

PROBABILITY-BASED STRUCTURAL FIRE LOAD

In the structural design of airframes and buildings, probability-based procedures are used to mitigate the risk of failure as well as produce cost-effective designs. This book introduces the subject of probabilistic analysis to structural and fire protection engineers and can also be used as a reference to guide those applying this technology.

In addition to providing an understanding of how fire affects structures and how to optimize the performance of structural framing systems, *Probability-Based Structural Fire Load* provides guidance for design professionals and is a resource for educators.

The goal of this book is to bridge the gap between prescriptive and probability-based performance design methods and to simplify very complex and comprehensive computer analyses to the point that stochastic structural fire loads have a simple, approximate analytical expression that can be used in structural analysis and design on a day-to-day basis. Numerous practical examples are presented in step-by-step computational form.

Leo Razdolsky has more than fifty years of experience in structural engineering, including the design of high-rise and mid-rise buildings, field inspections, and construction management. His specialty projects include stadiums, cable structures, exhibition halls and pavilions, restoration and rehabilitation of buildings, power plants, cooling towers, and bridges. He also has experience in computer modeling, wind-tunnel-testing analysis, dynamic analysis of structures, seismic design, and complex-foundation-systems analysis and design. Dr. Razdolsky has been teaching various structural engineering courses for more than fifteen years at the University of Illinois at Chicago and at Northwestern University. For the past ten years, he has been conducting research on the analytical methods of obtaining the structural fire load and on high-rise building designs subjected to abnormal fire conditions. He is currently a member of the Fire & Safety Working Group at the Council on Tall Buildings and Urban Habitat (CTBUH).

I dedicate this book to the memory of my parents

Probability-Based Structural Fire Load

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Contents

<i>Foreword by William F. Baker</i>	<i>page ix</i>
<i>Preface</i>	xi
<i>Acknowledgments</i>	xv
1 Introduction	1
1.1 Deterministic Approach to Structural Fire Protection Engineering	3
1.2 Probability-Based Approach	8
1.2.1 Potential Failure Modes	11
1.2.2 Definition of Acceptable Probability of Failure (Target Probability)	11
1.2.3 Statistically Characterized Design Variables and Random Functions	12
1.2.4 Structural Reliability Assessment	12
1.2.5 Limit State Design	13
Example 1.1	16
1.2.6 Partial Safety Factor ψ and Reliability Index β	20
Example 1.2	24
1.3 Probability-Based Structural Fire Load as a Stochastic Process	26
References	29
2 Introduction to Probability Theory	31
2.1 Random Variables: Definition of a Probability	31
2.1.1 The Classical Definition of Probability	32
2.1.2 Mutually Exclusive Events	35
Exercises 2.1 through 2.9	36
2.1.3 Conditional Probability	40
Exercise 2.10	40
2.1.4 Independent and Mutually Exclusive Events	40
Exercises 2.11 through 2.13	41
2.2 Complete Probability Formula	42
Exercise 2.14 Moving Fire	42
2.2.1 Bayes' Theorem	43
Exercises 2.15 through 2.22	43

2.3	Probability Distributions – Concept of Random Variable	51
2.3.1	The Binomial Probability Distribution	51
	Exercises 2.23 through 2.33	53
2.3.2	The Poisson Probability Distribution	59
	Exercises 2.34 through 2.38	60
2.3.3	Correlation and Dependence	62
2.4	Continuous Probability Distributions	64
2.4.1	Normal Probability Distributions	64
	Exercises 2.39 through 2.42	67
2.4.2	Weibull Distribution	69
2.4.3	Rayleigh Distribution	70
2.4.4	Chi-Squared Distribution	71
2.5	Joint Probability Distribution	73
2.6	Characteristic Functions	74
2.7	Functions of Random Variables and Their Distribution	76
	Exercises 2.43 through 2.45	76
2.7.1	One-to-One Functions of an Absolutely Continuous Random Variable	78
2.7.2	Probabilistic Transformation (Linearization) Method	79
	Exercise 2.46	81
2.8	Confidence Interval	82
	Exercise 2.47	83
2.8.1	Confidence Interval (Exponential Distribution)	85
2.8.2	Confidence Interval (Poisson Distribution)	86
2.8.3	Binomial Proportion Confidence Interval	86
	References	87
3	Random Processes	89
3.1	Definitions	89
3.1.1	Mean Value	90
3.2	Properties and Estimations	92
3.3	Stationary Random Processes	95
	Exercise 3.1	95
3.3.1	Stationary and Ergodic Random Processes	96
	Exercises 3.2 and 3.3	96
3.4	Power Spectrum	97
	Exercises 3.4 through 3.12	99
3.5	Exponential Distribution Flow	102
	Exercises 3.13 through 3.15	102
3.6	Poisson Distribution	104
3.7	Erlang Distribution	105
3.8	Markov Process: A Class of Random Processes	107
3.8.1	Definitions	108
3.8.2	Queuing Theory (Markov Chain)	109
	Exercises 3.16 through 3.25	113
3.8.3	Engset Formula	122
	Exercises 3.26 through 3.31	124
	References	135

4	Very Fast Fire Severity: Probabilistic Structural Fire Resistance Design	137
4.1	Introduction	137
4.1.1	Very Fast Fire: Statistical Data ($0 < \gamma < 0.05$)	140
4.2	The First-Order Reliability Method (FORM)	153
4.2.1	Most Probable Point Methods	154
4.3	Limit State Approximation	155
4.4	Partial Safety Factor ψ and Reliability Index β	161
4.5	Confidence Interval – Maximum Dimensionless Temperature	165
	Example 4.1	169
	Example 4.2	170
4.6	Confidence Interval – Dimensionless Parameter “ γ ”	171
4.7	Confidence Interval – Dimensionless Time “ $\tau_{\max}$ ”	174
4.8	Flashover Point (T^2 Method – Probabilistic Approach)	175
4.9	Structural Failures in Time	177
4.9.1	Structural Fire Load as a Stochastic Process	177
4.10	Ergodicity	184
4.11	The First-Occurrence Time Problem and the Probability Density $P(a, t)$	187
	Example 4.3	190
	Example 4.4	190
	References	191
5	Fast Fire and Life-Cycle Cost Analysis	193
5.1	Fire Load and Severity of a Real Fire	193
5.2	Fast Fire: Statistical Data ($0.05 < \gamma < 0.175$)	198
5.3	Reliability Index	202
5.4	Confidence Interval – Maximum Dimensionless Temperature	204
	Example 5.1	206
	Example 5.2	207
	Example 5.3	207
	Example 5.4	208
5.5	Confidence Interval – Dimensionless Parameter “ γ ”	210
5.6	Confidence Interval – Dimensionless Time “ $\tau_{\max}$ ”	211
5.7	Flashover Point (T^2 Method – Probabilistic Approach)	212
5.8	Structural Failures in Time	213
5.8.1	Structural Fire Load as a Stochastic Process	213
5.9	The First-Occurrence Time Problem and the Probability Density $P(a, t)$	216
5.10	Life-Cycle Cost Analysis (Probability-Based Structural Damage Due to Fire)	218
5.10.1	Introduction	218
5.10.2	Developing CERs	222
	Example 5.5	224
	References	226

6	Medium Fire Severity and Thermal Diffusivity Analysis	228
6.1	Introduction	228
6.2	Medium Fire: Maximum Temperature Statistical Data ($0.175 < \gamma < 0.275$)	231
6.3	Reliability Index β	234
	Example 6.1	236
6.4	Confidence Interval – Dimensionless Parameter “ γ ”	237
6.5	Random Variable “ $\tau_{\max}$ ”: Confidence Interval (Dimensionless Time “ $\tau_{\max}$ ”)	238
6.6	Structural Failures in Time	239
6.7	Spectral Function	245
6.8	The First-Occurrence Time Problem and the Probability Density $P(a, t)$	246
6.9	Probability-Based Thermal Analyses	248
	6.9.1 Introduction	248
	6.9.2 Autocorrelation Function – Output (Method #1)	254
	6.9.3 Autocorrelation Function – Output (Method #2)	261
	6.9.4 Autocorrelation Function – Output (Method #3)	263
6.10	Definitions of Random Fire Rating and Practical Applications	265
	6.10.1 Case 1: Probability $P(\theta_{11} < 4)$ – Stochastic Process	265
	6.10.2 Case 2: Reliability Method	265
	6.10.3 Case 3: Structural Design (Reliability Method)	266
	Example 6.2	266
	6.10.4 Case 4: Statistical Linearization Method	268
	References	269
7	Slow Fire Severity and Structural Analysis/Design	270
7.1	Introduction	270
7.2	Structural Systems under Stochastic Temperature-Time Fire Load	271
7.3	Autocorrelation Function: Dynamic Analysis	275
7.4	Slow Fire: Maximum Temperature Statistical Data ($0.275 < \gamma < 1.0$)	282
7.5	Reliability Index β	287
7.6	Random Variable “ $\gamma_{\max}$ ”: Confidence Interval (Dimensionless Time “ $\tau_{\max}$ ”)	289
7.7	Random Variable “ $\tau_{\max}$ ”: Dimensionless Time	290
7.8	Autocorrelation and Spectral Functions	291
7.9	The First-Occurrence Time Problem and the Probability Density $P(a, t)$	295
7.10	Applications: ODOF Structural Systems	298
	Example 7.1	298
	Example 7.2	300
	Example 7.3: 15 Story Building	302
	References	306
	<i>Annex 1</i>	309
	<i>Annex 2</i>	313
	<i>Annex 3</i>	318
	<i>Annex 4</i>	330
	<i>Index</i>	335

Foreword

As building designers, we constantly look for ways to improve the economy and reliability of our designs. In recent years, this has led to a move away from prescriptive criteria and toward a performance-based design methodology.

Application of performance-based design methodologies is evolving quickly in several fields of structural engineering, for example, seismic engineering. Advances in computational tools and capabilities, the continued collection of data from natural events, and the continued physical testing of components subjected to these conditions have made possible the development of more rigorous analysis and simulation techniques. These techniques provide a means to more accurately and rationally evaluate the performance of structures under a set of imposed circumstances.

In buildings, fire has always been a major concern. Engineers working in the field of structural fire loads and fire protection engineering in the evaluations of structures continue to advance the field through research, testing, and more sophisticated performance-based methodologies and acceptance criteria. Traditionally, the fire protection of structures has been one of the most prescriptive of all the processes used in the design of a building. Fire engineering will benefit greatly from a transition to a performance-based approach. In recent years, there have been great strides in this field, but there is still a long way to go.

Performance-based design within fire protection engineering has been a topic of in-depth study for Dr. Leo Razdolsky. I have known Dr. Razdolsky for more than 25 years. We first met when he was with the City of Chicago Department of Buildings, where he eventually led the city's structural engineering team. He has a strong theoretical background but is well grounded in the practical aspects of the profession. *Probability-Based Structural Fire Load* is his latest book. In conjunction with its predecessor, *Structural Fire Loads*, it is his addition to the progress of performance-based design in the fire protection engineering field.

William F. Baker, P.E., S.E.

Preface

The author is pleased to present *Probability-Based Structural Fire Load*. This book will serve a wide range of readers, in particular, graduate students, professors, scientists, and engineers. Thus, the book should be considered not only as a graduate textbook but also as a reference handbook to those working or interested in areas of applied probability in continuum mechanics, stress analysis, and fire protection design. In addition, the book provides extensive coverage of a great many practical problems and numerous references to the literature.

The goal of the book is to bridge the gap between prescriptive and probability-based performance design methods and to simplify very complex and comprehensive computer analyses to the point that stochastic structural fire loads have a simple approximate analytical expression that can be used in structural analysis and design on a day-by-day basis. The main audience is practicing structural and fire prevention engineers. The scope of the work is broad enough to be useful to practicing and research engineers and to serve as a teaching tool in colleges and universities.

The book contains a large amount of original research material as well as substantially modified and increased material from his previously published articles. At the same time the book contains many other results obtained by other researchers, primarily reflecting the most important data in the area of statistical structural analysis. It is worthwhile to underline here that the structural fire load in general as part of the performance-based method is evolving very rapidly, and the author has limited himself to only very few research papers connected with the probabilistic structural fire load. The main portion of the book is devoted to the applied probability methods and simplifications that are specifically tailored to the structural fire load problem only.

This book is based on many years of experience of teaching applied probability theory at higher educational establishments. It contains many of the problems the author himself has encountered in his research and consulting work.

The text is divided into seven chapters, each of which begins with a short theoretical introduction followed by relevant formulas. The problems differ both in the fields of application and in difficulty. At the beginning of each chapter, the reader will find comparatively simple problems whose purpose is to help the reader grasp the fundamental concepts and acquire and consolidate the experience of applying probabilistic methods. More complicated applied problems, which can be solved

only after the requisite theoretical knowledge has been acquired and the necessary techniques mastered, then follow.

The author has avoided the standard typical problems that can be solved mechanically. Many problems may prove difficult for both beginners and experienced readers alike. In the interest of the reader, most of the problems have both answers and detailed solutions, given immediately after the problem rather than at the end of the book. This structure is very convenient and justified itself in *Structural Fire Load*, which was written in 2012.

The author believes that statements and detailed solutions of nontrivial problems that demonstrate certain techniques of problem solving are particularly interesting. The aim is not just to solve a problem but to use the simplest and most general techniques. Some problems have been given several different solutions. In many cases a method of solution used has a general nature and can be applied in several fields. The author has paid particular attention to the fact that the statistical database is presented in dimensionless form (the original deterministic equations of conservation of energy, mass, and momentum are dimensionless!), therefore the probability-based results (such as mean and standard deviation values and other numerical characteristics of stochastic structural fire load) make it possible to solve a number of problems with exceptional simplicity. The applied problems using the theory of Markov stochastic processes have been given the greatest consideration.

Probabilistic structural analysis methods provide a means to quantify the inherent risk of a design and assess the sensitivities of design variables. This book is intended to introduce the subject of probabilistic analysis to the structural and fire protection engineering industry as well as to act as a reference to guide those applying this technology. The solutions in many cases are presented in the “best-to-fit” analytical forms ready to be used for practical computations.

The deterministic structural analysis approach to the mathematical modeling of structural fire loads is described in Chapter 1, and its shortcomings are pointed out. Probability-based first- and second-order reliability methods (FORM/SORM) development is described in detail, along with associated structural safety design applications. An in-depth explanation of the necessity of implementation of such methods in the structural engineering industry is given, along with some practical examples. The chapter concludes with a consensus of potential benefits as well as potential issues of concern that must be addressed by those using these stochastic analyses. When using a probabilistic approach to design, the designer should no longer think of each variable as a single value or number. Instead, each variable should be viewed as a probability distribution. From this perspective, probabilistic design predicts the flow of variability (or distributions) through a system. By considering this flow, a designer can make adjustments to reduce the flow of random variability and improve quality. Proponents of this approach contend that many quality problems can be predicted and rectified during the early design stages and at a much reduced cost.

Chapter 2 presents the required information regarding probability space definition, probability of a fire event, mutually exclusive events, discrete and continuous random variables distributions, etc. This chapter is written as an introduction to applied probability theory enriched with numerous examples, exercises, and applications, mostly related to the structural and fire protection engineering fields.

Functions of random variables and their distribution, the stochastic equivalent linearization procedure for estimating the mean and variance of a nonlinear system to random variables, and a confidence interval definition as a type of interval that estimates population of random parameters and is often used in reliability computations (along with some statistical data processing information) are also presented in this chapter.

In Chapter 3 a fire event is presented as stochastic (random) processes. If in Chapter 2 the discrete and continuous random variable distributions are the tools for analyzing the maximum values of dimensionless temperature load due to a fire event (the probabilistic approach used often in wind and seismic structural analyses and practical design calculations), then the random autocorrelation functions approach in this chapter is a necessary tool to solve the probabilistic problems connected with the so-called “first-occurrence time problem and the probability density $P(a, t)$. This type of random function problem allows the structural engineer to compute also the number of times the dimensionless temperature accedes a given level a (the area between this level and the curve above) – all additional statistical information needed to describe in detail the structural fire load due to given fire event.

In the simple case of discrete time, a stochastic process amounts to a sequence of random variables known as a time series (for example, a Markov chain). This is a very powerful tool that is used not only in theory of stochastic processes but also in a mass service theory, part of which comprises the probabilistic approach to the building evacuation process in case of fire or any other emergency situation (for instance, a bio-chemical terror attack), in moving fires investigations and analysis, progressive (or partial) collapse of the structural system analysis, probabilistic optimum reliability design etc. The large portion of this chapter is devoted to Markov chain problems via numerous examples from structural and fire prevention engineering fields.

Chapters 4, 5, 6, and 7 are similar from an ideological point of view, since they describe the practical aspects of creating the statistical data from the deterministic solutions of conservation of energy, mass, and momentum equations (for each fire severity case: very fast, fast, medium, and slow fire, respectively). The main topics that are discussed in these chapters are as follows: the C.A. Cornell methodology of application of first-order reliability method (FORM) and second-order reliability method (SORM) in case of fire; probability-based limit states design; the structural reliability; probability of failure; another interpretation of the reliability index β based on applied theory of random functions and its application to the fire development process modeling; the confidence interval and its application to risk management design etc. However, not all of the topics are repeated in each of these chapters. The author has decided to have core elements (such as creating the statistical data, the (FORM) method, the confidence interval method, autocorrelation functions application, etc.) in each chapter (for each fire severity case). However, some of them such as life-cycle cost (LCC) parametric dimensional analysis are located only in Chapter 5; proof of ergodicity is presented in Chapters 4 and 5 only; and in Chapters 6 and 7 autocorrelation functions are constructed based on the fact that the stochastic processes in these cases are in fact ergodic; the application of probability-based method to the thermo-diffusion problem (passive insulation design) is presented in Chapter 6 only; the application of probability-based structural

fire load to regular structural engineering analysis (standard structural systems: beams; frames; trusses and arches) are presented in Chapter 7. Obviously the author assumes that the professional engineer can easily adapt the methodology to any particular fire severity scenario case.

Overall, the book has a large number of practical examples (for fire protection and structural engineering design) that are presented in a simple step-by-step computational form.

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