Long-Range Dependence and Self-Similarity

This modern and comprehensive guide to long-range dependence and self-similarity starts with rigorous coverage of the basics, then moves on to cover more specialized, up-to-date topics central to current research. These topics concern, but are not limited to, physical models that give rise to long-range dependence and self-similarity; central and non-central limit theorems for long-range dependent series, and the limiting Hermite processes; fractional Brownian motion and its stochastic calculus; several celebrated decompositions of fractional Brownian motion; multidimensional models for long-range dependence and self-similarity; and maximum likelihood estimation methods for long-range dependent time series. Designed for graduate students and researchers, each chapter of the book is supplemented by numerous exercises, some designed to test the reader's understanding, while others invite the reader to consider some of the open research problems in the field today.

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To Natércia and Filipa
and
to Rachelle, Yael, Jonathan, Noah, Kai and Olivia

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Abbreviations

ACVF	autocovariance function
ACF	autocorrelation function
AIC	Akaike information criterion
AR	autoregressive
ARMA	autoregressive moving average
BIC	Bayesian information criterion
biFBM	bifractional Brownian motion
BLUE	best linear unbiased estimator
BM	Brownian motion
CDF	cumulative distribution function
CMF	conjugate mirror filters
CRW	correlated random walk
FARIMA	fractionally integrated autoregressive moving average
FBM	fractional Brownian motion
FBF	fractional Brownian field
FBS	fractional Brownian sheet
FGN	fractional Gaussian noise
FFT	fast Fourier transform
FGN	fractional Gaussian noise
LFSM	linear fractional stable motion
LFSN	linear fractional stable noise
LRD	long-range dependence, long-range dependent
MA	moving average
MLE	maximum likelihood estimation
MRA	multi-resolution analysis
MSE	mean squared error
ODE	ordinary differential equation
OFBF	operator fractional Brownian field
OFBM	operator fractional Brownian motion
OU	Ornstein–Uhlenbeck

PDF	probability density function
RKHS	reproducing kernel Hilbert space
SDE	stochastic differential equation
SPDE	stochastic partial differential equation
SRD	short-range dependence, short-range dependent
SS	self-similar
SSSI	self-similar with stationary increments
VFBM	vector fractional Brownian motion
WN	white noise

Notation

Numbers and sequences

$n!$	n factorial
$\mathbb{Z}$	set of integers $\{\dots, -1, 0, 1, \dots\}$
$\mathbb{R}_+$	half-axis $(0, \infty)$
$\|x\|_2$	Euclidean norm $\|x\|_2 = (\sum_{j=1}^q \|x_j\|^2)^{1/2}$ for $x = (x_1, \dots, x_q)' \in \mathbb{C}^q$
x_+, x_-	$\max\{x, 0\}$, $\max\{-x, 0\}$, respectively
$\Re, \Im$	real and imaginary parts, respectively
$\overline{z}$	complex conjugate of $z \in \mathbb{C}$
$x \wedge y, x \vee y$	$\min\{x, y\}$, $\max\{x, y\}$, respectively
$[x], \lceil x \rceil$	(floor) integer part and (ceiling) integer part of x, respectively
$\widehat{a}$	Fourier transform of sequence a
$a^\vee$	time reversion of sequence a
$a * b$	convolution of sequences a and b
$\downarrow_2, \uparrow_2$	downsampling and upsampling by 2 operations, respectively
$\ell^p(\mathbb{Z})$	space of sequences $\{a_n\}_{n\in\mathbb{Z}}$ such that $\sum_{n=-\infty}^{\infty} \|a_n\|^p < \infty$

Functions

$\widehat{f}$	Fourier transform of function f
$f * g$	convolution of functions f and g
$f \otimes g$	tensor product of functions f and g
$f^{\otimes k}$	kth tensor product of a function f
1_A	indicator function of a set A
$\log, \log_2$	natural logarithm (base e) and logarithm base 2, respectively
e^{ix}	complex exponential, $e^{ix} = \cos x + i \sin x$
$\Gamma(\cdot)$	gamma function
$B(\cdot, \cdot)$	beta function
$H_k(\cdot)$	Hermite polynomial of order k
$C^k(I)$	space of k–times continuously differentiable functions on an interval I
$C_b^k(I)$	space of k–times continuously differentiable functions on an interval I with the first k derivatives bounded
$L^p(\mathbb{R}^q)$	space of functions $f : \mathbb{R}^q \to \mathbb{R}$ (or $\mathbb{C}$) such that $\int_{\mathbb{R}^q} \|f(x)\|^p dx < \infty$
$\|f\|_{L^p(\mathbb{R}^q)}$	(semi-)norm $\left(\int_{\mathbb{R}^q} \|f(x)\|^p dx \right)^{1/p}$
$L^p(E, m)$	space of functions $f : \mathbb{E} \to \mathbb{R}$ (or $\mathbb{C}$) such that $\int_E \|f(x)\|^p m(dx) < \infty$
$f^{\leftarrow}, f^{-1}$	(generalized) inverse of non-decreasing function f

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Matrices

$\mathbb{R}^{p\times p}$, $\mathbb{C}^{p\times p}$	collections of $p \times p$ matrices with entries in $\mathbb{R}$ and $\mathbb{C}$, respectively
A', A^T	transpose of a matrix A
A^*	Hermitian transpose of a matrix A
$\det(A)$	determinant of a matrix A
$\mathrm{Pf}A$	Pfaffian of a matrix A
$\|A\|$	matrix (operator) norm
$A \otimes B$	Kronecker product of matrices A and B
$A^{1/2}$	square root of positive semidefinite matrix A
$\mathrm{tr}(A)$	trace of matrix A

Probability

Var, Cov	variance, covariance, respectively
$\xrightarrow{fdd}$	convergence in the sense of finite-dimensional distributions
$\xrightarrow{d}$, $\xrightarrow{p}$	convergence in distribution and in probability, respectively
$\stackrel{d}{=}$, $\sim$	equality in distribution, as in $Z \sim \mathcal{N}(0,1)$
$S\alpha S$	symmetric α-stable
$\|\xi\|_p$	$(\mathbb{E}\lvert\xi\rvert^p)^{1/p}$ for a random variable ξ
$\mathcal{N}(\mu,\sigma^2)$	normal (Gaussian) distribution with mean μ and variance σ^2
$\chi(X_1,\ldots,X_p)$	joint cumulant of random variables $X_1,\ldots,X_p$
$\chi_p(X)$	pth cumulant of a variable X

Time series and stochastic processes

$\gamma_X(\cdot)$	ACVF of series X
$f_X(\cdot)$	spectral density of series X
$I_X(\cdot)$	periodogram of series X
B	backshift operator
B_H or B^κ	FBM, $H = \kappa + 1/2$; vector OFBM
$Z_H^{(k)}$	Hermite process of order k
$B_{E,H}$	OFBF

Fractional calculus, Malliavin calculus and FBM

I^α_{a+}, I^α_{b-}	fractional integrals on interval
D^α_{a+}, $\mathcal{D}^\alpha_{b-}$	fractional derivatives on interval
$I^\alpha_\pm$, $\mathcal{D}^\alpha_\pm$, $\mathbf{D}^\alpha_\pm$	fractional integrals and derivatives on real line
span, $\overline{\mathrm{span}}$	linar span and closure of linear span, respectively
$\mathcal{I}^\kappa_a(\cdot)$	fractional Wiener integral
Λ^κ_a, $\lvert\Lambda\rvert^\kappa_a$	spaces of integrands
$(\cdot,\cdot)_{\mathcal{H}}$	inner product on Hilbert space $\mathcal{H}$
$\delta(\cdot)$	divergence integral ($\delta^\kappa_a(\cdot)$, in the case of FBM B^κ on an interval $[0,a]$)
D	Malliavin derivative
$\mathbb{D}^{1,p}$, $\mathbb{D}^{k,p}$	domains of the Malliavin derivative
$I_n(\cdot)$, $\widehat{I}_n(\cdot)$	multiple integrals of order n

Miscellaneous

a.e., a.s.	almost everywhere, almost surely, respectively
$\sim$	asymptotic equivalence; that is, $a(x) \sim b(x)$ if $a(x)/b(x) \to 1$
$o(\cdot)$, $O(\cdot)$	small and big O, respectively
$\mathcal{B}(A)$	σ–field of Borel sets of a set A
$\delta_a(v)$	point mass at $v = a$
mod	modulo

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Preface

We focus in this book on long-range dependence and self-similarity. The notion of long-range dependence is associated with time series whose autocovariance function decays slowly like a power function as the lag between two observations increases. Such time series emerged more than half a century ago. They have been studied extensively and have been applied in numerous fields, including hydrology, economics and finance, computer science and elsewhere. What makes them unique is that they stand in sharp contrast to Markovian-like or short-range dependent time series, in that, for example, they often call for special techniques of analysis, they involve different normalizations and they yield new limiting objects.

Long-range dependent time series are closely related to self-similar processes, which by definition are statistically alike at different time scales. Self-similar processes arise as large scale limits of long-range dependent time series, and vice versa; they can give rise to long-range dependent time series through their increments. The celebrated Brownian motion is an example of a self-similar process, but it is commonly associated with independence and, more generally, with short-range dependence. The most studied and well-known self-similar process associated with long-range dependence is fractional Brownian motion, though many other self-similar processes will also be presented in this book. Self-similar processes have become one of the central objects of study in probability theory, and are often of interest in their own right.

This volume is a modern and rigorous introduction to the subjects of long-range dependence and self-similarity, together with a number of more specialized up-to-date topics at the center of this research area. Our goal has been to write a very readable text which will be useful to graduate students as well as to researchers in Probability, Statistics, Physics and other fields. Proofs are presented in detail. A precise reference to the literature is given in cases where a proof is omitted. Chapter 2 is fundamental. It develops the basics of long-range dependence and self-similarity and should be read by everyone, as it allows the reader to gain quickly a basic familiarity with the main themes of the research area. We assume that the reader has a background in basic time series analysis (e.g., at the level of Brockwell and Davis [186]) and stochastic processes. The reader without this background may want to start with Chapter 1, which provides a brief and elementary introduction to time series analysis and stochastic processes.

The rest of the volume, namely Chapters 3–10, introduces the more specialized and advanced topics on long-range dependence and self-similarity. Chapter 3 concerns physical models that give rise to long-range dependence and/or self-similarity. Chapters 4 and 5 focus on central and non-central limit theorems for long-range dependent series, and introduce the limiting Hermite processes. Chapters 6 and 7 are on fractional Brownian motion and its stochastic calculus, and their connection to the so-called fractional calculus, the area of real analysis which extends the usual derivatives and integrals to fractional orders. Chapter 8 pursues the discussion on fractional Brownian motion by introducing several of its celebrated decompositions. Chapter 9 concerns multidimensional models, and Chapter 10 reviews the maximum likelihood estimation methods for long-range dependent time series. Chapters 3 through 10 may be read somewhat separately. They are meant to serve both as a learning tool and as a reference. Finally, Appendices A, B and C are used for reference throughout the book.

Each chapter starts with a brief overview and ends with exercises and bibliographical notes. In Appendix D, "Other notes and topics" contains further bibliographical information. The reader will find the notes extremely useful as they provide further perspectives on the subject as well as suggestions for future research.

A number of new books on long-range dependence and self-similarity, listed in Appendix D, have been published in the last ten years or so. Many features set this book apart. First, a number of topics are not treated elsewhere, including most of Chapter 8 (on decompositions) and Chapter 9 (on multidimensional models). Second, other topics provide a more systematic treatment of the area than is otherwise presently available; for example, in Chapter 3 (on physical models). Third, some specialized topics, such as in Chapters 6 and 7 (on stochastic analysis for fractional Brownian motion), reflect our perspective. Fourth, most specialized topics are up to date; for example, the classical results of Chapters 4 and 5 (on Hermite processes and non-central limit theorems) have been supplemented by recent work in the area. Finally, the book contains a substantial number of early and up-to-date references. Though even with this large number of references (more than a thousand), we had to be quite selective and could not include every single work that was relevant.

We would like to conclude this preface with acknowledgments and a tribute. A number of our former and present students have read carefully and commented on excerpts of this book, including Changryong Baek, Stefanos Kechagias and Sandeep Sarangi at the University of North Carolina, Shuyang Bai, Long Tao and Mark Veillette at Boston University, Yong Bum Cho, Heng Liu, Xuan Yang, Pengfei Zhang, Yu Gu, Junyi Zhang, Emilio Seijo and Oliver Pfaffel at Columbia University. Various parts of this volume have already been used in teaching by the authors at these institutions. As the book was taking its final shape, a number of researchers read carefully individual chapters and provided invaluable feedback. We thus express our gratitude to Solesne Bourguin, Gustavo Didier, Liudas Giraitis, Kostas Spiliopoulos, Stilian Stoev, Yizao Wang. We are grateful to Diana Gillooly at Cambridge University Press for her support and encouragement throughout the process of writing the book. We also thank the National Science Foundation and the National Security Agency for their support. We are responsible for the remaining errors and typos, and would be grateful to the readers for a quick email to either author if such are found.

Finally, we would like to pay tribute to Benoit B. Mandelbrot,[1] a pioneer in the study of long-range dependence and self-similarity, among his many other interests. He was the

[1] A fascinating account of Benoit B. Mandelbrot's life can be found in [681].

first to greatly popularize and draw attention to these subjects in the 1970s. It was Benoit B. Mandelbrot who introduced one of the authors (M.S.T.) to this area, who in turn passed on his interests and passion to the other author (V.P.). This volume, including the presented specialized topics, are direct fruits of the work started by Benoit B. Mandelbrot. With his passing in 2010, the scientific community lost one of its truly bright stars.