1 Introduction

Notation

k – Thermal conductivity that has the dimensions W/m*K or J/m*s*K
T – Temperature
d – Thickness in the direction if heat flow.
ρ – Air density
c – Specific heat capacity
K – Number of collisions that result in a reaction per second
A – Total number of collisions
E – Activation energy
R – Ideal gas constant
P – Losses of heat due to thermal radiation
e – Emissivity factor
T_o – Ambient temperature
A_v – Area of openings
c_p – Average specific heat at constant pressure
t – Time
$\vec{v}(u;v;w)$ – Velocity vector
D – Diffusion coefficient [m^2/sec]
p – Pressure
ν – Kinematic viscosity [$V = \mu/\rho$]
θ – Dimensionless temperature
τ – Dimensionless time
h – Height of the compartment [m]
a – Thermal diffusivity [m^2/sec]

Time – $t = \frac{h^2}{a}\tau$ [sec]

Temperature – $T = \frac{RT_*^2}{E}\theta + T_*$ [K], where $T_* = 600°K$ is the baseline temperature

Coordinates – $\bar{x} = x/h$ and $\bar{z} = z/h$ – x and z – dimensionless coordinates

Velocities – $\bar{u} = \frac{\nu}{h}u$ [m/sec] and $\bar{w} = \frac{\nu}{h}w$ [m/sec] – horizontal and vertical components velocity accordingly

ν – Kinematic viscosity [m^2/sec] – u and w – dimensionless velocities
Pr = ν/a – Prandtl number

$Fr = \frac{gh^3}{\nu a}$ – Froude number

g – Gravitational acceleration
$Le = a / D = Sc / Pr$ – Lewis number
$Sc = \nu / D$ –Schmidt number

$\beta = \frac{RT_*}{E}$ – Dimensionless parameter

$\gamma = \frac{c_p RT_*^2}{QE}$ – Dimensionless parameter

$P = \frac{e\sigma K_v(\beta T_*)^3 h}{\lambda}$ – Thermal radiation dimensionless coefficient

σ = 5.67(10^–8) [watt/m^2K4] [$\sigma = 5.6703(10^{-8})$ watt/m^2K^4] – Stefan-Boltzman constant
$K_v = A_o h/V$ – Dimensionless opening factor
A_o – Total area of vertical and horizontal openings

$\delta = \left(\frac{E}{RT_*^2}\right) Qz \left(\exp\left(-\frac{E}{RT_*}\right)\right)$ – Frank-Kamenetskii's parameter

$C = [\,1\text{-}P(t)/P_O\,]$ – Concentration of burned fuel product in fire compartment

$\bar{W} = \frac{v}{h} W$ – Vertical component of gas velocity

$\bar{U} = \frac{v}{h} U$ – Horizontal component of gas velocity

b = L/h – L and h – length (width) and height of fire compartment accordingly
W; U – Dimensionless velocities
R_c – Characteristic value for resistance
A – Design variable (e.g., cross- sectional area of the steel rod considered previously)
G_c – Characteristic value for permanent load
S – Characteristic value for variable load
φ – Partial safety factor for resistance
ψ – Partial safety factor for permanent load
ψ_2 – Partial safety factor for variable load
β – Reliability index
S – Probability space
A – Set of outcomes (events) to which a probability is assigned
$P(E_2|E_1)$ – Conditional probability

Introduction

It appears inevitable that the structural engineering community, as well as fire protection and many other engineering communities that are ultimately responsible for life safety issues, will eventually incorporate probabilistic analysis methods to some degree. Probabilistic analysis methods, unlike traditional deterministic methods, provide a means to quantify the inherent risk of a structural design and to quantify the sensitivities of most important parts of the design in the overall reliability of a structural system as a whole. The degree to which these methods are successfully applied depends on addressing the issues and concerns discussed in this book. Certainly, one issue is to disseminate familiarity and basic understanding of the principles and assumptions made in probability-based structural design.

This book is intended to introduce the subject of probabilistic analysis (also known as probabilistic design) to engineers in the building design industry as well as act as a reference to guide those applying these methods to other branches of the fire protection industry, such as (but not limited to) egress design and reliability of sprinkler systems. The level of mathematical complexity is aimed at those with limited statistical training; numerous references are given throughout that point to more elaborate details of the probabilistic methods.

The field of probability-based structural fire load lies at the crossroads of stress analysis, fire protection and structural engineering, probability theory, thermodynamics, heat conduction theory, and advanced methods of applied mathematics. Each of these areas is covered to the extent necessary. The book starts from basic concepts and principles, and these are developed to more advanced levels as the text progresses. Nevertheless, some basic preparation in structural analysis/design and mathematics is expected of the reader.

While selecting material for the book, the author made every effort to present both classical topics and methods and modern, or more recent, developments in the field.

1.1 Deterministic Approach to Structural Fire Protection Engineering

History shows that fire is a frequent and deadly event that strikes structural systems. During the late 1960s and 1970s, a number of natural disasters occurred worldwide that caused extensive loss of life and property damage and focused the attention of the structural engineering community and the public on the need to advance structural design practices for disaster mitigation. Among the more notable of these were the structural failure investigations that followed the San Fernando, California, earthquake of 1971; the Managua, Nicaragua, earthquake of 1972; and the Miyagi-ken-oki earthquake of 1978; the investigation of snow and rain load conditions prior to the collapse of the Hartford Civic Arena roof in 1978; and the evaluations of wind loads, wind load effects, and building performance following Hurricane Camille on the Gulf Coast (1969) and Cyclone Tracy in Darwin, Australia (1974); the First Interstate Bank Building in Los Angeles; One Meridian Plaza in Philadelphia; and Buildings 5 and 7 at the World Trade Center after 9/11 show that burnouts can occur in buildings. When a burnout occurs, there is a potential for partial or even complete collapse of the structure.

Performance-based procedures can be used to help mitigate the risk of collapse and, at the same time, produce a cost-effective design. These and other investigations of structural systems performances revealed a number of deficiencies in the provisions for structural safety appearing in the codes of practice of the time and emphasized the need for improvements in design for natural hazards. Performance-based fire codes and associated analysis will eventually find universal acceptance, but not as quickly and easily as other types of performance-based codes have in the past. For example, earthquake codes and seismic structural analysis were quickly accepted since they were unrestrained by previous practice. Buildings had essentially not been designed specifically for earthquakes, and engineers, architects, and building officials gratefully adopted the new methods as they found their way into engineering literature and the building codes. Performance-based fire analysis methods, however, find the field already occupied by a long-established prescriptive code based on a hundred years of furnace tests and engineering practice. The new methods must be highly developed, extensively verified, and carefully peer-reviewed before they can supplement or replace the traditional methods. The following types of efforts would aid in this process:

- Development of peer-review protocols for the transitional period when performance-based analysis is first being presented to building officials.
- More exposure of engineering students and practitioners to the basics of structural fire performance and analytical methods to predict it; sponsorship of workshops and seminars for nonspecialists
- Some sort of codification of methods to calculate fire curves for the most common fire scenarios so design engineers do not have to engage a specialist for routine structural design; an effort in this area is currently being made by the Society of Fire Protection Engineers (SFPE) and American Society of Civil Engineers (ASCE)
- Incorporation into commercial structural computer codes of the basic capabilities to conduct fire analysis, especially as nonlinear programs come into greater use; ideally, fire should be treated as an additional design load case, just as other infrequent loading conditions such as wind or earthquake are

The structural engineer ultimately is responsible to check the building structure subjected to the structural fire load (SFL) and to quantify the response of the originally proposed structural system in realistic fire scenarios in order to determine whether this response is acceptable. Strengths and weaknesses then can be clearly identified and addressed within the structural design, as appropriate. Behavior of the structural system under SFL should be considered an integral part of the structural design process. The role of a structural engineer today involves a significant understanding of both static and dynamic loading and the structures that are available to resist them. The complexity of modern structures often requires a great deal of creativity from the engineer in order to ensure that the structures support and resist the loads to which they are subjected.

Fire engineering begins with the development of a design fire exposure to the structure. This normally takes the form of a time-temperature curve based upon the fire load, ventilation, and thermal properties of the bounding surfaces (walls, floor, and ceiling). Design fire loads are dependent upon the occupancy and other fire protection features of the building. The analysis involves the definition of the design fire exposure and the thermal response of the structural system. In annex E of Eurocode 1 the fuel load densities per floor area for different occupancies are presented and illustrated in Table 5.1. In some other European documents the fuel load is presented as a density per the total enclosed area of a compartment.

The corresponding values are given in columns 4 and 5 of Table 1.1.

Table 1.1. *Fire load densities (of floor area)*

Fire Load Densities $q_{f,k}$	(MJ/m² Floor Area)	(MJ/m² Floor Area)	(MJ/m² Enclosing Area)	
Occupancy	Average	80% fractile	80% fractile	
Floor area	–	–	20 m²	50 m²
Dwelling	780	948	225	270
Hospital (room)	230	280	66	80
Hotel (room)	310	377	89	108
Library	1500	1824	120	146
Office	420	511	82	99
Classroom of a school	285	347		104
Shopping center	600	730		35
Theatre (cinema)	300	365		
Transport (public space)	100	122		

On the basis of NFPA 557 Standard [1] and "Swedish" fire curves [2] [3] for the postflashover realistic fire exposure we can standardize fires as indicated in Table 1.2.

Table 1.2. *Fire severity*

Category	Fuel Load L[MJ/m²]	Maximum Temperature T_{max} [°K]	Maximum Dimensionless Temperature θ_{max}	Parameter γ From Table 5.4
Ultrafast	$500 < L < 700$	$1020 < T_{max} < 1300$	$7.0 < \theta_{max} < 11.67$	$0 < \gamma < 0.05$
Fast	$300 < L < 500$	$880 < T_{max} < 1020$	$4.67 < \theta_{max} < 7.0$	$0.05 < \gamma < 0.175$
Medium	$100 < L < 300$	$820 < T_{max} < 880$	$3.67 < \theta_{max} < 4.67$	$0.175 < \gamma < 0.275$
Slow	$50 < L < 100$	$715 < T_{max} < 820$	$1.92 < \theta_{max} < 3.67$	$0.275 < \gamma < 1.0$

Note: If fuel load $L > 700$, select $\gamma = 0$

At the root of the structural fire safety problem is the uncertain nature of the man-made and environmental forces that act on structures, of material properties that are changing quite rapidly under high temperature conditions, and of structural analysis procedures that, even in this computer age, are no more than models of reality. The severity of the fire and, as a result, the structural fire loading (deterministic approach) conditions can be determined by the conservation of energy, mass m and momentum equations as follows [4]:

$$c_p\rho\frac{\partial T}{\partial t} = \mathrm{div}(\lambda \mathrm{grad}T - c_p\rho\vec{v}\nabla T) + Qze^{-E/RT} - \frac{e\sigma A_v(T^4 - T_o^4)}{V} \tag{1.1}$$

$$\frac{\partial C_i}{\partial t} = D_i\Delta C_i - \mathrm{div}\vec{v}C_i - \frac{\nu_i}{\nu_1}Qze^{-E/RT} \tag{1.2}$$

The mass fractions are defined as follows:

$$C_{mi} = \frac{M_iC_i}{\sum_k M_kC_k} = \frac{M_iC_i}{\rho} \tag{1.3}$$

where I and k are gas component numbers and M_k indicates molecular weights.

For the binary mixture of gas species:

$$C_{m1} + C_{m2} = 1 \tag{1.4}$$

Fick's law for the multimass fractions mixtures diffusion process can be written as follows:

$$g = -D\rho \operatorname{grad} C_{mi} + \vec{v} C_{mi} \tag{1.5}$$

However, if the density of the mixture is assumed to be constant or the diffusivity coefficients for gas components are approximately equal, then one can assume that the diffusion process is independent for each component, and therefore Fick's law can be written as

$$g = -D \operatorname{grad} C + C \tag{1.6}$$

Where C is the mass fraction (concentration) of a component of a one-step chemical reaction (reactant or product of chemical reaction). This assumption simplifies considerably the number of partial differential equations (1.5). Instead it will have only one equation (1.6). The most general form of the Navier–Stokes equation (conservation of momentum) is

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\bullet\nabla\vec{v}\right) = -\nabla p + \nabla S_{ij} + \vec{f} \tag{1.7}$$

This is a statement of the conservation of momentum in a fluid and it is an application of Newton's second law to a continuum. A very significant feature of the Navier–Stokes equations is the presence of convective acceleration: the effect of time independent acceleration of a fluid with respect to space, represented by the quantity $\vec{v}\nabla\vec{v}$, where $\nabla\vec{v}$ is the tensor derivative of the velocity vector, equal in Cartesian coordinates to the component by component gradient. The vector $\vec{f}$ represents body forces. Typically this is only gravity forces, but it may include other fields (such as centrifugal force). Often, these forces may be represented as the gradient of some scalar quantity. Gravity in the z coordinate direction, for example, is the gradient of $-\rho gz$.

The vast majority of work on the Navier–Stokes equations is done under an incompressible flow assumption for Newtonian fluids. The incompressible flow assumption typically holds well even when dealing with a "compressible" fluid, such as air at room temperature (even when flowing up to about Mach number 0.3). Taking the incompressible flow assumption into account and assuming constant viscosity, the Navier–Stokes equations will read (in vector form) [4]

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\bullet\nabla\vec{v}\right) = -\nabla p + \nu\rho\nabla^2\vec{v} + \vec{f} \tag{1.8}$$

Under the incompressible assumption, density is a constant and it follows that the mass continuity equation will simplify to

$$\nabla\vec{v} = 0 \tag{1.9}$$

Scale analysis is a powerful tool used in the combustion theory for the simplification of differential equations with many parameters. It allows us also to identify some small parameters that approximate the magnitude of individual terms in the equations and their impact on the solution as a whole. Let's introduce the dimensionless parameters and variables in conservation of energy, mass, and momentum equations [4]:

$$\frac{\partial\theta}{\partial\tau}+\mathrm{Pr}\left(U\frac{\partial\theta}{\partial x}+W\frac{\partial\theta}{\partial z}\right)=\delta(1-C)^{k}\exp\left(\frac{\theta}{1+\beta\theta}\right)-P\theta^{4} \qquad (1.10)$$

$$\frac{\partial C}{\partial\tau}+\mathrm{Pr}\left(U\frac{\partial C}{\partial x}+W\frac{\partial C}{\partial z}\right)=\gamma\delta(1-C)^{k}\exp\left(\frac{\theta}{1+\beta\theta}\right) \qquad (1.11)$$

where U and W are horizontal and vertical velocities, respectively, that should be obtained from the Navier–Stokes equations (1.8) and (1.9).

The direct solution of equations (1.10) and (1.11) is the "deterministic" way of solving the problem (obtaining the temperature-time function in a fire compartment). However, in the case of fully developed fire in a large building volume the mathematical modeling of the physical and chemical transformations of real materials is known only with a small degree of confidence. At the same time on the basis of many fire test results data one can expect that curtain parameters such as the maximum temperature, type of temperature-time function, and others are well known. On the other hand, some other parameters (for example, parameter γ from equation (1.11)) are known with some degree of approximation. From a physical point of view this parameter characterizes the ratio of heat loses during the development stages of a fire (incipient and free-burning) divided by total energy released (heat rate) [5]: $\gamma=\frac{c_p R T_*^2}{QE}$

If, for example, the heat rate of a chemical reaction is large, then the parameter γ is small. Therefore, parameter γ has a bounded variation between 0 and 1.

It is also important to underline here that for any given value of parameter γ from the interval [0;1] only one solution of equations (1.10) and (1.11) exists and the temperature-time function in this case has only one maximum value. It can be seen by observation that this maximum temperature value increases when the parameter γ decreases from 1 to 0. On the other hand, the maximum gas temperature in a real fire compartment and the fuel load are defining the category of the fire severity (see Table 1.2); therefore there is a correlation between the fire severity category and the value of parameter γ. In order to establish this correlation first the statistical data for each fire severity case will be created.

In the case of fully developed fire in a large building volume the physical and chemical transformations of real materials occur in very small flame zone under very high temperatures (much higher than the average gas temperature in a fire compartment); therefore, it is very hard (if not almost impossible) to obtain these data (specific heat, thermal conductivity parameter, thermal diffusivity, etc.) in regular laboratory conditions. The fire engineering community is fully aware of this fact, and corresponding tasks and recommendations regarding possible improvements in this area of expertise are provided in the report [6]. The NIST Special Publication (Mathematical Model of FDS) [7] calls them "uncertain parameters." Therefore, in our case any solution of differential equations (1.10) and (1.11) is a function of two independent variables: τ time and γ from an interval [0;1]. Therefore, for any given γ

there is the realization of the dimensionless temperature-time stochastic process $\theta(\tau)$. The probability-based mathematical model of a real fire in a compartment can be formulated now as follows:

1. For each fixed number of γ_i from the interval [0;1] find the discrete number of solutions of differential equations (1.10) and (1.11) – temperature-time curves – statistical data of functions $\theta_i(\tau)$. For the interval of γ_i [0,1] find a discrete number of solutions of differential equations (1.10) and (1.11), which may be represented in the form of temperature-time curves which are in turn raw statistical data (realizations) for the construction of random functions $\theta_i(\tau)$.
2. Find the maximum values of dimensionless temperatures and compose the matrixes of statistical data for each fire severity case.
3. Since a very large number of different parameters are influencing the final result, the stochastic temperature-time curves, the normal probability distribution for each ordinate will be assumed here. Obviously all solutions of differential equations have to be obtained in dimensionless forms (temperature θ and time τ).
4. The solutions of equations (1.10) and (1.11) (using the simple mathematical software POLYMATH) can be presented in tabular and analytical forms. The reason for presenting the results in both forms is that the tabular solution allows the "user" to analyze some other regimes of fire development, such as: fire growth period, decay, and flashover period. However, it is logical to call the uncertain parameter γ a random variable from the interval [0; 1]. To summarize the preceding discussion, this is the mechanism of creating statistical data for the probability-based analyses of structural fire load (SFL) and its application to the stochastic methods of structural design.

1.2 Probability-Based Approach

Dealing with this uncertainty is one of the main roles of codes and standards. The foundation of probabilistic structural design for fire safety involves basing design criteria on reliability targets instead of deterministic criteria. Design parameters such as applied SFL, material strength, and operational parameters are researched and then statistically defined. A probabilistic analysis model is developed for the entire system and solutions performed to yield failure probabilities.

The solution includes a number of locations and failure modes. Each location requires corresponding applied SFL and material strength distributions. Mathematically, the applied load and material strength distributions are generally assumed to be independent. The general concept is to integrate the joint probability of applied stress and material strength over the region where stress exceeds strength. The result of this integration is the probability of structural failure. Sensitivity analysis can reveal the major contributors to risk; this allows the analyst to vary the design parameters to produce acceptable reliability at minimum cost, for example.

Today, the challenge to structural engineers and analysts is to define what data are obtainable accurately, assess the degree of confidence (confidence probability) to which these data apply to the current situation, statistically define the data, and predict performance. One must accept the notion that there is a finite (however small) probability of structural failure. One must determine level(s) of structural failure that can be tolerated, in concert with construction technology, economic, performance, and other constraints.

The concept of probabilistic structural risk assessment has been around for quite some time. Back in 1945 Professor Alfred Freudenthal wrote [8]: "The true character of the safety factor is disclosed by the introduction of a statistical concept of physical qualities, according to which the individual properties composing strain and resistance are represented by frequency distributions, instead of by individual values.… By application of the theory of probability, the concept of safety can be rationalized." Freudenthal's paper sparked international interest in structural safety; structural reliability theory was discussed and formulations presented in papers from British, French, Russian, Spanish, and Swedish authors during the early 1950s. The theory was fueled by Weibull's success [9] in developing robust statistical representations of material strength.

In 1967, C. A. Cornell [10] proposed a second-moment format for evaluation of structural reliability. This approach generates a "safety index" calculated from the means and variances of the parameter distributions. The safety index is considered to be a measure of reliability and is an alternative to integrating the joint probability density function numerically to determine a probability of failure. In 1973, Lind [11] demonstrated that Cornell's safety index could be used to derive safety factors on applied loads and resistance. This was a milestone; reliability analysis was at long last related to accepted (civil engineering) methods of design. Subsequent refinements were made by Hasofer and Lind [12], whose method (1974) is considered to be the foundation of probabilistic design theory.

Fire is among the most unpredictable of hazards and really should be considered in a probabilistic framework. Here are some examples how the uncertainty associated with natural or man-made fire is characterized by Howard Baum, NIST Fellow Emeritus [7]: "The idea that the dynamics of a fire might be studied numerically dates back to the beginning of the computer age. Indeed, the fundamental conservation equations governing fluid dynamics, heat transfer, and combustion were first written down over a century ago. Despite this, practical mathematical models of fire (as distinct from controlled combustion) are relatively recent due to the inherent complexity of the problem. The difficulties revolve about three issues: First, there are enormous number of possible fire scenarios to consider because of their accidental nature. Second, the physical insight and computing power required to perform all the necessary calculations for most fire scenarios are limited. Any fundamentally based study of fires must consider at least some aspects of bluff body aerodynamics, multiphase flow, turbulent mixing and combustion, radiative transport, and conjugate heat transfer, all of which are active research areas in their own right. Finally, the 'fuel' in most fires was never intended as such. Thus, the mathematical models and the data needed to characterize the degradation of the condensed phase materials that supply the fuel may not be available. Indeed, the mathematical modeling of the physical and chemical transformations of real materials as they burn is still in its infancy.

In order to make progress, the questions that are asked have to be greatly simplified. To begin with, instead of seeking a methodology that can be applied to all fire problems, we begin by looking at a few scenarios that seem to be most amenable to analysis. Hopefully, the methods developed to study these "simple" problems can be generalized over time so that more complex scenarios can be analyzed. Second, we must learn to live with idealized descriptions of fires and approximate solutions to our idealized equations. Finally, the methods should be capable of systematic improvement. As our physical insight and computing power grow more powerful, the methods of analysis can grow with them.

Now it could be, of course, that the mathematical modeling of fire dynamics is just incomplete, and, therefore, it gives a coarse description of a reality that is actually much finer. If that were the case, we should join the large number of people in their search for a finer mathematical model of physical reality. However, it has become clear that the search for such underlying "hidden variable" models runs into certain difficulties: They must at least allow us to see the "chemical transformations of real materials as they burn," which is very close to impossible. And even if that would not disturb us (which it does), they have not been very successful in the prediction of new phenomena.

The beauty of the probabilistic approach is that probability-based structural fire protection engineering does not predict the result of physical experiments with certainty, but yields probabilities for their possible outcomes; therefore, we do not have to search for a 100 percent guaranteed answer, which does not exist anyway. However, even if it does exist (with very good approximation of a real fire scenario in any particular case), still it would not have any practical value in the general population of such fires, because it is limited to this event, which will not be repeatable again. Therefore, the mathematical Fire Dynamics Simulator (FDS) modeling is applicable to the structural fire investigation processes, but not to the structural fire design stage, when the precise value of heat release rate (HRR) is not known in advance. This statement is supported again in the same reference [7]: "Because the model was originally designed to analyze industrial-scale fires, it can be used reliably when the heat release rate (HRR) of the fire is specified and the transport of heat and exhaust products is the principal aim of the simulation. Current research is aimed at improving this situation, but it is safe to say that modeling fire growth and spread will always require a higher level of user skill and judgment than that required for modeling the transport of smoke and heat from specified fires can be used reliably when the heat release rate (HRR) of the fire is specified and the transport of heat and exhaust products is the principal aim of the simulation. In these cases, the model predicts flow velocities and temperatures to accuracy within 10% to 20% of experimental measurements, depending on the resolution of the numerical grid 2. However, for fire scenarios where the heat release rate is predicted rather than specified, the uncertainty of the model is higher. There are several reasons for this: (1) properties of real materials and real fuels are often unknown or difficult to obtain, (2) the physical processes of combustion, radiation and solid phase heat transfer are more complicated than their mathematical representations in FDS, (3) the results of calculations are sensitive to both."

On the other hand, experimental data are available from a very limited number of real fire test results and it is fair to say that one cannot expect to have a large quantity of reliable statistical information regarding structural fire load in tall and supertall buildings or other structural systems (it is not practical and is cost prohibitive). That is true also of aerospace engineering systems, nuclear power plants, and other examples. The main goal of the mathematical theory of probability (as we know) is to use "means and methods" that require minimal statistical data in order