

Complexity of Infinite-Domain Constraint Satisfaction

Constraint Satisfaction Problems (CSPs) are natural computational problems that appear in many areas of theoretical computer science. Exploring which CSPs are solvable in polynomial time and which are NP-hard reveals a surprising link with central questions in universal algebra.

This monograph presents a self-contained introduction to the universal-algebraic approach to complexity classification, treating both finite and infinite-domain CSPs. It includes the required background from logic and combinatorics, particularly model theory and Ramsey theory, and explains the recently discovered link between Ramsey theory and topological dynamics and its implications for CSPs. The book will be of interest to graduate students and researchers in theoretical computer science and to mathematicians in logic, combinatorics, and dynamics who wish to learn about the applications of their work in complexity theory.

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***Complexity of Infinite-Domain
Constraint Satisfaction***

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Technische Universität Dresden



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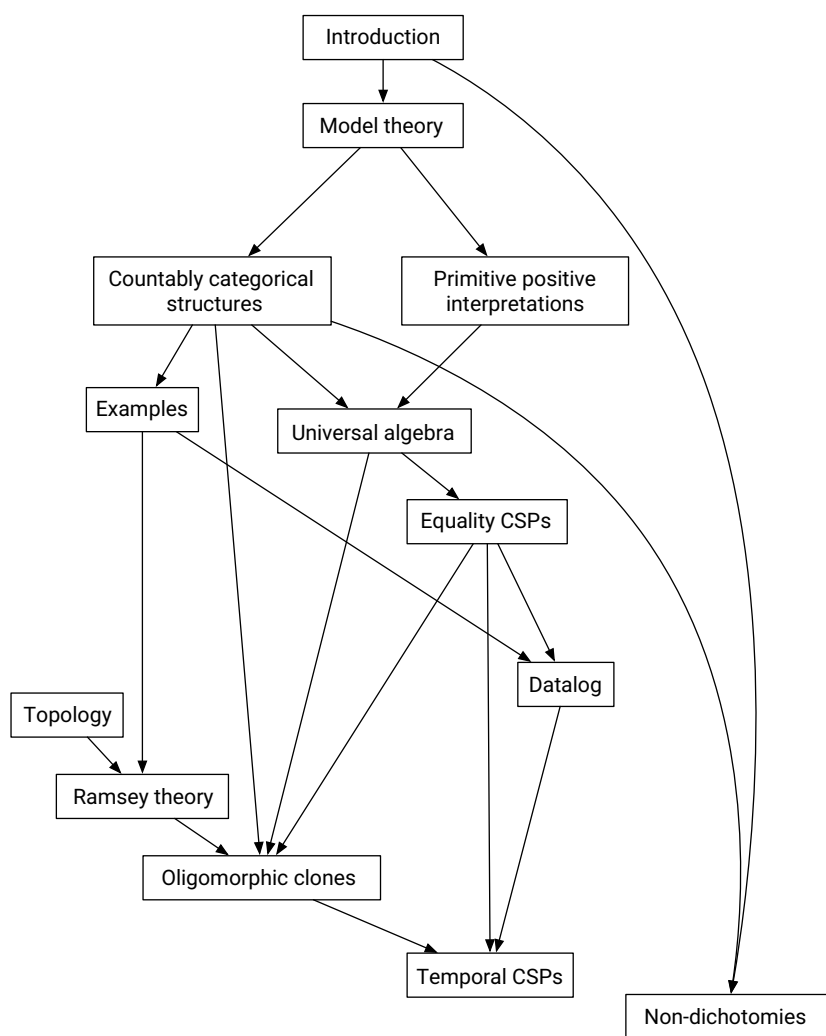
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