraic norm function, 181
- algebraically  $J$ -unitary element, 513
- algebraically unitary element, 102, 367
- almost norming subspace, 99
- almost transitive normed space, 302
- $\alpha$ -property, 135–137
- alternative algebra, 152
- alternative bimodule, 643
- alternative  $C^*$ -algebra, 153
- alternative  $C^*$ -complexification, 524
- alternative  $C^*$ -representation, 610
- alternative  $W^*$ -algebra, 409
- annihilator of an algebra, 4
- approximate unit, 404
- approximation problem, 90
- approximation property, 90
- $\mathfrak{A}$ -radical, 580
- Arens regular bilinear mapping, 126
- Arens regular normed algebra, 126
- Artin theorem, 153
- associative algebra, 1
- associative and commutative bimodule, 644
- associative bimodule, 643
- associator, 151
- $A$ -submodule (of a left  $A$ -module), 436
- automorphism of an  $n$ -algebra, 371
- Baer chain, 601
- Baer radical, 599
- Banach Jordan  $*$ -triple, 465
- Banach–Steinhaus closure theorem, 74
- Banach–Stone theorem, 151
- Bergmann operator, 509
- bicommutant, 24
- big point, 333
- Birkhoff–Witt theorem, 581
- Bishop–Phelps–Bollobás theorem, 287
- bounded below (operator), 27
- bounded index, 265
- Brown–McCoy radical, 20
- Calkin algebra, 93
- canonical derivation of  $Ea(K)$ , 651
- canonical involution
  - of the complexification, 31
  - of a matrix algebra, 167
- carrier space, 22
- Cayley algebra, 176
- Cayley numbers, 176
- Cayley–Dickson algebra, 199
- Cayley–Dickson doubling (of a Cayley algebra), 176
- Cayley–Dickson doubling process, 176
- central algebra over  $\mathbb{K}$ , 4
- central element, 192
- centralizer (on an algebra), 4
- centralizer set for a left  $A$ -module, 439
- centre, 192
- centre modulo the radical, 597
- centroid, 4
- character, 20
- closeable operator, 651
- closed curve, 58
- closed  $J$ -full subalgebra generated by a subset, 483
- closed operator, 641

- closed  $*$ -subalgebra generated by a subset, 419
- closed subalgebra generated by a subset, 9
- closed subtriple generated by a subset, 466
- closure of a closable operator, 651
- commutant, 24
- commutative algebra, 1
- commutative subset, 24
- commutator, 126
- compact operator, 70
- complete holomorphic vector field, 174
- complete normed algebra, 2
- complete tripotent, 517
- complex extreme point, 321
- complexification, 31
- composition algebra, 186
- cone, 49
- continuous functional calculus, 46, 479
- contour, 58
- contour surrounds  $K$  in  $\Omega$ , 58
- convex cone, 49
- convex-transitive normed space, 333
- core of a subspace (of an algebra), 429
- cross-product algebra, 187
- $CS$ -closed set, 294
- $C^*$ -algebra, 39
- $C^*$ -algebra of multipliers, 126
- $C^*$ -complexification, 524
- $C^*$ -equivalent algebra, 632
- $C^*$ -isotope algebra, 415
- $C^*$ -norm, 141
- $C^*$ -representation, 610
- $C^*$ -seminorm, 141
- $C^*$ -unital extension, 609
- curve, 58
- cyclic vector, 437
- degree of a non-associative word
  - global, 258
  - in each indeterminate, 373
- degree of an algebra, 212
- densely defined operator, 641
- density character, 257
- denting point, 118
- derivation
  - of an algebra, 122
  - of an  $n$ -algebra, 371
- descending chain condition, 583
- direct product of algebras, 33
- disc algebra, 315
- dissipative element, 97
- distinguished element (of a numerical range space), 94
- division algebra, 192
- division alternative algebra, 188
- division associative algebra, 15
- divisor of zero (joint, left, one-sided, right, two-sided), 27
- duality mapping, 284
- $e$ -homotope algebra, 465
- eigenvalue, 80
- eigenvector, 80
- element acting weakly as a unit, 316
- equivalent non-commutative  $JB^*$ -representations, 618
- essential ideal, 149
- exponential, 10, 342
- extended centroid, 195
- extremal algebra of  $K$ , 647
- finite-rank operator, 73
  - (first) Arens extension, 125
  - (first) Arens product, 125
- flexible algebra, 149
- flexible quadratic algebra of a pre- $H$ -algebra, 204
- (Fréchet) derivative of a function at a point, 8
- (Fréchet) differentiable function at a point, 8
- free complete normed non-associative algebra, 261
- free non-associative algebra, 258
- free (non-associative) monad, 258
- free normed non-associative algebra, 259
- Frobenius–Zorn theorem, 191
- full subalgebra, 22, 480
- fundamental formula
  - for Jordan algebras, 364
  - for Jordan  $*$ -triples, 508
- Gâteaux derivative of the norm, 204
- Gelfand homomorphism theorem
  - non-unital version, 428
  - unital version, 23
- Gelfand representation
  - of a complete normed unital associative and commutative complex algebra, 22
  - of a complex Banach Jordan  $*$ -triple, 500
- Gelfand space, 22
- Gelfand theory, 22
- Gelfand topology, 22
- Gelfand transform of an element, 22
- Gelfand–Beurling formula
  - associative, 15
  - Jordan, 458
- Gelfand–Mazur theorem
  - complex, 15
  - real, 194
- Gelfand–Mazur–Kaplansky theorem, 197
- Gelfand–Naimark theorem
  - commutative, 40
  - non-commutative, 40
  - non-unital non-associative, 415
  - unital non-associative, 343
- generalized standard algebra, 278
- generated as a normed algebra by a subset, 25
- generated as a normed  $*$ -algebra by a subset, 538
- generator of  $Ea(K)$ , 647
- geometric functional calculus, 648
- geometrically unitary element, 100

- $H$ -algebra, 208  
 hereditarily indecomposable Banach space, 247  
 hermitian Banach Jordan  $*$ -triple, 465  
 hermitian element, 97  
 hermitian Jordan-admissible complex  $*$ -algebra, 613  
 Hilbert tensor product, 417  
 hole, 29  
 holomorphic functional calculus, 64, 485  
 holomorphic vector field, 174  
 $H^*$ -algebra, 222  
 Hurwitz theorem, 217
- ideal (left, right, two-sided), 16  
 ideal generated by a subset, 583  
 idempotent, 3  
 identity, 406  
 index of a point with respect to a contour, 58  
 index of nilpotency, 265  
 inner ideal, 594  
 intrinsic numerical range, 308  
 inverse element, 5, 187  
 invertible element, 5, 187  
 involution on a set, 39  
 irreducible left  $A$ -module, 437  
 irreducible representation, 437  
 isomorphic left  $A$ -modules, 439  
 isotropic element, 179  
 $i$ -special Jordan algebra, 425
- Jacobson density theorem, 445  
 Jacobson radical  
   of an associative algebra, 429  
   of a Jordan-admissible algebra, 569  
 $JB$ -algebra, 319  
 $JB$ -algebra of multipliers, 325  
 $JB^*$ -admissible algebra, 406  
 $JB^*$ -algebra, 345  
 $JB^*$ -complexification, 524  
 $JB^*$ -representation, 610  
 $JB^*$ -triple, 130  
 $JB^*$ -triple complexification, 524  
 $JBW$ -algebra, 323  
 $JBW^*$ -triple, 528  
 $JC$ -algebra, 320  
 $JC^*$ -algebra, 345  
 $J$ -division Jordan algebra, 457  
 $J$ -division Jordan-admissible algebra, 475  
 $J$ -divisor of zero, 460, 478, 496  
 $J$ -full subalgebra, 476  
 $J$ -full subalgebra generated by a subset, 483  
 $J$ -inverse element, 451, 473  
 $J$ -invertible element, 451, 473  
 Johnson uniqueness-of-norm theorem, 565  
 Johnson–Aupetit–Ransford theorem, 570  
 Jordan  $A$ -bimodule, 637  
 Jordan-admissible algebra, 163  
 Jordan algebra, 162  
 Jordan derivation, 122  
 Jordan homomorphism, 122  
 Jordan identity, 162  
 Jordan  $*$ -triple, 463  
 Jordan triple identity, 463  
 $J$ -primitive ideal, 594  
 $J$ -primitive Jordan algebra, 594  
 $J$ -semisimple Jordan-admissible algebra, 569  
 $J$ -spectrum, 456  
 $J$ -unitary element, 512
- $\mathbb{K}$ -extreme point, 321  
 Kadison isometry theorem, 131  
 Kadison–Paterson–Sinclair theorem, 127  
 Kernel of a numerical-range space, 351  
 Kleinecke–Shirokov theorem, 442  
 Kurosh’s problem, 276
- left  $A$ -module, 436  
 left  $A$ -module corresponding to a representation, 436  
 left centralizer, 254  
 left-division algebra, 192  
 left multiplication operator, 13  
 left powers, 665  
 left semi- $H^*$ -algebra, 237  
 left standard representation, 436  
 left unit, 219  
 Lie algebra, 581  
 locally  $C^*$ -equivalent algebra, 632  
 locally finite algebra, 276  
 locally nilpotent algebra, 277  
 logarithm (of an element of an algebra), 68  
 $L$ -summand, 314
- Macdonald’s theorem, 389  
 matricial  $L_\infty$ -property, 170  
 matricial  $L_\infty^2$ -property, 170  
 maximal ideal (left, right, two-sided), 17  
 maximal modular ideal (left, right, two-sided), 427  
 maximal modular inner ideal, 594  
 $M$ -ideal, 315  
 minimal ideal (left, right, two-sided), 179  
 minimality of norm, 576  
 minimality of norm topology, 572  
 minimum norm, 596  
 minimum norm topology, 596  
 minimum polynomial, 180  
 modular ideal (left, right, two-sided), 426  
 modular unit (left, right), 426  
 module homomorphism, 439  
 module multiplication, 436  
 modulus of midpoint local convexity, 111  
 monad, 258  
 multilinear identity, 406  
 multiplication, 1  
 multiplication ideal, 443  
 multiplicatively nil ideal, 601

- Nagata–Higman theorem, 267  
 $n$ -algebra, 371  
 natural involution of a  $V$ -algebra, 134  
 $n$ -contractive operator, 169  
 nearly absolute-valued algebra, 198  
 nice algebra, 122  
 nil algebra, 265  
 nil algebra of bounded index, 265  
 nilpotent subset, 265  
 $n$ -linear non-associative word, 373  
 non-associative  $C^*$ -algebra, 170  
 non-associative polynomial, 262  
 non-associative word, 258  
 non-commutative  $JB^*$ -algebra, 345  
 non-commutative  $JB^*$ -complexification, 524  
 non-commutative  $JB^*$ -representation, 610  
     that factors through another, 618  
 non-commutative  $JB^*$ -unital extension, 609  
 non-commutative  $JBW^*$ -algebra, 531  
 non-commutative Jordan  $A$ -bimodule, 637  
 non-commutative Jordan algebra, 163  
 non-thin set at a point, 612  
 norm-unital normed algebra, 34  
 normal element, 42, 365  
 normal subset, 418  
 normed  $A$ -bimodule, 638  
 normed algebra, 2  
 (normed) algebra completion, 35  
 normed complexification, 31  
 normed  $n$ -algebra, 371  
 normed  $Q$ -algebra  
     associative, 440  
     Jordan-admissible, 572  
 normed unital extension, 609  
 norming subspace, 99  
 nowhere dense subset, 302  
 numerical index, 98  
 numerical radius, 98  
 numerical range, 94  
 numerical-range order, 143  
 numerical-range space, 94  
  
 octonions, 176  
     complex, 205  
 one-parameter semigroup, 10  
 one-sided division algebra, 192  
 one-sided semi- $H^*$ -algebra, 252  
 operator algebra, 173  
 operator space, 175  
 operator system, 175  
 operator that factors through a space, 87  
 opposite algebra, 13  
 order defined by a proper convex cone, 49  
 orthogonal idempotents, 54  
 orthogonal subtriples, 514  
  
 partial isometry, 552  
 partially defined centralizer, 194  
 partially defined derivation, 642  
  
 partially defined linear operator, 640  
 Peirce decomposition  
     of a Jordan  $*$ -triple, 505  
     of a power-associative algebra, 179  
 plenary powers, 566  
 polynomial function, 263  
 polynomial functional calculus, 54  
 positive element  
     of a  $C^*$ -algebra, 47  
     of a  $JB$ -algebra, 328  
     of a non-commutative  $JB^*$ -algebra, 383  
 positive hermitian Banach Jordan  $*$ -triple, 465  
 positive linear functional, 141  
 power-associative  $A$ -bimodule, 651  
 power-associative algebra, 164  
 power-commutative algebra, 165  
 pre-duality mapping, 285  
 pre- $H$ -algebra, 204  
 prime algebra, 194  
 prime ideal, 430  
 primitive algebra, 429  
 primitive ideal, 429  
 product, 1  
 product of an  $n$ -algebra, 371  
 projective tensor norm, 31  
 projective tensor product, 31  
 proper cone, 49  
 proper ideal, 16  
 pseudo-octonions, 220  
  
 quadratic algebra, 180  
 quadratic commutative algebra of a real  
     pre-Hilbert space, 232  
 quadratic form admitting composition, 183  
 quadratic operator, 506  
 quasi-division algebra, 192  
 quasi-full multiplication algebra, 578  
 quasi-full subalgebra, 440  
 quasi-full subalgebra generated by a subset, 578  
 quasi-inverse, 431  
 quasi-invertible element, 431  
 quasi-invertible subset, 431  
 quasi- $J$ -full subalgebra, 594  
 quasi- $J$ -inverse element, 585  
 quasi- $J$ -invertible element, 568  
 quasi- $J$ -invertible subset, 568  
 quaternions, 176  
 quotient algebra, 18  
 quotient involution, 145  
  
 radical, 429  
 radical algebra, 429  
 rational functional calculus, 57  
 real alternative  $C^*$ -algebra, 521  
 real  $C^*$ -algebra, 521  
 real  $JB^*$ -algebra, 521  
 real  $JB^*$ -triple, 522  
 real non-commutative  $JB^*$ -algebra, 521  
 real numerical index, 353

Cambridge University Press

978-1-107-04306-0- Encyclopedia of Mathematics and its Applications: Non-Associative Normed Algebras: Volume 1: The Vidav–Palmer and Gelfand–Naimark Theorems

Miguel Cabrera García and Ángel Rodríguez Palacios

Index

[More information](#)*Subject index*

711

- regular  $A$ -bimodule, 639
- regular left  $A$ -module, 436
- representation (of an associative algebra), 436
- representation corresponding to a left  $A$ -module, 436
- Rickart's dense-range-homomorphism theorem
  - non-associative to Jordan-admissible, 458
  - non-associative to non-unital associative, 427
  - non-associative to unital associative, 20
- Riesz–Schauder theory, 86
- right-division algebra, 192
- right multiplication operator, 13
- right semi- $H^*$ -algebra, 252
- Russo–Dye theorem, 140
- Russo–Dye–Palmer theorem, 141
  
- scalar-plus-compact property, 248
- scalar-plus-strictly-singular property, 248
- Schoenberg theorem, 216
- Schur lemma, 445
- second Arens extension, 126
- second Arens product, 126
- second commutant, 24
- sedenions, 199
- self-adjoint element, 42
- semi- $H^*$ -algebra, 254
- semi- $L$ -summand, 314
- semi- $M$ -ideal, 315
- semiprime algebra, 128
- semiprime ideal, 430
- semisimple algebra, 229
- separating points (family of mappings), 22
- separating space (of an operator), 18
- Shirshov–Cohn theorem, 337
  - with inverses, 491
- simple algebra, 18
- Singer–Wermer theorem, 391, 443
- smooth normed space at a norm-one element, 203
- smooth-normed algebra, 204
- solvable algebra, 269
- spatial numerical index, 105
- spatial numerical range, 107, 116, 308
- special Jordan algebra, 337
- spectral mapping theorem
  - for the continuous functional calculus, 47, 479
  - for the holomorphic functional calculus, 64, 484
- spectral radius, 6, 381
- spectrum of an element, 12
- split null  $A$ -extension, 639
- split null  $X$ -extension, 636
- standard involution
  - of a Cayley algebra, 176
  - of a free non-associative algebra, 258
- standard left  $A$ -module, 436
- standard normed unital extension, 609
- $*$ -algebra, 39
- $*$ -mapping, 39
- $*$ -subalgebra, 39
- state of  $X$  relative to  $u$ , 94
  
- Stone–Weierstrass theorem
  - unital version, 41
  - unit-free version, 53
- strict inner ideal, 594
- strictly singular operator, 248
- strong radical, 20, 427
- strong subdifferentiability of the norm, 299
- strongly associative subalgebra of a Jordan algebra, 356
- strongly exposed point, 118
- strongly exposed subset, 299
- strongly extreme point, 111
- strongly semisimple algebra, 20, 427
- subalgebra, 2
- subalgebra generated by a subset, 9
- subharmonic function, 611
- submean inequality, 611
- subtriple, 465
- subtriple generated by a subset, 466
- super-trigonometric algebra, 201
- symmetry (of a unital  $JB$ -algebra), 321
  
- $\tau$ -point, 299
- three-dimensional real spin factor, 560
- three-dimensional spin factor, 553
- topological divisor of zero (joint, left, one-sided, right, two-sided), 27
- topological group, 6
- topological  $J$ -divisor of zero, 460, 478, 496
- topologically nilpotent algebra, 604
- topologically simple algebra, 82
- totally disconnected, 399
- trace function, 181
- transitive normed space, 217
- transpose mapping of a continuous mapping, 45
- transpose of an involution, 146
- transpose of an operator, 29
- trigonometric algebra, 200
- triple homomorphism, 471
- triple powers, 468
- triple product, 127, 130, 324, 463
- triple spectrum, 504
- tripotent, 505
  
- $u$ -isotope  $JB^*$ -algebra, 519
- ultra-weak radical, 580
- ultrapower, 271
- ultraproduct, 271
- uniform Fréchet differentiability of the norm, 304
- uniform strongly subdifferentiability of the norm, 301
- uniformly non-square normed space, 230
- uniformly smooth normed space, 304
- unit, 2
- unital  $A$ -bimodule, 637
- unital algebra, 2
- unital extension, 33
- unital  $*$ -representation, 233
- unitary element, 43, 368, 471



Cambridge University Press

978-1-107-04306-0- Encyclopedia of Mathematics and its Applications: Non-Associative Normed Algebras: Volume 1: The Vidav–Palmer and Gelfand–Naimark Theorems

Miguel Cabrera García and Ángel Rodríguez Palacios

Index

[More information](#)

712

*Subject index*

- unitary normed algebra, 119
- upper semicontinuity (of a set-valued function), 284
- Urbanik–Wright theorem
  - commutative, 216
  - non-commutative, 216
- $V$ -algebra, 134
- variety of algebras, 424
- vertex, 99
- Vidav algebra, 134
- Vidav–Palmer theorem
  - associative, 142
  - alternative, 153
  - non-associative, 348
- $\mathcal{V}$ -normal element, 424
- $\mathcal{V}$ -normal subset, 424
- von Neumann inequality, 174
- von Neumann lemma, 7, 457
- Vowden theorem, 421
- weak radical, 578
- weakly compact operator, 70
- Weil algebra, 588
- $w^*$ -superbig point, 334
- $w^*$ -unitary element, 295
- $w^*$ -vertex, 295
- $x$ -modular strict inner ideal, 594
- $X$ -valued partially defined derivation, 640
- zero-annihilator ideal ( $z$ -ideal), 602
- zero-annihilator radical ( $z$ -radical), 602
- Zorn's vector matrices, 177