

# Index

- $\mathbb{C}$ , *xviii*  
 $\mathbb{C}^4$   
 interpolation followed by sampling in, 427  
 interpolation in, 423, 426  
 sampling followed by interpolation in, 424, 429  
 sampling in, 422, 425  
 $\mathbb{C}^N$ , 18–19, 31, 39, 163–164  
 1 norm on, 33  
 2 norm on, 33  
 basis for, 71  
 completeness of, 39, 172  
 equivalence of norms on, 33, 172  
 inner product on, 24, 170  
 norm on, 28, 170  
 $p$  norm on, 33  
 standard basis for, 71  
 $C^q([a, b])$ , 346  
 $C^q([a, b])$ , 32  
 as subspace of  $\mathbb{C}^{[a, b]}$ , 32  
 of  $\mathcal{L}^2([a, b])$ , 32  
 lack of completeness of, 40  
 polynomial functions as subspace of, 32  
 $\mathbb{C}^{\mathbb{R}}$ , 19  
 inner product on, 24  
 norm on, 28  
 $\mathbb{C}^{\mathbb{Z}}$ , 19  
 basis for, 71  
 inner product on, 24  
 norm on, 28  
 standard basis for, 71  
 $\lambda$ -tight frame, 104, 161  
 normalization, 104  
 $\ell^0(\mathbb{Z})$ , 39, 172  
 $\ell^0$  norm on, 172  
 $\mathcal{L}^1(\mathbb{R})$ , 35, 346  
 $\mathcal{L}^1$  norm on, 35  
 Fourier transform of functions in, 361, 405, 406  
 $\ell^1(\mathbb{Z})$ , 33, 35, 186  
 $\ell^1$  norm on, 33  
 DTFT of sequences in, 218  
 Fourier series of functions with spectra in, 383  
 $\mathcal{L}^2(\mathbb{R})$ , xix, 31–32, 35, 39, 346  
 $\mathcal{L}^2$  norm on, 31, 35  
 Fourier transform of functions in, 363  
 interpolation followed by sampling in, 472  
 interpolation in, 472  
 orthogonal projection operator in, 58  
 sampling followed by interpolation in, 475  
 $\mathcal{L}^2([a, b])$ , 31  
 $\mathcal{L}^2$  norm on, 31  
 Cauchy–Schwarz inequality, 31  
 inner product on, 31  
 $\ell^2(\mathbb{Z})$ , xix, 31, 33, 39, 185  
 $\ell^2$  norm on, 31, 33  
 convergence of inner product in, 171  
 DTFT of sequences in, 219  
 failure to belong to  $\ell^1(\mathbb{Z})$ , 35  
 Fourier series of functions with spectra in, 384  
 interpolation followed by sampling in, 445  
 interpolation in, 434  
 sampling followed by interpolation in, 436, 447  
 sampling in, 432, 443  
 $\mathcal{L}^\infty(\mathbb{R})$ , xix, 39, 346  
 $\mathcal{L}^\infty$  norm on, 35  
 $\ell^\infty(\mathbb{Z})$ , xix, 34, 39, 186  
 $\ell^\infty$  norm on, 34, 171, 172  
 $\mathcal{L}^p(\mathbb{R})$ , xix, 35, 39  
 $\mathcal{L}^p$  norm on, 35  
 completeness of, 40  
 $\ell^p(\mathbb{Z})$ , xix, 34, 39  
 $\ell^p$  norm on, 33  
 LSI system acting on sequence in, 329–330  
 Minkowski’s inequality, 164  
 nesting of spaces, 34, 172  
 $\mathbb{N}$ , *xviii*  
 $\omega$  independence, 160  
 $p$  norm, 33, 174  
 $\mathbb{Q}$ , *xviii*  
 lack of completeness of, 37, 38  
 $\mathbb{R}$ , *xviii*  
 $\mathbb{R}^+$ , *xviii*  
 $\mathbb{R}^2$ , 10–17  
 $p$  norm on, 174  
 basis for, 13–17, 169  
 analysis operator in, 16  
 Karhunen–Loève, 611

- standard, 13
  - synthesis operator in, 17
  - biorthogonal pair of bases for, 14–15
  - biorthogonality condition in, 14
  - change of basis in, 14
  - coordinates in, 13–17
  - distance in, 11
  - dual
    - basis in, 14
    - frame in, 15
    - pair of frames in, 107
  - expansion
    - coefficient in, 13
    - with frame in, 15
    - with orthonormal basis in, 13
    - with pair of biorthogonal bases in, 14
    - with tight frame, 15
  - frame for, 15–17, 103, 169
    - tight, 16, 104
  - inner product on, 10–11
  - Karhunen–Loève basis for, 611
  - matrix representation of bases and frames
    - in, 16–17
  - nonorthogonal vectors
    - interpolation in, 417–420
    - sampling in, 417–420
  - norm on, 10–11
  - oblique projection in, 12
  - orthogonal projection in, 12
  - orthogonality in, 11, 13
  - orthonormal
    - basis for, 13
    - set in, 13
  - Parseval equality in, 14
  - projection
    - oblique in, 12
    - orthogonal in, 12
    - theorem in, 11
  - Riesz basis for, 72
  - standard basis for, 13
  - subspace in, 11–12
  - tight frame for, 16, 104
  - vector space, 10
- $\mathbb{R}^3$
- best approximation in, 169
  - matrix representation of bases and frames
    - in, 170
  - normal equations in, 99
- $\mathbb{R}^N$ , 18–19, 39
- 1 norm on, 33
  - 2 norm on, 33
  - completeness of, 39
  - diagonalization of basis operator in, 113
  - equivalence of norms on, 33, 172
  - inner product on, 24
  - norm on, 28
  - $p$  norm on, 33
- $\mathbb{Z}$ , *xviii*
- $\mathbb{Z}^+$ , *xviii*
- 1-tight frame, 105, 161
- 1 norm, 33
- 2 norm, 33
- Absolute convergence, 137
- Accumulator, 200
  - difference equation of, 203
- Additive identity property
  - vector space axiom, 18
- Additive white Gaussian noise, 288, 397
- Additivity property
  - operator axiom, 40
- Adjoint operator, 43, 43–47, 58, 173, 174
  - generalization of Hermitian transpose, 43
  - matrix representation of, 118
  - of convolution
    - for functions on real line, 406
    - for infinite sequences, 210–211
  - of downsampling, 270
  - of downsampling preceded by filtering, 275
  - of DTFT, 227
  - of Fourier transform, 371
  - of local averaging operator, 45
  - of polynomial matrix, 322
  - of sampling and interpolation, 505
  - of upsampling, 270
  - of upsampling followed by filtering, 275
  - polyphase representation of, 282
  - properties of, 46
- Adjugate, 142
- Advance operator, 198
- Affine
  - function, 63, 176
  - subspace, 20, 21
- Algebra, 318–324
  - fundamental theorem of, 318
  - linear, 141, 324
  - polynomials, 318–324
- Algebraic
  - basis, 159
  - theory of signal processing, 328
- Aliasing, 267
  - of functions, 460–462
    - complex exponentials, 461
    - sinusoids, 461
  - of sequences, 440–441
- All Is Vanity*, 618
- Allpass filter
  - as orthonormal basis, 232
  - continuous-time, 373
  - discrete-time, 231, 251
  - energy conservation for, 231
  - orthogonality of impulse response
    - to all shifts, 231

- Parseval equality for, 231
- Anticausal filter, 208
- Antiderivative, 362
- Antidiagonal matrix, 150
- Approximation, 4–6, 82, 98–100, 508–576, 658
  - by polynomials, 513–537
  - by sampling followed by interpolation, 499
  - by series truncation, 560–576, 598
  - by splines, 510–511, 537–560, 597
  - Chebyshev equioscillation theorem, 525
  - de la Vallée-Poussin alternation theorem, 525
  - error of, 513
    - Lagrange interpolation, 519, 522, 597
    - Taylor series, 521, 597
  - filter design as, 529–537
  - least-squares, 530–532
  - minimax, 532–534
  - weighted minimax, 533–537
  - Hermite, 522–523, 607
  - Lagrange, 509–510, 517–520, 597, 600–601, 607
    - error of, 519, 522, 597
  - least-squares, 50, 175, 384, 508, 514–517, 597
    - in  $\mathbb{R}^2$ , 418
  - linear, 5, 510–511, 560–565, 598, 611
    - of AR-1 process, 570
    - with Fourier basis, 5, 510–511
    - with Haar basis, 6
  - minimax, 523–529, 597
  - near-minimax, 527–529, 597, 608
  - nonlinear, 6, 511–512, 560–565, 598, 611
    - operator, 610
    - with Haar basis, 6, 511–512
  - of functions, 510–511, 513–560
    - by polynomials, 513–537
    - by series truncation, 560–576, 598
    - by splines, 510–511, 537–560
  - of sequences
    - by series truncation, 560–576, 598
  - performance of, 611
  - piecewise-constant, 503
  - power of, 417
  - Taylor series, 509, 520–522, 597, 607
    - error of, 521, 597
  - to ideal filters, 464–467
  - Weierstrass theorem, 524
  - with Bernstein polynomials, 607, 608
  - with Chebyshev polynomials, 528
  - with Fourier basis, 5, 510–511
  - with Haar basis, 6, 511–512
  - with Legendre polynomials, 515
  - with local Fourier basis, 646, 652
  - with wavelet basis, 649, 655
- Argument
  - of complex number, 313
  - of function, 136
- Arithmetic
  - fixed-point, 123
  - floating-point, 123–126
- Arithmetic–geometric sequence
  - DTFT pair, 222
  - $z$ -transform pair, 243
- Associativity property
  - failure of, 208
  - of convolution
    - for functions on real line, 356, 404, 405
    - for infinite sequences, 207
  - vector space axiom, 18
- Asymptotic notation, xviii, 121
  - $\Omega(\cdot)$ , xviii, 121
  - $O(\cdot)$ , big O, xviii, 121
  - $o(\cdot)$ , little o, xviii, 121
  - $\Theta(\cdot)$ , xviii, 121
- Atkinson, Kendall E., 599
- Autocorrelation of function
  - deterministic, 350–351
  - relation to convolution, 356
  - Fourier transform pair, 369
  - of white noise, 397
  - stochastic, 395, 399
- Autocorrelation of sequence
  - deterministic, 190, 327, 336, 338
  - relation to convolution, 207
  - DTFT pair, 224
  - of polyphase components, 279
  - of white noise, 288
  - rational, 245
  - stochastic, 286, 292, 341
  - $z$ -transform pair, 245
- Autocorrelation of vector sequence
  - deterministic, 191–192
  - DTFT pair, 225
  - $z$ -transform pair, 246
- Autoregressive process, 291
  - first-order, 291, 293, 296
  - linear approximation of, 570
- MA, 290
- Averaging operator
  - continuous-time, 354
  - local, 45
  - discrete-time, 201
- B-splines, 541–545, 597, 609
  - orthogonalizing, 610
  - uncertainty principle for, 668
- Balakrishnan, A. Venkataraman, 499
- Balian, Roger, 668
- Banach space, 38
- Band matrix, 150
- Bandlimited
  - continuous-time stochastic process, 470, 505

- finite-dimensional vector, 503
- function, 452
  - aliasing of, 460–462
  - interpolation for, 452–470
  - projection to subspace, 453–454
  - sampling followed by interpolation for, 455–456
  - sampling for, 452–470
  - subspace of, 452
- periodic function, 481
  - interpolation for, 481–489
  - projection to subspace, 483–486
  - sampling followed by interpolation for, 486–489
  - sampling for, 481–489
  - subspace of, 481
- sequence, 437
  - aliasing of, 440–441
  - interpolation for, 437–442
  - oversampling of, 441
  - projection to subspace, 438
  - sampling followed by interpolation for, 440–442
  - sampling for, 437–442
  - subspace of, 437
- Bandpass
  - filter
    - continuous-time, 374
    - discrete-time, 228
  - sampling, 460, 505
- Bandwidth
  - of finite-dimensional vector, 503
  - of function, 452
  - of periodic function, 481
  - of sequence, 437
- Barbotin, Yann, 667
- Base spectrum, 441
- Basis, xix, 70, 69–119, 159–160, 162
  - algebraic, 159
  - analysis operator, 75
    - in  $\mathbb{R}^2$ , 16
  - best approximation with, 82
  - change in  $\mathbb{R}^2$ , 14
  - completeness of, 70
  - decomposition with, 97
  - diagonalization of operator in  $\mathbb{R}^N$ , 113
  - dual, 161, 175
  - existence of, 657
  - for  $\mathbb{C}^N$ , 71
  - for  $\mathbb{C}^Z$ , 71
  - for  $\mathbb{R}^2$ , 13–17
  - Gram matrix for, 108
  - Haar, 3–5
    - for functions, 648–650
    - for sequences, 653–656
  - Hamel, 159
  - local Fourier, 644–656
    - for functions, 645–647
    - for sequences, 651–653
  - nonorthogonal, 74
  - nonunit norm, 74
  - orthogonal decomposition with, 82
  - orthonormal, 76
  - Riesz, 72, 69–76, 174
  - Schauder, 160
  - standard, xix
    - for  $\mathbb{R}^2$ , 13
  - successive approximation, 82
  - synthesis operator, 75
    - in  $\mathbb{R}^2$ , 17
  - unconditional, 70, 160, 162
  - wavelet, 644–656
    - for functions, 648–650
    - for sequences, 653–656
- Battle-Lemarié
  - scaling function, 610
  - wavelet, 610
- Bayes' rule, 151
- Bayesian estimation, 156, 179
- Belyaev, Yuri K., 499
- Bernstein polynomials, 607, 608
- Bertsekas, Dimitri P., 162
- Bessel's inequality, 82
- Best approximation, 50–51
  - by ramps, 436
  - by sampling followed by interpolation, 499
  - in  $\mathbb{R}^3$ , 169
  - in splines spaces, 548
  - piecewise-constant, 499, 503
  - projection theorem, 54
  - pseudoinverse operator, 60
  - with
    - basis, 82
    - pair of biorthogonal bases, 98–100
- Bézout identity, 319
- Biased estimation, 157
- Bijjective function, 136
- Biorthogonal pair of bases, 86, 86–101, 161
  - change of, 111
  - finite-dimensional, 87
  - for  $\mathbb{R}^2$ , 14–15
  - matrix representation of operators with, 116
  - of cosine functions, 87, 175
  - series truncation with, 610
  - successive approximation with, 175
- Biorthogonality condition, 87
  - in  $\mathbb{R}^2$ , 14
- Bit allocation, 587–588, 612
- Blahut, Richard E., 328
- Block
  - average filter, 202
  - transform, 652
- Blu, Thierry, 498

- Bounded operator, 41
- Bounded-input bounded-output stable system  
 discrete-time, 208, 250
- Bounded-input, bounded-output stable system  
 continuous-time, 353, 357, 406  
 discrete-time, 198
- Box  
 function, 349, 402  
   centered and normalized, 349  
   Fourier series pair, 386  
   Fourier transform pair, 361, 366, 372  
   frequency center and spread, 622  
   relation to sinc function, 349  
   spectral decay, 376  
   time center and spread, 621  
   unit-width, 376  
 sequence, 188, 402  
   centered and normalized, 188  
   DFT pair, 256  
   DTFT pair, 222, 229  
   frequency center and spread, 631, 671  
   right-sided, 188  
   time center and spread, 629  
   uncertainty principle, 636
- Bracewell, Ronald N., 403
- Brémaud, Pierre, 403
- Calvez, Léon Claude, 667
- Canonical dual  
 frame, 108, 176  
 spline, 544, 609
- Cardinal spline, 547
- Carleson, Lennart, 403
- Cauchy sequence of vectors, 38, 173
- Cauchy–Schwarz inequality, 29, 31, 139  
 proof of, 171
- Causal  
 elementary B-splines, 542, 609  
 filter, 208  
 system  
   continuous-time, 353  
   discrete-time, 197
- Central limit theorem, 155
- Centroid decoding, 612
- Change  
 of basis  
   biorthogonal pair, 111  
   in  $\mathbb{R}^2$ , 14  
 of orthonormal basis, 109  
   by rotation in  $\mathbb{R}^2$ , 110  
   to standard in  $\ell^2(\mathbb{Z})$ , 111
- Characteristic polynomial, 145
- Chebira, Amina, 162
- Chebyshev  
 equioscillation theorem, 525  
 polynomial approximation with, 528  
 polynomials, 527, 601–603
- Chebyshev, Pafnuty L., 598, 674
- Cheney, Ward, 499
- Christensen, Ole, 162
- Circulant matrix, 150, 332–333  
 as circular convolution operator, 215  
 block, 338  
 diagonalization of, 150, 332–333
- Circular  
 shift in frequency  
   DFT pair, 257  
   DTFT pair, 223  
 shift in time  
   DFT pair, 255  
   Fourier series pair, 385  
 time reversal  
   DFT pair, 257
- Circular autocorrelation  
 of function  
   Fourier series pair, 388  
 of sequence  
   DFT pair, 257  
 of vector sequence  
   DFT pair, 258
- Circular convolution for functions, xx, 359,  
 358–359  
 definition, 359  
 diagonalization of, 394  
 eigenfunctions of, xx, 381  
 Fourier series pair, 387, 409  
 operator, 359
- Circular convolution for sequences, xx, 213,  
 211–216  
 as polynomial product, 337  
 circulant matrix as operator for, 215  
 cost of computing, 307  
 definition, 213  
 DFT pair, 257  
 diagonalization of, 259  
 eigensequences of, xx, 252, 325  
 equivalence with linear, 213  
 in frequency  
   DFT pair, 257  
   DTFT pair, 224, 337  
 matrix representation of, 214–216  
 operator, 213, 325
- Circular crosscorrelation  
 of function  
   Fourier series pair, 388  
 of sequence  
   DFT pair, 258
- Circular extension, 183  
 for functions, 358  
 for sequences, 211
- Classical estimation, 157  
 oracle scaling, 573–575

- Closed  
 set, 135  
 subspace, 37
- Closure of set, 135
- Code  
 fixed-rate lossless, 577  
 Shannon–Fano–Elias, 578
- Codebook optimization, 593
- Coding  
 gain, 590  
 Huffman, 591–592  
 lossless, 576  
 lossy, 576  
 nearest-neighbor, 612  
 source, 576  
 transform, 584–591
- Codomain of function, 136
- Coefficient  
 expansion, 70, 161  
   in  $\mathbb{R}^2$ , 13  
 Fourier, 70, 381  
   generalized, 70  
 Haar wavelet  
   continuous, 648  
   discrete, 654  
 local Fourier  
   continuous, 645  
   discrete, 651  
 subband, 70  
 transform, 70, 161  
 wavelet  
   continuous, 648  
   discrete, 654
- Cofactor, 141
- Column  
 space, 142, 144  
 vector, 141
- Commutativity property  
 of convolution  
   for functions on real line, 356  
   for infinite sequences, 207  
 of upsampling and downsampling, 271, 340  
 vector space axiom, 18
- Completeness, 35, 38, 37–40  
 lack of, 164–165, 172  
 of  $\mathbb{C}^N$ , 39, 172  
 of  $C^q([a, b])$ , 40  
 of  $\mathcal{L}^p(\mathbb{R})$ , 40  
 of  $\mathbb{R}^N$ , 39  
 of basis, 70, 172  
 of Fourier series, 408  
 of Hilbert space, 35, 37–40
- Complex  
 analysis, 313–314  
 exponential function, 354  
   aliasing, 461  
   Fourier transform pair, 365  
 exponential sequence, 193, 329  
 number(s), 313  
   argument of, 313  
   complex conjugate of, 313  
   Euler’s formula, 313  
   imaginary part of, 313  
   magnitude of, 313  
   modulus of, 313  
   phase of, 313  
   polar form of, 313  
   real part of, 313
- Complexity, 120
- Composition of functions, 136
- Compression, 511, 576–591, 658  
 image, 663–664  
 JPEG, 664, 668  
 JPEG2000, 664, 668  
 lossless, 577–579  
 lossy, 576
- Computational aspects, 119–134, 303–312, 489–495, 591–596, 658
- Condition number  
 of Hermitian matrix, 127  
 of normal matrix, 127
- Conditional  
 convergence, 137  
 expectation, 153  
 PDF, 153  
 probability, 151
- Conditioning of matrix, 126–128
- Consistent operator, 427, 445, 472, 497
- Constant  
 function  
   Fourier transform pair, 365  
   piecewise, 413, 503  
 sequence, 199  
   DFT pair, 256  
   DTFT pair, 220, 222
- Continuity of inner product, 170
- Continuous random variable, 152
- Continuous-time signal processing, 343–403  
 using discrete-time operators, 462  
 modulation, 505
- Continuous-time stochastic process, 395–397  
 autocorrelation of, 395  
 bandlimited, 470, 505  
 crosscorrelation of, 395  
 filtering, 400  
 Fourier transform, 399–400  
 Gaussian, 397  
 mean of, 395  
 power spectral density, 399  
 sampling for, 470  
 standard deviation of, 395  
 stationary, 396  
 variance of, 395  
 white noise, 397, 400

- WSS, 396
- Continuous-time system, 351–359
  - adjoint of convolution, 358
  - averaging operator, 354
  - BIBO-stable, 353, 406
  - causal, 353
  - circular convolution, 359, 358–359
  - convolution for functions on real line, 356, 356–359
  - differential equations, 355
  - impulse response of, 355
  - integrator, 354
  - linear, 352
  - LSI, 353, 355–359
  - maximum operator, 354
  - memoryless, 352
  - modulator, 354
  - properties of, 352–354
  - shift, 353
  - shift-invariant, 353
  - stable, 353
  - stochastic, 397–399
- Convergence, 36–37, 136–137
  - absolute, 137
  - conditional, 137
  - in normed vector space, 36–37
  - mean-square, 219
  - of convolution sum, 316
  - of DTFT, 218–221
  - of Fourier series, 383–385
  - of Fourier transform, 360–365
  - of inner product in  $\ell^2(\mathbb{Z})$ , 171
  - of sequence
    - of functions, 137
    - of numbers, 136–137, 177
    - of vectors, 36
  - of series, 137
  - of  $z$ -transform, 235–240
  - pointwise, 137
  - tests, 177
  - uniform, 137
- Convex set, 20
  - projection onto, 491–495
- Convolution
  - adjoint of, 406
  - circular
    - for finite-length sequences, 213
    - for periodic functions, 359
  - of derivative and primitive, 407
  - of PDFs, 406
  - of PMFs, 340
- Convolution for functions on real line, xx, 356, 356–358
  - adjoint of, 358
  - associativity, 356, 404, 405
  - commutativity, 356
  - connection to inner product, 356
  - deterministic autocorrelation, 356
  - differentiation, 369
  - eigenfunctions of, xx, 359
  - Fourier transform pair, 368
  - in frequency
    - Fourier series pair, 388
    - Fourier transform pair, 369
  - operator, 356
  - properties of, 356–357
  - smoothing effect of, 357
  - via discrete-time processing, 462
  - with circularly extended signal, 358–359
- Convolution for infinite sequences, xx, 206, 205–211
  - adjoint of, 210–211
  - associativity, 207
  - commutativity, 207
  - computation of using
    - overlap–add algorithm, 310
    - overlap–save algorithm, 311, 341
  - connection to inner product, 207
  - convergence of sum, 316
  - cost of computing, 307–311
  - deterministic autocorrelation, 207
  - DTFT pair, 224
  - eigensequences of, xx, 216, 234, 325
  - failure of the associative property, 208
  - matrix representation of, 209
  - operator, 206, 325
  - properties of, 207–208
  - with circularly extended signal, 211–213
  - $z$ -transform pair, 242
- Convolution theorem
  - for finite-length sequences, 257
  - for functions on real line, 368
  - for infinite sequences, 224, 242
  - for periodic functions, 387
- Conway, John H., 600
- Cooley, James W., 328
- Cooley–Tukey FFT, 306, 328
- Coordinates in  $\mathbb{R}^2$ , 13–17
- Correlation
  - coefficient, 32, 153
  - of function
    - deterministic, 350–351
    - stochastic, 395
  - of sequence
    - deterministic, 189–192
    - stochastic, 286
- Cosine
  - function
    - biorthogonal pair of bases of, 87, 175
    - frame of, 102, 175
    - orthonormal basis of, 76
  - sequence
    - DTFT pair, 221
- Cost of computing, 120–123

- circular convolution, 307
- complex multiplication, 176
- convolution, 307–311, 338
- downsampling preceded by filtering, 311
- FFT, 303–307
- Gaussian elimination, 131
- linear convolution, 307–311, 338
- matrix multiplication, 122, 167–168
- multirate operations, 282, 311–312
- polynomial evaluation, 120
- solution of system of linear equations, 134
- upsampling followed by filtering, 312
- Walsh–Hadamard transform, 335
- Coulot, Lionel, 498
- Courant, Richard, 162
- Covariance, 153
  - matrix, 154
- Cover, Thomas M., 599
- Cramer's formula, 142
- Crochiere, Ronald E., 328
- Cross spectral density, 400
- Crosscorrelation
  - of function
    - deterministic, 351
    - Fourier transform pair, 371
    - stochastic, 395
  - of sequence
    - deterministic, 190–191, 327, 336
    - DTFT pair, 225
    - stochastic, 286, 341
    - $z$ -transform pair, 246
- Cumulative distribution function, 152
- Daubechies, Ingrid, 162, 664
- Davis, Philip J., 599
- DCT, 328, 587, 664
- de la Vallée-Poussin alternation theorem, 525
- Decay
  - of function, 407
  - spectral
    - of Fourier series, 393, 409
    - of Fourier transform, 374–379
- Decoding
  - centroid, 612
- Decomposition, 2, 50–51, 60, 60–63, 82, 161
  - polyphase, 278
  - with basis, 97
- Decorrelation, 288
- Definite linear operator, 50
  - eigenvalue of, 173
- Degenerate Gaussian PDF, 154
- Delay operator, 198
- Dense span, 160
- Derivative
  - discrete, 556
- Descartes, René, 1
- Determinant, 141
- Cramer's formula, 142
- Deterministic
  - autocorrelation
    - of function, 350–351, 356, 369
    - of sequence, 190, 207, 224, 245, 279, 327, 336, 338
    - of vector sequence, 191–192, 225, 246
  - correlation
    - of function, 350–351
    - of sequence, 189–192
  - crosscorrelation
    - of function, 351, 371
    - of sequence, 190–191, 225, 246, 327, 336
- DFT, 659
- Diagonalization
  - of basis operator in  $\mathbb{R}^N$ , 113
  - of circular convolution operator, 259
  - of matrix, 146
    - circulant, 150
- Difference equations, 202, 315–316
  - homogeneous solution to, 315
  - impulse response from, 205
  - initial conditions to, 205
  - linear, 202
    - constant-coefficient, 202
- LSI, 336
  - solution to, 206
- of accumulator, 203
- particular solution to, 315
- transfer function, 249
- with finite number of coefficients, 249
- Differential entropy, 584
- Differential equations, 355
  - homogeneous solution to, 355
  - initial conditions to, 355
  - linear, 355
    - constant-coefficient, 355
    - particular solution to, 355
- Differentiation
  - convolution of derivative and primitive, 407
  - in  $z$ 
    - $z$ -transform pair, 241
  - in frequency
    - DTFT pair, 223
    - Fourier transform pair, 367
  - in time
    - Fourier series pair, 385
    - Fourier transform pair, 367
  - of convolution for functions on real line, 369
- Digital communications, 411, 659–663
- Dimension of vector space, 22
- Dirac
  - comb, 390, 392
    - Fourier series pair, 386, 390, 505
    - Fourier transform pair, 366, 409
  - weighted, 415, 456



- delta function, xviii, 316, 404
  - derivative of, 406
  - Fourier transform pair, 365, 366, 372
  - properties of, 318
  - relation to Heaviside function, 349
- Direct sum, 60, 60–63
  - decomposition, 61
  - failure of, 61
- Dirichlet kernel, 482–483
  - properties, 502–503
  - properties of, 502
- Discrete
  - derivative, 556
  - frequency, 253, 381
  - integral, 559
  - spline
    - uncertainty principle for, 672
- Discrete cosine transform, 328, 587, 664
- Discrete Fourier transform, xxi, 253, 256, 252–264, 325, 401
  - analysis
    - of infinite sequences, 260–264
    - with orthonormal basis, 254
  - circular convolution
    - in frequency, 257
    - in time, 257
  - circular deterministic autocorrelation, 257
    - of vector sequence, 258
  - circular shift
    - in frequency, 257
    - in time, 255
  - definition, 252–255
  - deterministic crosscorrelation, 258
  - diagonalization
    - of circulant matrix, 150, 332–333
    - of circular convolution operator, 259
  - FFT, 303
  - frequency, 253
  - frequency response, xx, 253, 259–264
  - inverse, 253
  - linearity, 255
  - matrix representation of, 254
  - modulation, 257
  - operator, 253, 255
    - Vandermonde matrix, 254
  - Parseval equality for, 258
  - properties of, 255–258, 338
  - relation to DTFT, 255
  - spectrum, 253
  - time reversal, 257
- Discrete-time Fourier transform, xxi, 217, 222, 216–233, 325, 401
  - adjoint of, 227
  - angular frequency, 217
  - circular convolution in frequency, 224, 337
  - convergence of, 218–221
  - convolution in time, 224
    - definition, 216–218
  - deterministic
    - autocorrelation, 224
    - autocorrelation of vector sequence, 225
    - crosscorrelation, 225
  - differentiation, 223
  - downsampling
    - by 2, 267
    - by  $N$ , 268
    - preceded by filtering, 272
  - downsampling followed by upsampling
    - by 2, 270
    - by  $N$ , 271
  - duality with Fourier series, 382, 412
  - existence of, 218–221
  - failure of convergence, 220
  - for stochastic processes, 292–294
  - frequency, 199, 217
  - frequency response, xx, 217, 227–233
  - Gibbs phenomenon, 219
  - inverse, 217
  - linearity, 221
  - mean-square convergence of, 219
  - modulation, 223
  - moments, 223
  - of sequences
    - constant, 220
    - cosine, 221
    - in  $\ell^1(\mathbb{Z})$ , 218
    - in  $\ell^2(\mathbb{Z})$ , 219
    - sinc, 219
  - Parseval equality for, 226, 231
  - properties of, 221–227
  - relation to DFT, 255
  - sampling of, 504
  - scaling in time, 223
  - shift
    - in frequency, 223
    - in time, 221
  - spectrum, 217
  - time reversal, 223
  - upsampling
    - by 2, 269
    - by  $N$ , 270
    - followed by filtering, 274
- Discrete-time operator
  - for continuous-time signal processing, 462
  - modulation, 505
- Discrete-time signal processing, 325, 181–328
- Discrete-time stochastic process, 285–288
  - ARMA, 290
  - autocorrelation of, 286, 341
  - crosscorrelation of, 286, 341
  - downsampling, 297
    - preceded by filtering, 299
  - DTFT, 292–294
  - filtering, 294, 298

- Gaussian, 288
- i.i.d., 286
- mean of, 286
- MMSE estimation, 300–303
- multirate system, 294–303
- orthogonal, 300
- power spectral density, 292
- rational sampling rate change, 300
- standard deviation of, 286
- stationary, 287
- upsampling, 298
  - followed by filtering, 299
- variance of, 286
- white noise, 288, 294
- Wiener filtering, 300, 334
- WSCS, 294
- WSS, 287
- Discrete-time system, 195–216
  - accumulator, 200, 203
  - adjoint of convolution, 210–211
  - advance, 198
  - averaging operator, 201
  - BIBO-stable, 198, 250
  - causal, 197
  - circular convolution, 213, 211–216
  - convolution, 211–213
    - for infinite sequences, 206, 205–211
  - difference equations, 202–205, 249
  - equivalence of circular and linear convolutions, 213
  - impulse response of, 205
  - Laplacian, 336
  - linear, 195
  - LPSV, 265, 340
  - LSI, 197, 205–211
  - matrix representation
    - of circular convolution, 214–216
    - of convolution, 209
  - maximum operator, 202
  - memoryless, 196
  - modulator, 199
  - multirate, 264–285
  - periodically shift-varying, 266
  - properties of, 195–202
  - shift, 198
  - shift-invariant, 197
  - stable, 197
  - stochastic, 288–292
- Discrete wavelet transform, 587, 655, 664
- Distance, 30
  - in  $\mathbb{R}^2$ , 11
  - not induced by norm, 171
- Distortion, 580
- Distributivity property
  - inner product axiom, 23
  - vector space axiom, 18
- Divergent sequence, 136
- Domain of function, 136
- Dominated convergence theorem, 139
- Donoho, David L., 599, 667
- Doubly infinite series, 137
- Downsampling, 265–268, 339, 628, 630, 632, 634
  - and upsampling, 270–271, 340
    - and filtering, 275, 272–278
  - by 2, 265, 265
  - by  $N$ , 268, 339, 504
  - commutativity with upsampling, 271, 340
  - DTFT pair, 223, 267, 268, 339
  - followed by upsampling, 270
  - matrix representation of, 266
  - of discrete-time stochastic process, 297
  - operator, 266
    - preceded by filtering, 272
      - cost of computing, 311
      - of discrete-time stochastic process, 299
    - preceded by upsampling, 270
  - $z$ -transform pair, 241, 267, 268, 339
- Dragotti, Pier Luigi, 498
- Draščić, Biserka, 499
- Dual
  - basis, 86, 92–96, 118, 161, 175
    - in  $\mathbb{R}^2$ , 14
  - expansion coefficients, 96
  - frame, 108, 161, 176
    - in  $\mathbb{R}^2$ , 15
  - pair of frames, 107, 107–109, 161
    - in  $\mathbb{R}^2$ , 107
    - properties of, 176
  - spline, 544, 609
    - canonical, 544, 609
- Dudgeon, Dan E., 328
- DWT, 587, 655, 664
- Dyadic
  - Haar basis
    - for functions, 648
    - for sequences, 654
  - tiling
    - for functions, 640
    - for sequences, 643
- Eigenfunction, xx, 49
  - of circular convolution for functions, 381
  - of convolution for functions on real line, 359
- Eigenpair
  - of matrix, 145
  - of operator, 49
- Eigensequence, xx, 49
  - of circular convolution for sequences, 252, 325
  - of convolution for infinite sequences, 216, 234, 325
- Eigenvalue

- and operator norm, 173
- of linear operator, 173
- of matrix, 145, 178
- of operator, 49, 179
- of positive definite matrix, 149
- of self-adjoint operator, 49
- real, 49
- Eigenvector, *xx*
  - of matrix, 145, 178
  - of operator, 49
  - of self-adjoint operator, 49
  - orthogonal, 49
- Elementary B-spline, 541–542, 609
  - orthogonalizing, 610
- Elias, Peter, 599
- Energy, 225, 293, 370, 400
  - conservation, 226
  - spectral density, 225, 293, 370, 400
- Entropy
  - bound on optimal code length, 579
  - constrained quantization, 582
  - differential, 584
  - joint, 611
- Equivalence of norms, 172
- Error
  - loss of significance, 125
  - of approximation, 513
    - by Lagrange interpolation, 519, 522, 597
    - by Taylor series, 521, 597
  - of estimation, 157
  - system, 532
  - weighted, 533
- Essential
  - infimum, 135
  - lower bound, 135
  - supremum, 135
  - upper bound, 135
- Estimation, 155–159
  - Bayesian, 179
  - classical, 157
    - oracle scaling, 573–575
  - error of, 157
  - from quantized frame expansion, 594–595
  - hard threshold, 576
  - linear, 63–66, 68
    - diagonal, 571–576
  - LMMSE, 63, 64, 179
  - MAP, 156
  - minimum-variance unbiased, 158
  - ML, 158
  - MMSE, 63–69, 156, 158, 300–303
  - MSE of, 157
  - nonlinear
    - diagonal, 571–576
  - of random vectors, 67–69
  - optimal, 63, 66–69
  - oracle, 575
    - performance of, 611
    - (un)biased, 157
- Euclid, 162, 673
  - Elements*, 29, 162
- Euclidean
  - geometry, 162
  - norm, 28
  - space, 10, 18
  - square norm, 33
- Euler's formula, 313
- Even function, 174
- Event, 151
- Existence
  - of DTFT, 218–221
  - of Fourier series, 383–385
  - of Fourier transform, 360–365
  - of *z*-transform, 235–240
- Expansion, 161
  - coefficient, 70, 161
    - in  $\mathbb{R}^2$ , 13
  - matrix representation in  $\mathbb{R}^2$ , 17
  - of operators, 173
  - Taylor, 509, 520–522, 597, 607
    - error of, 521, 597
  - with frame in  $\mathbb{R}^2$ , 15
  - with orthonormal basis, 76–79
    - in  $\mathbb{R}^2$ , 13
  - with pair of biorthogonal bases, 88–90
    - in  $\mathbb{R}^2$ , 14
  - with tight frame in  $\mathbb{R}^2$ , 15
- Expectation, 152–153
- Exponential function
  - Fourier transform pair, 366
- Extension, 577
- Fano, Robert M., 599
- Fast algorithms, 328
- Fast Fourier transform algorithm, 303–307
  - computing
    - circular convolution, 307
    - cost of, 303–307
    - linear convolution, 307–311, 338
  - Cooley–Tukey, 306, 328
  - Good–Thomas, 306
  - Rader's, 306
  - radix-2, 304
  - split-radix, 307
  - Winograd, 307
- FFT, 658
- Fibonacci filter, 331, 332
- Filter
  - allpass
    - continuous-time, 373
    - discrete-time, 231, 232, 251
  - anticausal, 208
  - block average, 202
  - causal, 208

- Daubechies, 664
- design, 599
- Fibonacci, 331, 332
- FIR, 208, 229, 251
- half-band
  - design, 531, 533
- ideal
  - approximation to, 464–467
  - continuous-time, 373
  - discrete-time, 228, 229
- ideal bandpass
  - continuous-time, 374
  - discrete-time, 228
- ideal half-band, 229
  - continuous-time, 402
  - discrete-time, 229
  - DTFT pair, 219
- ideal highpass
  - continuous-time, 374
  - discrete-time, 228
- ideal lowpass
  - continuous-time, 373
  - discrete-time, 228, 229
- ideal  $N$ th-band
  - discrete-time, 229
- ideal third-band, 337
- IIR, 208
- linear-phase, 230, 251
  - continuous-time, 373
  - design, 529–537
  - discrete-time, 227
  - generalized, 227
- lowpass design, 535
- moving-average
  - continuous-time, 354
  - discrete-time, 184, 201, 229
- orthogonal, 278
- orthogonality of impulse response
  - to all shifts, 337
  - to even shifts, 277
- passband
  - continuous-time, 373
  - discrete-time, 228
- post, 433, 444, 450, 471, 479
- pre, 430, 442, 449, 471, 478
- stopband
  - continuous-time, 373
  - discrete-time, 228
- transition band, 442
- two-sided, 208
- zero-phase
  - continuous-time, 373
  - design, 529–537
  - discrete-time, 227
- Filtering
  - as projection, 330
  - followed by downsampling, 272
  - cost of computing, 311
  - of discrete-time stochastic process, 299
- interchange of multirate operations, 340
- interchange of multirate operations and, 275
- of discrete-time stochastic process, 298
- preceded by upsampling, 273
  - cost of computing, 312
  - of discrete-time stochastic process, 299
- to remove uncorrelated additive noise, 302
- Wiener, 300, 334
- Finite
  - geometric series, 178
  - impulse response filter, 208, 229, 251
- Finite-dimensional
  - biorthogonal pair of bases, 87
  - nonorthogonal vectors
    - interpolation followed by sampling for, 427
    - interpolation for, 426
    - sampling followed by interpolation for, 427–429
    - sampling for, 425–426
  - orthonormal basis, 76
  - orthonormal vectors
    - interpolation followed by sampling for, 423
    - interpolation for, 422–423
    - sampling followed by interpolation for, 423–424
    - sampling for, 421–422
  - space, 18
  - vectors
    - bandlimited, 503
    - full-band, 503
    - interpolation for, 420–429
    - sampling for, 420–429
- Finite-length sequence, 183, 192–193
  - recovery, 672
  - uncertainty principle for, 636–637, 672
- Fix, George, 599
- Fixed-point arithmetic, 123
- Fixed-rate
  - lossless code, 577
  - quantization, 582
- Fleischer, P. E., 599
- Floating-point arithmetic, 123–126
- Folding frequency, 457
- Folland, Gerald B., 403
- Fourier coefficient, 70, 381
  - generalized, 70
- Fourier representations, 48, 401, 657
  - for finite-length sequences, 253
  - for functions on real line, 360
  - for infinite sequences, 217
  - for periodic functions, 381

- Fourier series, xxi, 2–3, 5, 381, 386, 380–394, 401  
 as orthonormal basis expansion, 383  
 circular convolution in time, 387, 409  
 circular deterministic  
 autocorrelation, 388  
 crosscorrelation, 388  
 completeness of, 408  
 convergence of, 383–385  
 convolution in frequency, 388  
 definition, 381–382  
 diagonalization of circular convolution operator, 394  
 differentiation in time, 385  
 Dirac comb, 390, 505  
 duality with DTFT, 382  
 existence of, 383–385  
 frequency, 381  
 fundamental, 381  
 frequency response, xx, 394  
 Gibbs phenomenon, 5  
 integration, 387  
 real, 408  
 inversion  
 in  $\mathcal{L}^2$ , 384  
 least-squares approximation, 384  
 linearity, 385  
 modulation, 385  
 norm conservation, 384  
 of functions with spectra  
 in  $\ell^1(\mathbb{Z})$ , 383  
 in  $\ell^2(\mathbb{Z})$ , 384  
 Parseval equality for, 384  
 Poisson sum formula, 392, 409  
 properties of, 385–394  
 real, 408  
 integration, 408  
 regularity of, 393, 409  
 relation to Fourier transform, 382, 406  
 sawtooth wave, 386, 409  
 shift  
 in frequency, 385  
 in time, 385  
 spectral decay of, 393, 409  
 spectrum, 381  
 square wave, 386, 388  
 time reversal, 385  
 triangle wave, 386, 389, 409  
 Fourier transform, xxi, 360, 366, 359–380, 401  
 $C^q$  regularity, 375  
 adjoint of, 371  
 angular frequency, 359  
 box function, 372  
 convergence of, 360–365  
 convolution  
 in frequency, 369  
 in time, 368  
 definition, 359–360  
 deterministic  
 autocorrelation, 369  
 crosscorrelation, 371  
 differentiation  
 in frequency, 367  
 in time, 367  
 Dirac  
 comb, 409  
 delta function, 372  
 existence of, 360–365  
 for stochastic processes, 399–400  
 frequency, 359  
 frequency response, xx, 360, 373–374  
 integration, 368  
 inverse, 360  
 inversion, 364  
 linearity, 365  
 Lipschitz regularity, 378, 408  
 modulation, 365  
 moments, 368  
 no existence, 364  
 of functions  
 in  $\mathcal{L}^1(\mathbb{R})$ , 361, 405, 406  
 in  $\mathcal{L}^2(\mathbb{R})$ , 363  
 of periodic functions, 391  
 Parseval equality for, 371  
 properties of, 365–372  
 regularity of, 374–379  
 relation to Fourier series, 382, 406  
 scaling  
 in frequency, 367  
 in time, 367  
 shift  
 in frequency, 365  
 in time, 365  
 spectral decay of, 374–379, 407  
 spectrum, 360  
 time reversal, 367  
 Fourier, J. B. Joseph, 2, 403, 673  
 Frame, 101, 101–109, 161, 162, 175  
 1-tight, 105  
 analysis operator, 103  
 bounds, 101  
 canonical dual, 108, 176  
 dual, 176  
 pair of, 107  
 estimation from quantized, 594–595  
 for  $\mathbb{R}^2$ , 15–17, 103  
 Gram matrix for, 108  
 inverse  
 relationship for operators, 106  
 synthesis and analysis, 108  
 oblique projection with, 108  
 of cosine functions, 102, 175  
 Parseval tight, 106

- redundancy of, 106
  - synthesis operator, 103
  - tight, 104
  - versus Riesz basis, 101
- Frequency, 199, 217, 359
  - angular, 217, 359
  - center and spread
    - for box function, 622
    - for box sequence, 631, 671
    - for functions, 622, 666, 669
    - for Gaussian function, 622
    - for sequences, 630, 666, 672
    - for sinc function, 622
    - for sinc sequence, 631
  - discrete, 253, 381
  - division, 663
  - folding, 457
  - fundamental, 381
  - localization, 616
    - for functions, 622–623
    - for sequences, 630–633
  - Nyquist, 457
  - one-sided center and spread
    - for functions, 623
    - for sequences, 632
  - response, xx
    - DFT of filter, 259
    - DTFT of filter, 227
    - Fourier series of filter, 394
    - Fourier transform of filter, 373
    - inverse, 227, 259, 373, 394
    - magnitude response, 227, 259, 373, 394
    - phase response, 227, 259, 373, 394
  - sampling, 457
- Frobenius norm, 147
- Fubini's theorem, 138
- Full-band
  - finite-dimensional vector, 503
  - function, 452
  - periodic function, 481
  - sequence, 437
- Full-rank matrix, 161
- Function, 135–137, 345–351
  - affine, 63, 176
  - antiderivative of, 362
  - argument of, 136
  - bandlimited, 452
    - aliasing of, 460–462
    - interpolation for, 452–470
    - sampling followed by interpolation for, 455–456
    - sampling for, 452–470
    - subspace of, 452
  - bandlimited periodic
    - interpolation for, 481–489
    - sampling for, 481–489
  - bandwidth of, 452
  - bijjective, 136
  - box, 349, 402
    - centered and normalized, 349
    - Fourier transform pair, 361, 372
    - frequency center and spread, 622
    - relation to sinc function, 349
    - time center and spread, 621
  - circular extension, 358
  - codomain of, 136
  - complex
    - uncertainty principle for, 671
  - complex exponential
    - aliasing, 461
    - Fourier transform pair, 365
  - composition of, 136
  - continuous, 346
  - cosine, 76, 87, 102, 175
  - decay of, 407
  - Dirac comb, 390, 392, 505
    - weighted, 415, 456
  - Dirac delta, xviii, 316, 404
    - derivative of, 406
    - Fourier transform pair, 365, 372
    - relation to Heaviside function, 349
  - domain of, 136
  - even, 174
  - frequency center and spread, 622, 666, 669
  - full-band, 452
  - Gaussian, 350
    - as PDF, 350
    - Fourier transform pair, 372, 407
    - frequency center and spread, 622
    - time center and spread, 621
  - generalized, 317
  - Heaviside, 348, 380
    - Fourier transform pair, 372
    - Laplace transform pair, 380
    - relation to Dirac delta function, 349
  - image of, 136
  - improper, 317
  - indicator, xviii, 413
  - infinite bandwidth, 452
  - injective, 136
  - interpolation for, 447–477
  - inverse, 136
  - Lebesgue
    - integrable, 24
    - measurable, 24
  - Lipschitz
    - pointwise, 378
    - uniformly, 378
  - localization for, 619–627
    - in frequency, 622–623
    - in time, 620–621
  - modified
    - frequency center and spread, 627, 671
    - time center and spread, 627, 671

- nonorthogonal
  - interpolation followed by sampling for, 472–474
  - interpolation for, 471–472
  - sampling followed by interpolation for, 474–477
  - sampling for, 471
- odd, 174
- on finite interval, 345
- on real line, 345, 345–351
- one-sided frequency center and spread, 623
- orthonormal
  - interpolation followed by sampling for, 451
  - interpolation for, 414, 450–451, 500
  - sampling followed by interpolation for, 415, 451–452
  - sampling for, 414, 449–450, 500
- orthonormal periodic
  - interpolation followed by sampling for, 480
  - interpolation for, 479–480
  - sampling followed by interpolation for, 480–481
  - sampling for, 478–479, 505
- periodic, 351
  - bandlimited, 481
  - bandlimited subspace of, 481
  - bandwidth of, 481
  - full-band, 481
  - interpolation for, 477–489
  - sampling for, 477–489
  - sampling theorem for, 489
  - shift-invariant subspace of, 478, 505
- piecewise
  - constant, 413, 503
  - linear, 346
- primitive of, 362
- projection to bandlimited subspace of, 453–454
  - periodic, 483–486
- range of, 136
- rational, 319
- recovery for, 415
- regularity of, 374–379
- Riemann integrable, 24
- sampling for, 447–477
- sampling theorem for, 457, 456–460
- shift-invariant subspace of, 449
- sinc, 186, 402
  - Fourier transform pair, 363, 372
  - frequency center and spread, 622
  - relation to box function, 349
  - time center and spread, 621
- sinc squared
  - Fourier transform pair, 458
  - Nyquist sampling, 458
  - undersampling of, 459
- sinusoidal, 461
- surjective, 136
- time center and spread, 621, 666, 669
- triangle
  - Fourier transform pair, 362
  - in frequency, 458, 459, 506
  - periodic, 94
  - uncertainty principle for, 624–627
    - complex, 671
    - unit-step, 348
- Fundamental
  - frequency, 381
  - theorem of algebra, 318
- Furukawa, Keiichi, 667
- Gabor, Dennis, 667, 674
- Gallager, Robert G., 403
- Gauss, Carl F., 328
- Gaussian
  - continuous-time stochastic process, 397
  - degenerate PDF, 154
  - discrete-time stochastic process, 288
  - distribution, 154
  - elimination, 129–131, 176
    - cost of computing, 131
  - function, 350
    - as PDF, 350
    - Fourier transform pair, 366, 372, 407
    - frequency center and spread, 622
    - time center and spread, 621
  - PDF, 154
  - random
    - variable, 154–155
    - vector, 154–155, 179
- General frame, 107
- Generalized
  - function, 317
  - Parseval equality, 77
    - for DTFT, 227
    - for Fourier transform, 407
- Generator of shift-invariant subspace
  - of  $\mathcal{L}^2(\mathbb{R})$ , 449
  - of  $\mathcal{L}^2(-\frac{1}{2}T, \frac{1}{2}T)$ , 478
  - of  $\ell^2(\mathbb{Z})$ , 430
- Geometric
  - sequence, 186
    - DFT pair, 256
    - DTFT pair, 222
    - properties of, 336
    - ROC of left-sided, 237, 337
    - ROC of right-sided, 236, 337
    - $z$ -transform pair, 243
  - series, 178
- Geometric view of signals and spaces, 657
- Gerchberg, R. W., 499
- Gersho, Allen, 599



- Gibbs phenomenon  
 for DTFT, 219  
 for Fourier series, 5  
 sawtooth wave, 409
- Gibbs, Josiah W., 5, 403, 673
- Gilbert, Alan. C  
*All Is Vanity*, 618
- Givens rotations, 162
- Gohberg, Israel, 162
- Goldberg, Seymour, 162
- Goldstein, Allen A., 499
- Good–Thomas FFT, 306
- Goyal, Vivek K, 185, 232, 600, 668
- Gram matrix, 90  
 for basis, 108  
 for frame, 108  
 for Riesz basis, 98
- Gram–Schmidt orthogonalization, 84–86  
 of Legendre polynomials, 86
- Grant, Edward, 1
- Gray, Robert M., 599
- Grimmett, Geoffrey R., 162
- Haar  
 basis, 3–5  
 for functions, 648–650  
 for sequences, 653–656  
 scaling  
 function, 3, 649  
 sequence, 655  
 wavelet  
 continuous-time, 3, 648  
 discrete-time, 654
- Haar, Alfred, 3
- Half-band filter  
 DTFT pair, 219  
 impulse response of, 229  
 least-squares design, 531  
 minimax design, 533
- Hamel basis, 159
- Hamming, Richard W., 24
- Heaviside  
 function, 348  
 Fourier transform pair, 372  
 Laplace transform pair, 380  
 pointwise multiplication by, 349  
 relation to Dirac delta function, 349  
 sequence, 187  
 pointwise multiplication by, 188
- Heil, Christopher E., 162
- Heisenberg box  
 for functions, 624  
 uncertainty principle, 625  
 for sequences, 633  
 uncertainty principle, 634
- Heisenberg, Werner K., 667, 674
- Hermite interpolation, 522–523, 607
- Hermitian  
 matrix, 127, 148, 191  
 condition number of, 127  
 operator, 43  
 symmetry property  
 inner product axiom, 23  
 of autocorrelation, 190  
 transpose, 43, 141, 277
- Highpass filter  
 continuous-time, 374  
 discrete-time, 228
- Hilbert space, xix, 38, 35–50, 79–80  
 isometry of, 79
- Hilbert, David, 162, 673
- Hölder  
 conjugates, 139  
 exponent, 378  
 inequality, 139
- Householder building blocks, 162
- Huffman coding, 591–592
- Huo, Xiaoming, 667
- Ideal filter  
 approximation to, 464–467  
 bandpass  
 continuous-time, 374  
 discrete-time, 228  
 continuous-time, 373  
 discrete-time, 228, 229  
 half-band  
 discrete-time, 229  
 highpass  
 continuous-time, 374  
 discrete-time, 228  
 impulse response of, 228  
 lowpass  
 continuous-time, 373  
 discrete-time, 228, 229  
 magnitude response of, 228  
*N*th-band  
 discrete-time, 229  
 third-band, 337
- Ideally matched operator, 428, 446, 474, 497, 504
- Idempotency property, 51
- Idempotent operator, 55
- Identity matrix, 141
- Image of function, 136
- Images, 663–664
- Imaginary part of complex number, 313
- Improper function, 317
- Improper rotation matrix, 148
- Impulse response  
 and periodized version, 212  
 from difference equation, 205  
 of continuous-time system, 355  
 of discrete-time system, 205



- orthogonality of
  - to all shifts, 337
  - to even shifts, 277
- Inconsistent operator, 500
- Independent events, 151
  - random variables, 153
- Indicator function, xviii, 56, 413
  - via pointwise multiplication by Heaviside function, 349
  - sequence, 56
  - via pointwise multiplication by Heaviside sequence, 188
- Inequality, 139–140
  - Bessel's, 82
  - Cauchy–Schwarz, 29, 31, 139, 171
  - Hölder's, 139
  - integral, 140
  - Jensen's, 611
  - Minkowski's, 139, 164
  - triangle, 27, 171
- Infimum, 135
- Infinite
  - bandwidth function, 452
  - impulse response filter, 208
  - length sequence, 183, 185–192
- Injective function, 136
- Inner product, xix, 23, 23–30
  - computation, 76–79, 88–91
  - in spline spaces, 610
  - conjugate linearity in second argument property, 23
  - continuity of, 170
  - convergence in  $\ell^2(\mathbb{Z})$ , 171
  - distributivity property, 23
  - Hermitian symmetry property, 23
  - linearity in first argument property, 23
  - on  $\mathbb{C}^N$ , 24, 170
  - on  $\mathbb{C}^{\mathbb{R}}$ , 24
  - on  $\mathbb{C}^{\mathbb{Z}}$ , 24
  - on  $\mathcal{L}^2([a, b])$ , 31
  - on  $\mathbb{R}^2$ , 10–11
  - on  $\mathbb{R}^N$ , 24
  - on random variables, 32
  - vectors, 179
  - positive definiteness property, 23
  - preservation of, 48
  - relation to convolution
    - for functions on real line, 356
    - for infinite sequences, 207
- Inner product space, 27, 30–32
  - of complex-valued finite-dimensional vectors, 31
  - of continuous functions with  $q$  continuous derivatives, 32
  - of polynomials, 172
  - of random variables, 32
  - of square-integrable functions, 31
  - of square-summable sequences, 31
- Integral
  - discrete, 559
  - inequalities, 140
- Integration
  - by parts, 140
  - Fourier series pair, 387
  - real, 408
  - Fourier transform pair, 368
- Integrator, 354
- Interchange
  - of multirate operations and filtering, 275, 340
  - of summation and integration, 138
  - theorems, 138–139
- Interior point, 135
- Interpolation, 658
  - for finite-dimensional vectors, 420–429
    - in  $\mathbb{C}^4$ , 423, 426
    - in  $\mathbb{R}^2$ , 417–420
    - nonorthogonal, 426, 497
    - operator, 421, 422, 425, 426
    - orthonormal, 422–423, 496
  - for functions, 447–477
    - adjoint operator of, 505
    - bandlimited, 452–470
    - error of Lagrange, 519
    - error of Lagrange interpolation, 522, 597
    - Hermite, 522–523, 607
    - in  $\mathcal{L}^2(\mathbb{R})$ , 472
    - Lagrange, 509–510, 517–520, 597, 600–601, 607
    - nonorthogonal, 471–472, 497
    - operator, 414, 449, 450, 471
    - orthonormal, 414, 450–451, 496, 500
    - oversampled, 501–502
  - for periodic functions, 477–489
    - bandlimited, 481–489
    - nonorthogonal, 497
    - operator, 478, 479
    - orthonormal, 479–480, 496
  - for sequences, 429–447
    - bandlimited, 437–442
    - in  $\ell^2(\mathbb{Z})$ , 434
    - nonorthogonal, 444, 497
    - operator, 430, 433, 442, 444
    - orthonormal, 433–434, 496
- node, 509
- operator
  - consistent with sampling operator, 427, 445, 472, 497

- for finite-dimensional vectors, 421, 422, 425, 426
  - for functions, 414, 449, 450, 471
  - for periodic functions, 478, 479
  - for sequences, 430, 433, 442, 444
  - ideally matched with sampling operator, 428, 446, 474, 497, 504
  - inconsistent with sampling operator, 500
  - range of, 500
- Interpolation followed by sampling
  - for finite-dimensional vectors
    - in  $\mathbb{C}^4$ , 427
    - nonorthogonal, 427, 497
    - orthonormal, 423, 497
  - for functions
    - in  $\mathcal{L}^2(\mathbb{R})$ , 472
    - nonorthogonal, 472–474, 497
    - orthonormal, 451, 497
  - for periodic functions
    - nonorthogonal, 497
    - orthonormal, 480, 497
  - for sequences
    - in  $\ell^2(\mathbb{Z})$ , 445
    - nonorthogonal, 444–446, 497
    - orthonormal, 415, 434, 497
  - operator
    - for finite-dimensional vectors, 423, 427
    - for functions, 451, 472
    - for periodic functions, 480
    - for sequences, 415, 434, 444
- Intersymbol interference, 662
- Inverse
  - DFT, 253
  - DTFT, 217
  - Fourier transform, 360
  - function, 136
  - matrix, 141
  - operator, 41, 58, 174
  - synthesis and analysis, 91–92
- Inversion
  - of Fourier series, 384
  - of Fourier transform, 364
- Ishii, Rokuya, 667
- Isometry of separable Hilbert space, 79–80
- Iterative projection, 596
  
- Jensen's inequality, 611
- Jerri, Abdul J., 498
- Johnstone, Iain M., 599
- Joint
  - CDF, 153
  - entropy, 611
  - PDF, 153
- Jointly
  - distributed random variables, 153–154
  - Gaussian
    - PDF, 154
  - random vector, 179
- Jones, M. C., 499
- JPEG, 664, 668
- JPEG2000, 664, 668
  
- Karhunen–Loève
  - basis for  $\mathbb{R}^2$ , 611
  - transform, 567, 604–606
  - optimality of, 591
- Kaczmarz's algorithm, 132, 177
- Kailath, Thomas, 162
- Kamilov, Ulugbek, 600
- Kernel
  - Dirichlet, 482, 502–503
  - matrix, 142, 144
  - operator, 40
- Knot, 537
- Kolmogorov, Andrey N., 403
- Konsbruck, Richard, 499
- Kotelnikov, Vladimir A., 457, 498
- Kovačević, Jelena, 162, 185, 232, 328, 668
- Kronecker
  - delta sequence, xviii, 13, 186, 636
  - DFT pair, 256
  - DTFT pair, 222
  - periodic, 192
  - $z$ -transform pair, 243
  - product, 324, 334, 335
  
- Lagrange
  - interpolating polynomial, 518, 607
  - interpolation, 509–510, 517–520, 597, 600–601, 607
  - error of, 519, 522, 597
- Laplace transform, 379, 380, 379–380
  - of Heaviside function, 380
  - ROC of, 379
- Laplacian operator, 336
- Least-squares, 143–144
  - approximation, 50, 384, 508, 514–517, 597
  - filter design as, 530–532
  - in  $\mathbb{R}^2$ , 418
  - solution, 144, 179
- Lebesgue
  - integrable function, 24
  - measurable function, 24
- Left
  - inverse, 41, 141
  - null space, 142
  - singular vector, 146
- Legendre polynomials, 86, 166, 515–516, 606
- Likelihood, 156, 157
- Lindberg, David C., 1
- Linear
  - approximation, 5, 510–511, 560–565, 598, 611
  - comparison with nonlinear, 562–563

- of AR-1 process, 570
  - of random vector, 566–573
  - of square wave, 564
  - of stochastic process, 566–571, 573
  - of triangle wave, 564, 611
- diagonal estimation, 571–576
- difference equations, 202
  - constant-coefficient, 202
  - LSI, 336
- differential equations, 355
  - constant-coefficient, 355
- estimation, 63–66, 68
  - MMSE, 63, 64
- independence, 160, 170
- operator, 40
- programming, 595–596
- Linear algebra, 10, 141–151, 162
  - definitions, 141–147
  - Kronecker product, 324
  - special matrices, 147–151
- Linear shift-invariant system, 197, 353
  - acting on sequences in  $\ell^p(\mathbb{Z})$ , 329–330
  - anticausal filter, 208
  - BIBO stability, 406
    - continuous-time, 357
    - discrete-time, 208
  - causal filter, 208
  - circular convolution, 213, 359
    - for functions, 358–359
    - for sequences, 211–216
  - continuous-time, 353, 355–359
  - convolution
    - for functions on real line, 356–358
    - for infinite sequences, 205–211
    - with circularly extended signal, 211–213, 358–359
  - difference equations, 206
  - discrete-time, 197, 205–211
  - equivalence of circular and linear convolutions, 213
  - filter, 208, 357
  - FIR filter, 208
  - IIR filter, 208
  - impulse response of
    - continuous-time, 355
    - discrete-time, 205
  - operator norm of, 603
  - stability
    - continuous-time, 357
    - discrete-time, 208–209
  - two-sided filter, 208
- Linear system
  - continuous-time, 352
  - discrete-time, 195
  - periodically shift-varying, 265, 340
- Linear-phase filter
  - antisymmetric, 230
  - continuous-time, 373
  - discrete-time, 230, 251
  - symmetric, 230
- Linearity property
  - DFT, 255
  - DTFT, 221
  - Fourier series, 385
  - Fourier transform, 365
  - inner product axiom, 23
  - $z$ -transform, 241
- Linearly independent set, 21
- Lipschitz
  - exponent, 378
  - regularity, 378, 408
    - pointwise, 378
    - uniform, 378
- Lloyd, Stuart P., 499, 599
- Local Fourier
  - basis, 644–656
    - approximation with, 646, 652
    - for functions, 645–647
    - for sequences, 651–653
  - time–frequency localization
    - for functions, 647
    - for sequences, 652
- Localization, 658
  - for functions, 619–627
    - in frequency, 622–623
    - in time, 620–621
  - for sequences, 627–637
    - in frequency, 630–633
    - in time, 629–630
  - for structured sets
    - of functions, 638–640
    - of sequences, 640–643
- Loss of significance error, 125
- Lossless
  - code, 577
    - entropy bound on optimal code length, 579
    - fixed-rate, 577
    - length, 577
    - optimal, 577
    - prefix, 577
    - uniquely decodable, 577
  - coding, 576
  - compression, 577–579
- Lossy
  - coding, 576
    - distortion, 580
    - rate, 580
  - encoder, 581
- Low, Francis E., 668
- Lower bound, 135
- Lowpass filter
  - continuous-time, 373
  - discrete-time, 228, 229

- weighted minimax design, 535
- LU decomposition, 131
- Luenberger, David G., 162
- MacLaurin series, 178
  - expansions, 178
- Magnitude
  - of complex number, 313
  - response
    - DFT, 259
    - DTFT, 227
    - Fourier series, 394
    - Fourier transform, 373
  - of allpass filter, 231
- Mallat, Stéphane, 403, 599
- Manhattan norm, 33
- Mantissa, 123
- Marcellin, Michael W., 668
- Marginal
  - CDF, 153
  - PDF, 153
- Marks, Robert J., 498
- Marziliano, Pina, 498
- Matrix, 141–151
  - adjoint of polynomial, 322
  - adjugate, 142
  - antidiagonal, 150
  - band, 150
  - block circulant, 338
  - characteristic polynomial of, 145
  - circulant, 150, 215, 332–333
  - cofactor, 141
  - column space of, 142, 144
  - condition number of, 127
  - covariance, 154
  - determinant of, 141
  - DFT, 254
  - diagonalization of, 146
  - eigenpair of, 145
  - eigenvalue of, 145, 178
  - eigenvector of, 145, 178
  - Frobenius norm of, 147
  - Gram, 90
  - Hermitian, 127, 148, 191
  - identity, 141
  - improper rotation, 148
  - inverse, 141
  - kernel of, 142–143, 144
  - Kronecker product of, 324
  - Laurent unimodular polynomial, 322
  - left inverse, 141
  - lossless, 324
  - minor, 141
  - multiplication, 169
    - cost of computing, 122, 167–168, 176
  - norm, 147
  - normal, 127, 149
  - null space of, 142–143, 144
  - of Laurent polynomials, 322
  - of polynomials, 321–324
  - of ratios of polynomials, 322
  - operations, 141
  - orthogonal, 148, 169
  - paraunitary, 323
  - polynomial, 321
  - positive definite, 149
  - power of, 179
  - pseudocirculant polynomial, 324
  - pseudoinverse, 144
  - range of, 142–143
  - rank of, 144
  - rectangular, 141
  - right inverse, 141
  - rotation, 148
  - rotoinversion, 148
  - row space of, 144
  - self-adjoint, 148
  - short, 141
  - singular, 141
    - value of, 146
    - vector of, 146
  - sparse, 131–134
  - spectral
    - decomposition of, 145
    - theorem, 145
  - square, 141
  - SVD of, 146
  - symmetric, 149, 191
  - tall, 141
  - Toeplitz, 150, 197, 341
  - trace of, 146
  - transpose, 141
  - unimodular polynomial, 321
  - unit antidiagonal, 150
  - unitary, 147
  - Vandermonde, 150–151, 254
  - Walsh–Hadamard, 334, 335
- Matrix representation
  - of accumulator, 200
  - of adjoint, 118
  - of advance operator, 199
  - of averaging, 201–202
  - of bases and frames
    - in  $\mathbb{R}^2$ , 16–17
    - in  $\mathbb{R}^3$ , 170
  - of basic systems, 203
  - of BIBO-stable system, 198
  - of causal system, 197
  - of circular convolution for sequences, 214–216
  - of convolution for infinite sequences, 209
  - of delay, 198
  - of DFT, 254
  - of linear system, 196

- of LSI system, 197
- of memoryless system, 196
- of modulator, 199
- of operator, 109–119
  - with biorthogonal pair of bases, 116
  - with orthonormal basis, 112
- of running average, 200
- of shift, 198
- Max, Joel, 599
- Maximum a posteriori probability, 156
  - estimation, 156
- Maximum likelihood, 158
  - estimation, 158
- Maximum operator
  - continuous-time, 354
  - discrete-time, 202
- McClellan, James H., 599
- Mean, 152
  - of continuous-time stochastic process, 395
  - of discrete-time stochastic process, 286
  - of white noise, 288, 397
- Mean-squared error, 63
  - of estimation, 157
- Measure, 317
- Memoryless system
  - continuous-time, 352
  - discrete-time, 196
- Mersereau, Russell M., 328
- Minimal set, 160
- Minimax approximation, 597
  - design
    - criterion, 532
    - filter, 532–534
    - near, 527–529, 608
- Minimum mean-squared error
  - estimation, 63–69, 156, 158
    - of discrete-time stochastic process, 300–303
- Minimum-phase solution to spectral factorization, 247
- Minimum relative perturbation, 127
- Minimum-variance unbiased estimation, 158
- Minkowski's inequality, 139, 164
- Minkowski, Hermann, 162
- Minor, 141
- Modified
  - frequency center and spread
    - for functions, 627, 671
  - time center and spread
    - for functions, 627, 671
- Modulation, 199, 620–622, 625, 627, 629, 631, 633, 638, 641
  - continuous-time, 354
    - using discrete-time operators, 505
  - DFT pair, 257
  - discrete-time, 199
  - DTFT pair, 223
    - Fourier series pair, 385
    - Fourier transform pair, 365
- Modulus of complex number, 313
- Moment, 152–153
  - DTFT pair, 223
  - Fourier transform pair, 368
  - $z$ -transform pair, 241
- Moura, José M. F., 328
- Moving-average
  - filter
    - continuous-time, 354
    - discrete-time, 184, 201, 229
    - process, 290, 293, 297
- Multichannel sampling, 467–470, 505
- Multidimensional signal processing, 328
- Multiplicative identity property
  - vector space axiom, 18
- Multirate processing, 264–285, 326, 328, 504
  - commutativity of upsampling and downsampling, 271, 340
  - cost of computing
    - downsampling preceded by filtering, 311
    - upsampling followed by filtering, 312
  - downsampling, 265–268, 339
    - and upsampling, 270–271, 340
    - by 2, 265, 265
    - by  $N$ , 268, 339
  - followed by upsampling, 270
  - of discrete-time stochastic process, 297
  - preceded by filtering, 272, 312
  - preceded by filtering of discrete-time stochastic process, 299
  - upsampling and filtering, 275, 272–278
- filtering of discrete-time stochastic process, 298
- identities, 340
- interchange of multirate operations and filtering, 275, 340
- operations, 265–285
- orthogonality of filter's impulse response to even shifts, 277
- polyphase representation, 278–285
  - of downsampling preceded by filtering with period 2, 283
  - of filtering with period 2, 280
  - of filtering with period  $N$ , 284
  - of sequences with period 2, 278
  - of sequences with period  $N$ , 284
  - of upsampling followed by filtering with period 2, 282
- rational sampling rate change of discrete-time stochastic process, 300
- system, 264–285
  - with stochastic input, 294–303
- upsampling
  - by 2, 265, 268
  - by  $N$ , 270

- followed by downsampling, 270
  - followed by filtering, 273
  - followed by filtering of discrete-time stochastic process, 299
  - of discrete-time stochastic process, 298
  - with different sampling rates, 339
- Multiset, 107
- Multivariate normal PDF, 154
- Music, 659
- Near-minimax approximation, 527–529, 597, 608
  - with Chebyshev polynomials, 527
- Nearest-neighbor coding, 612
- Negative (semi)definite linear operator, 50
- Neuhoff, David L., 599
- Newton, Isaac, 1
- Node, 509
- Noise
  - AWGN, 288, 397
  - uncorrelated additive, 302
  - white, 288, 397
  - whitening, 288
- Nonlinear
  - approximation, 6, 511–512, 560–565, 598, 611
    - comparison with linear, 562–563
    - of square wave, 564
    - of triangle wave, 564, 611
    - operator, 610
  - diagonal estimation, 571–576
  - operator, 202, 354
  - projection, 576
- Nonnegative/positive (semi)definite linear operator, 50
- Nonorthogonal
  - finite-dimensional vector
    - interpolation followed by sampling for, 427
    - interpolation for, 426
    - interpolation in  $\mathbb{R}^2$ , 417–420
    - recovery for, 428
    - sampling followed by interpolation for, 427–429
    - sampling for, 425–426
    - sampling in  $\mathbb{R}^2$ , 417–420
  - function
    - interpolation followed by sampling for, 472–474
    - interpolation for, 471–472
    - recovery for, 474
    - sampling followed by interpolation for, 474–477
    - sampling for, 471
  - sequence
    - interpolation followed by sampling for, 444–446
    - interpolation for, 444
    - recovery for, 446
    - sampling followed by interpolation for, 446–447
    - sampling for, 442–444
- Nonsingular matrix, 161
- Nonuniform sampling, 468
- Norm, 27, 27–30, 161
  - $\ell^1$ , 33
  - $\ell^p$ , 33
  - $\mathcal{L}^1$ , 35
  - $\mathcal{L}^2$ , 35
  - $\ell^2$ , 33
  - $\mathcal{L}^\infty$ , 35
  - $\ell^\infty$ , 34, 171, 172
  - $\mathcal{L}^p$ , 35
  - $p$ , 33
  - 1, 33
  - 2, 33
  - convergence in, 36
  - equivalence of, 33, 172
  - Euclidean, 28
  - Frobenius, 147
  - induced by inner product, 28–30, 171
  - Manhattan, 33
  - matrix, 147
  - on  $\mathbb{C}^N$ , 28, 170
  - on  $\mathbb{C}^{\mathbb{R}}$ , 28
  - on  $\mathbb{C}^{\mathbb{Z}}$ , 28
  - on  $\mathcal{L}^1(\mathbb{R})$ , 35
  - on  $\ell^1(\mathbb{Z})$ , 33
  - on  $\mathcal{L}^2(\mathbb{R})$ , 31, 35
  - on  $\mathcal{L}^2([a, b])$ , 31
  - on  $\ell^2(\mathbb{Z})$ , 31, 33
  - on  $\mathcal{L}^p(\mathbb{R})$ , 35
  - on  $\ell^p(\mathbb{Z})$ , 33
  - on  $C([0, 1])$ , 170
  - on  $\mathbb{R}^2$ , 10–11
  - on  $\mathbb{R}^N$ , 28
  - on random variables, 32
  - operator, 173
  - positive definiteness property, 27
  - positive scalability property, 27
  - preservation, 174
  - quasi, 172
  - taxicab, 33
  - triangle inequality property, 27
- Normal
  - distribution, 154
  - equations, 98, 98–100, 144
    - in  $\mathbb{R}^3$ , 99
  - matrix, 127, 149
  - PDF, 154
  - rank, 321
- Normalization property
  - of Dirac delta function, 318
  - of Kronecker delta sequence, 187

- Normed vector space, xix, 30, 33–35  
 of absolutely integrable functions, 35  
 of absolutely summable sequences, 33  
 of bounded  
   functions, 35  
   sequences, 34  
 of complex-valued  
   finite-dimensional vectors, 33  
 of functions with finite  $\mathcal{L}^p$  norm, 35  
 of real-valued finite-dimensional vectors, 33  
 of sequences with finite  $\ell^p$  norm, 33
- Null space  
 of matrix, 142–143, 144  
 of operator, 40  
   sampling, 500
- Nyquist  
 frequency, 457  
 rate, 457  
 sampling, 458
- Nyquist, Harry, 457, 498
- Oblique projection, 96–97, 175  
 in  $\mathbb{R}^2$ , 12  
 onto one-dimensional subspace, 62  
 operator, 55  
 with frame, 108  
 with pair of biorthogonal bases, 96–97
- Odd function, 174
- One-sided  
 frequency center and spread  
   for functions, 623  
   for sequences, 632
- Open set, 135
- Operator, 40–51, 136, 161  
 additivity property of, 40  
 adjoint, 43, 43–47, 173  
   of DTFT, 227  
   of Fourier transform, 371  
 advance, 198  
 averaging, 115, 201, 354  
 basis analysis, 75  
   in  $\mathbb{R}^2$ , 16  
 basis synthesis, 75  
   in  $\mathbb{R}^2$ , 17  
 bounded, 41  
 circular convolution  
   for functions, 359  
   for sequences, 213  
 consistent, 427, 428, 445, 472, 497  
 continuous-time in spline spaces, 554–560  
 convolution  
   for functions on real line, 356  
   for infinite sequences, 206  
 definite linear, 50  
 delay, 198  
 derivative, 117  
 DFT, 253  
 eigenpair of, 49  
 eigenvalue of, 49, 173, 179  
 eigenvector of, 49  
 expansion, 173  
 frame  
   analysis, 103  
   synthesis, 103  
 Hermitian, 43  
 ideally matched, 428, 446, 474, 497, 504  
 idempotent, 55  
 inconsistent, 500  
 interpolation  
   for finite-dimensional vectors, 421, 422, 425, 426  
   for functions, 414, 449, 450, 471  
   for periodic functions, 478, 479  
   for sequences, 430, 433, 442, 444  
 inverse, 41, 58, 174  
 invertible, 48  
 Laplacian, 336  
 matrix representation of, 40, 109–119  
   with biorthogonal pair of bases, 116  
   with orthonormal basis, 112  
 maximum, 202, 354  
 nonlinear, 202, 354  
   approximation, 610  
 nonnegative (semi)definite linear, 50  
 nonpositive (semi)definite linear, 50  
 norm, 41, 147, 173, 179  
   of LSI system, 603  
 null space of, 40  
   sampling, 500  
 oblique projection, 55  
 orthogonal projection, 55, 108  
   in  $\mathbb{R}^2$ , 11  
 orthonormal basis  
   analysis, 80  
   synthesis, 80  
 projection, 55, 54–60, 174  
   oblique, 55  
   orthogonal, 11, 55, 108  
 pseudoinverse, 59–60  
 range of, 40  
   interpolation, 500  
 restriction, 56, 188, 349  
 sampling followed by interpolation  
   for finite-dimensional vectors, 423, 427  
   for functions, 415, 451, 472, 474  
   for periodic functions, 480  
   for sequences, 415, 434, 435, 444, 446  
 scalability property of, 40  
 self-adjoint, 43, 55  
 shift, 198, 353  
 singular value of, 179  
 smoothing, 333  
 unbounded, 41



- unitary, 48, 47–48, 253
- Oppenheim, Alan V., 328
- Optimal
  - estimation, 63, 66–69
  - frame bounds, 101
  - quantization, 582–583
  - stability constants, 72
- Oracle
  - estimation, 575
  - projection, 575, 611
  - scaling, 573–575, 611
- Orthogonal
  - complement, 25
  - decomposition, 50–51, 60
    - with basis, 82
  - filter, 278
  - matrix, 148, 169
  - polynomials, 517, 606
    - root of, 606
    - weight function, 517
  - random vectors, 69, 67–69
  - set(s), 25
  - vectors, 25
- Orthogonal projection, 50–51, 59, 165–166
  - in  $\mathbb{R}^2$ , 11, 12
  - in coefficient space, 175
  - onto subspace, 81
    - one-dimensional, 56
  - operator, 55, 57, 108
    - in  $\mathbb{R}^2$ , 11
    - on  $\mathcal{L}^2(\mathbb{R})$ , 58
  - projection theorem, 51–54
  - via truncation, 608
  - with orthonormal basis, 80–82
- Orthogonality, 25, 25–27, 67–69, 301
  - condition, 76
  - in  $\mathbb{R}^2$ , 11, 13
  - of discrete-time stochastic process, 300
  - of filter's impulse response
    - to all shifts, 231, 337
    - to even shifts, 277
- Orthonormal
  - finite-dimensional vector
    - interpolation followed by sampling for, 423
    - interpolation for, 422–423
    - recovery for, 423
    - sampling followed by interpolation for, 423–424
    - sampling for, 421–422
  - function
    - interpolation followed by sampling for, 451
    - interpolation for, 414, 450–451, 500
    - recovery for, 452
    - sampling followed by interpolation for, 415, 451–452
    - sampling for, 414, 449–450, 500
  - periodic function
    - interpolation followed by sampling for, 480
    - interpolation for, 479–480
    - recovery for, 480
    - sampling followed by interpolation for, 480–481
    - sampling for, 478–479, 505
  - sequence
    - interpolation followed by sampling for, 415, 434
    - interpolation for, 433–434
    - recovery for, 435
    - sampling followed by interpolation for, 435–436, 504
    - sampling for, 430–433
  - set
    - in  $\mathbb{R}^2$ , 13
    - normal equations, 99
- Orthonormal basis, 76, 76–86, 160–161
  - allpass filter, 232
  - analysis operator, 80
  - change of, 109
    - by rotation in  $\mathbb{R}^2$ , 110
    - to standard in  $\ell^2(\mathbb{Z})$ , 111
  - DFT, 254
  - expansion, 77
  - finite-dimensional, 76
  - for  $\mathbb{R}^2$ , 13
  - for shift-invariant subspaces, 504
  - Fourier series, 3, 383
  - Haar, 4
  - least-squares approximation with, 175
  - matrix representation of operators with, 112
  - of cosine functions, 76
  - orthogonal projection with, 80–82
  - shift-invariant, 232
  - synthesis operator, 80
  - uniqueness of expansion, 76
- Outcome, 151
- Overflow, 123
- Overlap
  - add algorithm, 310
  - save algorithm, 311, 341
- Oversampling, 441
  - interpolation, 501–502
  - interpolation of, 501
- Pacifici, Giovanni
  - Fire Truck*, 664
- Pair of biorthogonal bases, 86
- Papoulis, Athanasios, 162, 403, 498
- Papoulis–Gerchberg algorithm, 491–495
- Parallelogram law, 29
  - proof of, 171



- Parhizkar, Reza, 667  
 Parks, Thomas W., 599  
 Parseval equality, 48, 77  
   for biorthogonal pair of bases, 89  
   for DFT, 258  
   for DTFT, 226, 231  
   for Fourier series, 384  
   for Fourier transform, 371, 407  
   for tight frames, 106  
   generalized, 77, 227, 407  
   in  $\mathbb{R}^2$ , 14  
 Parseval tight frame, 106  
 Partial sum, 137  
 Partition  
   cells, 581, 613  
   of unity, 550  
   optimization, 593  
 Passband  
   continuous-time, 373  
   discrete-time, 228  
 Pattern recognition, 338  
 Periodic function, 351  
   bandlimited, 481  
     sampling followed by interpolation for, 486–489  
     subspace of, 481, 483–486  
   bandwidth of, 481  
   Fourier series of sinc, 386  
   Fourier transform, 391  
   full-band, 481  
   interpolation for, 477–489  
   regularity of, 393, 409  
   sampling for, 477–489  
   sampling theorem for, 489  
   shift-invariant subspace of, 478, 505  
   sinc, 386  
 Periodic nonuniform sampling, 468  
 Periodic sequence, 183, 184, 192–193  
   complex exponential, 193  
   DFT of sinc, 256  
   Kronecker delta, 192  
   sinc, 256  
 Periodically shift-varying system  
   with deterministic input, 266  
   with stochastic input, 294  
 Phase  
   of complex number, 313  
   response  
     DFT, 259  
     DTFT, 227  
     Fourier series, 394  
     Fourier transform, 373  
 Picket-fence function, 390  
 Piecewise  
   constant  
     approximation, 499, 503  
     function, 413, 503  
   linear  
     function, 346  
     polynomials, 537  
 Pivot, 130  
 Pixel, 193  
 Pointwise  
   convergence, 137  
   Lipschitz function, 378  
   multiplication by Heaviside  
     function, 349  
     sequence, 188  
 Poisson sum formula, 392, 409  
 Polar form of complex number, 313  
 Polynomial(s), 318, 318–320  
   adjoint of vector or matrix, 322  
   approximation by, 513–537  
   B-spline  
     representation with, 549–551  
     reproduction with, 549  
 Bernstein, 607, 608  
 Bézout identity for, 319  
 Chebyshev, 527, 528, 601–603  
 constant sequence, 320  
 coprime, 319  
 cost of evaluation, 120  
 degree of, 318  
 discrete, 320  
 function, 318  
 inner product  
   computation with bases, 91  
   space of, 172  
 irreducible, 319  
 Lagrange, 518, 607  
 Laurent, 319  
   unimodular matrix, 322  
   vectors and matrices of, 322  
 Legendre, 86, 166, 515–516, 606  
 linear sequence, 320  
 lossless matrix, 324  
 matrix, 321  
   adjoint of, 322  
   lossless, 324  
   of Laurent, 322  
   paraunitary, 323  
   pseudocirculant, 324  
   rank, 321  
   unimodular, 321  
 minimax approximation by, 523–529  
 orthogonal, 517, 606  
   root of, 606  
   weight function, 517  
 paraunitary matrix, 323  
 piecewise, 537  
 pseudocirculant matrix, 324  
 quadratic, 319  
 quadratic sequence, 320  
 ratio of, 319

- vectors and matrices of, 322
- rational function, 319
- representing with B-splines, 549–551
- reproduction with B-splines, 549, 552
- root of, 318
- sequence, 320
- unimodular matrix, 321
- vector, 321
  - space of, 19
- Polyphase representation, 278–285
  - component
    - zero, 340
  - components, 278
  - decomposition, 278
  - deterministic autocorrelation
    - with period 2, 279
  - of downsampling preceded by filtering
    - with period 2, 283
  - of filtering
    - with period 2, 280
    - with period  $N$ , 284
  - of sequences
    - with period 2, 278
    - with period  $N$ , 284
  - of upsampling followed by filtering
    - with period 2, 282
  - polyphase matrix, 280
- Porat, Boaz, 328, 403, 499
- Positive
  - (semi)definite linear operator, 50
  - scalability property
    - norm axiom, 27
- Positive definite
  - linear operator, 50
  - matrix, 149
  - operator, 49–50
- Positive definiteness property
  - inner product axiom, 23
  - norm axiom, 27
- Posterior distribution, 156
- Postfilter, 433, 444, 450, 471, 479
- Powell, Alex M., 600
- Power, 225, 293, 370, 400
  - complementarity, 277
  - of approximation, 417
  - of matrix, 179
  - output, 294, 400
  - series, 178
  - spectral density, 225, 292–294, 370, 399, 400
    - and filtering, 294, 400
    - of white noise, 294, 400
- Prandoni, Paolo, 328
- Pre-Hilbert space, 27
- Precision, 123–126
  - of computing average, 125
- Prefilter, 430, 442, 449, 471, 478
- Prefix code, 577
- Primitive, 362
  - convolution with derivative, 407
- Prior distribution, 156
- Probability, 10, 151–159, 162
  - estimation, 155–159
  - law, 151
  - model, 151
  - standard distributions, 154–155
- Probability density function, 152, 153
  - conditional, 153
  - convolution of, 406
  - Gaussian, 154
    - degenerate, 154
    - jointly, 154
  - marginal, 153
  - multivariate normal, 154
  - normal, 154
  - uniform, 154
- Probability mass function, 340
  - convolution of, 340
- Projection, 50–54
  - filtering as, 330
  - in  $\mathbb{R}^2$ , 11–12
  - onto convex sets, 491–495
    - inpainting, 492–493
    - Papoulis–Gerchberg algorithm, 491–495
  - onto subspace, 58
  - operator, 55, 54–60, 174
  - oracle, 611
  - orthogonal, 608
  - to bandlimited subspace
    - of functions, 453–454
    - of periodic functions, 483–486
    - of sequences, 438
  - via domain restriction, 55, 173
  - via truncation, 608
- Projection theorem, 51, 51–54, 162
  - best approximation, 54
  - existence, 51
  - idempotency, 51
  - in  $\mathbb{R}^2$ , 11
  - linearity, 51
  - orthogonality, 51
  - self-adjointness, 51
  - uniqueness, 51
- Proper subspace, 21
- Pseudocirculant polynomial matrix, 324, 328
- Pseudoinverse
  - matrix, 144
  - operator, 59–60
- Püschel, Markus, 328
- Pythagoras, 29
- Pythagorean theorem, 29, 83
  - in  $\mathbb{R}^2$ , 14
- Quadrature mirror formula, 277

- Quantization  
 design, 593  
   codebook optimization, 593  
   partition optimization, 593  
 entropy-constrained, 582  
 estimation, 594–596  
 fixed-rate, 582  
 high-resolution analysis, 583–584  
 optimization, 612  
 scalar, 579–584  
   general, 581–582  
   optimal, 582–583  
   uniform, 580–581  
   variable-rate, 582  
 Quasinorm, 172
- Raabe, Herbert P., 457, 498  
 Rabiner, Lawrence R., 328  
 Rader's FFT, 306  
 Radix-2 FFT, 304  
 Rainbow  
   geometric explanation of, 1  
   spectral explanation of, 1  
 Raised cosine window, 189  
 Ramp sequence, 436  
 Random variable, 152–155  
   CDF, 152  
   conditional  
     expectation, 153  
     PDF, 153  
   continuous, 152, 406  
   correlation coefficient of, 153  
   covariance of, 153  
   discrete, 340  
   entropy of, 579  
   expectation of, 152–153  
   Gaussian, 154–155  
   independent, 153  
   joint  
     CDF, 153  
     PDF, 153  
   jointly distributed, 153–154  
   marginal  
     CDF, 153  
     PDF, 153  
   mean of, 152  
   moment of, 152–153  
   MSE, 63  
   PDF, 152  
   scalar, 67  
   space of, 32  
   standard, 154  
   sum of, 340, 406  
   uniform, 154  
   variance of, 152, 152–153  
 Random vector, 67  
   covariance matrix, 154  
   estimation of, 67–69  
   Gaussian, 154–155  
   inner product on, 179  
   linear approximation of, 566–573  
   orthogonal, 69, 67–69  
 Rangan, Sundeeep, 600  
 Range  
   of function, 136  
   of matrix, 142, 142–143  
   of operator, 40  
     interpolation, 500  
 Rank, 144  
   full, 161  
   normal, 321  
 Rate, 580  
   Nyquist, 457  
 Rational  
   autocorrelation of sequence, 245  
   function, 319  
   transfer function, 237  
 Ravel, Maurice  
   *Boléro*, 659  
 Real  
   Fourier series, 408  
   part of complex number, 313  
   plane, 10–17  
 Real analysis, 135–140  
   convergence, 136–137  
     of convolution sum, 316  
   definitions, 135–136  
   difference equations, 315–316  
   inequalities, 139–140  
   integration by parts, 140  
   interchange theorems, 138–139  
 Reconstruction, 161  
 Recovery  
   for functions, 415  
   for sequences, 415  
     finite-length, 672  
   with nonorthogonal  
     functions, 474  
     sequences, 446  
     vectors, 428  
   with orthonormal  
     functions, 452  
     periodic functions, 480  
     sequences, 435  
     vectors, 423  
 Rectangular  
   matrix, 141, 161  
   window, 188  
 Redundancy, 161  
 Region of convergence, 337  
   of  $z$ -transform, 235  
   of geometric sequence  
     left-sided, 237, 337  
     right-sided, 236, 337

- of Laplace transform, 379
  - of shift sequence, 236, 337
- Regularity
  - $C^q$ , 375
  - Lipschitz, 378, 408
  - of functions, 374–379
  - of periodic functions, 393, 409
- Remez, Evgeny Y., 599
- Representation, 161
- Reproduction
  - codebook, 581
  - decoder, 581
- Resolution, 618
- Restriction
  - operator, 56
  - property of Kronecker delta sequence, 187
- Riemann
  - integrable function, 24
  - series theorem, 137
- Riesz
  - basis, 72, 69–76, 162, 174
  - for  $\mathbb{R}^2$ , 72
  - condition for unconditional convergence of series, 25
  - failure of condition, 74, 174
  - invertibility of Gram matrix, 98
  - normal equations, 98
  - versus frame, 101
  - sequence, 101
- Right
  - inverse, 41, 141
  - singular vector, 146
- Root of unity of order  $N$ , xviii, 314
  - orthogonality of, 314
- Rotation matrix, 148
- Rotoinversion matrix, 148
- Row
  - space, 142, 144
  - vector, 141
- Sample space, 151
- Sampling, 658
  - bandpass, 460, 505
  - DTFT of finite-length sequence, 504
  - for continuous-time stochastic processes, 470
  - for finite-dimensional vectors, 420–429
    - in  $\mathbb{C}^4$ , 422, 425
    - in  $\mathbb{R}^2$ , 417–420
    - nonorthogonal, 425–426, 497
    - orthonormal, 421–422, 496
  - for functions, 447–477, 504
    - adjoint operator of, 505
    - and derivative, 469
    - bandlimited, 452–470
    - nonorthogonal, 471, 497
    - orthonormal, 414, 449–450, 496, 500
  - for periodic functions, 477–489
    - bandlimited, 481–489
    - nonorthogonal, 497
    - orthonormal, 478–479, 496
  - for sequences, 429–447
    - bandlimited, 437–442
    - in  $\ell^2(\mathbb{Z})$ , 432, 443
    - nonorthogonal, 442–444, 497
    - orthonormal, 430–433, 496
  - frequency, 457
  - function, 504
  - in movies, 505
  - multichannel, 467–470, 505
  - Nyquist, 458
  - operator
    - consistent with interpolation operator, 427, 445, 472, 497
    - ideally matched with interpolation operator, 428, 446, 474, 497, 504
    - inconsistent with interpolation operator, 500
    - null space of, 500
  - periodic nonuniform, 468
  - property of
    - Dirac delta function, 318
    - Kronecker delta sequence, 187
  - sinc squared, 458
  - triangle wave, 485
- Sampling followed by interpolation
  - for finite-dimensional vectors
    - in  $\mathbb{C}^4$ , 424, 429
    - nonorthogonal, 427–429, 497
    - operator, 423, 427
    - orthonormal, 423–424, 497
  - for functions
    - aliasing, 460–462
    - bandlimited, 455–456
    - best approximation by, 499
    - in  $\mathcal{L}^2(\mathbb{R})$ , 475
    - nonorthogonal, 474–477, 497
    - operator, 415, 451, 472, 474
    - orthonormal, 415, 451–452, 497
  - for periodic functions
    - bandlimited, 486–489
    - nonorthogonal, 497
    - operator, 480
    - orthonormal, 480–481, 497
  - for sequences
    - aliasing, 440–441
    - bandlimited, 440–442
    - in  $\ell^2(\mathbb{Z})$ , 436, 447
    - nonorthogonal, 446–447, 497
    - operator, 415, 434, 435, 444, 446
    - orthonormal, 435–436, 497, 504
    - oversampling, 441
  - operator
    - for finite-dimensional vectors, 423, 427

- for functions, 415, 451, 474
  - for periodic functions, 480
  - for sequences, 435, 446
- Sampling theorem
  - for functions, 457, 456–460
  - for periodic functions, 489
  - for sequences, 440
- Sathe, Vinay P., 328
- Sawtooth wave
  - Fourier series pair, 386, 409
  - Gibbs phenomenon, 409
- Scalability property
  - operator axiom, 40
- Scalar, 141
  - quantization, 581, 579–584
    - lossy encoder, 581
    - partition cells, 581
    - reproduction codebook, 581
    - reproduction decoder, 581
  - random variable, 67
- Scale, 617, 618
- Scaling
  - function
    - Battle–Lemarié, 610
  - in frequency, 620, 621, 623, 625, 638
    - Fourier transform pair, 367
  - in time, 620, 621, 623, 625, 638
    - DTFT pair, 223
    - Fourier transform pair, 367
    - $z$ -transform pair, 241
  - property of Dirac delta function, 318
- Schafer, Ronald W., 328
- Schauder basis, 160
- Schoenberg, Isaac J., 599
- Self-adjoint
  - matrix, 148
  - operator, 43, 51, 55
- Separable
  - Hilbert space, 35, 79–80
  - space, 40
- Sequence, 185–194
  - bandlimited, 437
    - aliasing of, 440–441
    - interpolation for, 437–442
    - oversampling of, 441
    - sampling followed by interpolation for, 440–442
    - sampling for, 437–442
    - subspace of, 437, 438
  - bandwidth of, 437
  - box, 188, 402
    - centered and normalized, 188
    - frequency center and spread, 631, 671
    - right-sided, 188
    - time center and spread, 629
    - uncertainty principle, 636
  - circular extension, 183, 211
    - complex exponential, 193, 329
    - constant, 220
    - cosine, 221
    - finite-length, 183, 192–193
      - sampling DTFT of, 504
      - uncertainty principle for, 636–637, 672
    - frequency center and spread, 630, 666, 672
    - full-band, 437
    - geometric, 186, 336
      - left-sided, 237, 337
      - right-sided, 236, 337
    - Heaviside, 187
    - infinite-length, 183, 185–192
    - interpolation for, 429–447
  - Kronecker delta, xviii, 13, 186
    - periodic, 192
  - localization for, 627–637
    - in frequency, 630–633
    - in time, 629–630
  - nonorthogonal
    - interpolation followed by sampling for, 444–446
    - interpolation for, 444
    - sampling followed by interpolation for, 446–447
    - sampling for, 442–444
  - of numbers convergence, 136–137
  - one-sided frequency center and spread, 632
  - orthogonal, 185
  - orthonormal
    - interpolation followed by sampling for, 415, 434
    - interpolation for, 433–434
    - sampling followed by interpolation for, 435–436, 504
    - sampling for, 430–433
  - periodic, 183, 192–193
  - periodization of, 183
  - ramp, 436
  - recovery for, 415
  - sampling for, 429–447
  - sampling theorem for, 440
  - shift-invariant subspace of, 430
  - sinc, 186, 219, 402
    - frequency center and spread, 631
    - time center and spread, 629
  - time center and spread, 629, 666, 672
  - two-dimensional, 193–194
  - uncertainty principle for, 633–636, 672
  - unit-norm, 185
  - unit-step, 187
  - window, 188
    - raised cosine, 189
    - rectangular, 188
  - with zero polyphase component, 340
- Series
  - absolute convergence of, 137

- conditional convergence of, 137
- convergence tests, 177
- doubly infinite, 137
- finite geometric, 178
- geometric, 178
- MacLaurin, 178
- partial sum of, 137
- power, 178
- Riemann series theorem, 137
- Taylor, 178
- truncation, 560–576
  - approximation by, 598
  - with biorthogonal pair of bases, 610
- Set, 135
  - closed, 135
  - closure of, 135
  - convex, 20
    - projection onto, 491–495
  - linearly independent, 21
  - minimal, 160
  - open, 135
  - orthogonal, 25
  - orthonormal
    - in  $\mathbb{R}^2$ , 13
  - span of, 21
- Shannon, Claude E., 457, 498, 599, 667, 673
  - A Mathematical Theory of Communication*, 498
- Shannon–Fano–Elias code, 578
- Shift
  - continuous-time, 353
  - discrete-time, 198
  - in frequency, 620–622, 625, 627, 629, 631, 633, 638, 641
    - DTFT pair, 223
    - Fourier series pair, 385
    - Fourier transform pair, 365
  - in time, 619, 621, 622, 625, 627, 629, 631, 633, 638, 641
    - DTFT pair, 221
    - Fourier series pair, 385
    - Fourier transform pair, 365
    - $z$ -transform pair, 241
- Shift-invariant
  - subspace
    - of functions, 449
    - of periodic functions, 478, 505
    - of sequences, 430
    - orthonormal basis for, 504
  - system
    - continuous-time, 353
    - continuous-time linear, 353
    - discrete-time, 197
    - discrete-time linear, 197
- Shifting property
  - of Dirac delta function, 318, 357
  - of Kronecker delta sequence, 187, 207
- Short matrix, 141
- Siebert, William M., 403
- Sifting property
  - of Dirac delta function, 318
  - of Kronecker delta sequence, 187
- Signal processing
  - algebraic theory, 328
  - continuous-time, 343–403
  - discrete-time, 181–328
  - fast algorithms for, 328
  - for communications, 328
  - multidimensional, 328
  - multirate, 328
  - statistical, 328
- Signal-to-noise ratio, 575
- Signaling
  - frequency division, 663
  - time division, 663
  - time–frequency division, 663
- Significant, 123
- Sinc
  - function, 186, 402
    - Fourier transform pair, 363, 366, 372
    - frequency center and spread, 622
    - relation to box function, 349
    - squared, 458, 459
    - time center and spread, 621
  - sequence, 186, 402
    - DTFT pair, 219, 222
    - frequency center and spread, 631
    - time center and spread, 629
- Singular
  - matrix, 141
  - value
    - decomposition, 146
    - of matrix, 146
    - of operator, 179
- Sinusoidal
  - function, 184, 461
  - sequence, 184, 336
- Sloane, Neil J. A., 600
- Smoothing operator, 333
- Solving systems of linear equations, 129–134, 144
- Someya, Isao, 457, 498
- Source coding, 576
- Space
  - $\mathcal{L}^p([0, 1])$ , 172
  - Banach, 38
  - Hilbert, 38
  - inner product, 27
  - normed vector, 30
  - of absolutely integrable functions, 35
  - of absolutely summable sequences, 33
  - of complex-valued
    - finite-dimensional vectors, 31

- of continuous functions with  $q$  continuous derivatives, 32
  - of finite-energy functions, 31
  - sequences, 31
  - of functions
    - of bounded variations, 348
    - with finite  $\mathcal{L}^p$  norm, 35
  - of random variables, 32
    - completeness of, 40
    - inner product on, 32
    - norm on, 32
  - of sequences with finite  $\ell^p$  norm, 33
  - of square-integrable functions, 31
  - of square-summable sequences, 31
  - pre-Hilbert, 27
  - vector, 18
- Span, 21
- failure of closure, 37
- Sparse matrix, 131–134
- Spectral decay
  - of Fourier series, 393, 409
  - of Fourier transform, 374–379, 407
- decomposition of matrix, 145
- density
  - cross, 400
  - energy, 225, 293, 370, 400
  - power, 225, 292–294, 370, 399, 400
- factorization, 247
  - minimum-phase solution, 247
- replica, 441
- theorem, 145
- Spectrogram, 659
- Spectrum
  - base, 441
  - for DFT, 253
  - for DTFT, 217
  - for Fourier series, 381
  - of triangle function, 506
  - for Fourier transform, 360
- Speech processing, 464–467
- Spline(s), 538–541
  - approximation by, 510–511, 537–560
- B-splines, 541–545, 597, 609
  - bases, 543–545
  - causal elementary, 542
  - elementary, 541–542, 609, 610
  - representing with, 549–551
  - uncertainty principle for, 668
- bases, 541–548
  - B-splines, 543–545
- best approximation by, 548
- canonical dual, 544, 609
- cardinal, 547
- computing in spline spaces
  - derivatives, 555–557
  - inner products, 610
  - integrals, 558–560
- discrete
  - uncertainty principle for, 672
- free-knot, 537
- knot, 537
- spaces, 538, 538–541, 608
  - best approximation in, 548
  - computing derivatives in, 555–557
  - computing inner products in, 610
  - computing integrals in, 558–560
  - orthonormal bases for, 546–547
- uniform, 537, 538
- Split-radix FFT, 307
- Square
  - matrix, 141, 161
  - wave
    - (non)linear approximation of, 564
    - Fourier series pair, 386, 388
- Stability constants, 72
- Stable system
  - continuous-time, 353
  - discrete-time, 197
- Standard
  - basis, xix
  - for  $\mathbb{R}^2$ , 13
  - deviation, 32
  - of continuous-time stochastic process, 395
  - of discrete-time stochastic process, 286
  - of white noise, 288
  - random variable, 154
- Stark, Philip B., 667
- Stationarity
  - of continuous-time stochastic process, 396
  - of discrete-time stochastic process, 287
- Statistical signal processing, 328
- Stirzaker, David R., 162
- Stochastic
  - autocorrelation, 341
  - continuous-time, 395, 399
  - discrete-time, 286, 292
  - continuous-time
    - bandlimited process, 470, 505
    - process, 395–397
    - system, 397–399
  - crosscorrelation, 341
  - continuous-time, 395
  - discrete-time, 286
  - discrete-time
    - process, 285–303, 566–571, 573
    - system, 288–292
- Stopband
  - continuous-time, 373
  - discrete-time, 228
- Strang, W. Gilbert, 162, 599
- Strang–Fix condition, 548–554, 603–604



- Strassen's algorithm, 122, 167–168  
 Subband coefficient, 70  
 Subnormal numbers, 124  
 Subspace, 20  
   affine, 20, 21  
   closed, 37, 172  
   decomposition, 60–63  
   in  $\mathbb{R}^2$ , 11–12  
   of bandlimited  
     functions, 452–454  
     periodic functions, 481, 483–486  
     sequences, 437, 438  
   proper, 21  
 Successive approximation, 82, 100–101  
   with biorthogonal pair of bases, 175  
 Supremum, 135  
 Surjective function, 136  
 Symmetric matrix, 149, 191  
 System  
   BIBO-stable, 198, 250, 353  
   causal, 197, 353  
   linear, 195, 352  
   LPSV, 265, 340  
   LSI, 197, 353  
   memoryless, 196, 352  
   of linear equations, 129–134, 143–144, 179  
     iterative solution, 131  
     Kaczmarz's algorithm, 132  
   periodically shift-varying, 266  
   shift-invariant, 197, 353  
   stable, 197, 353  
 Tall matrix, 141  
 Taubman, David S., 668  
 Taxicab norm, 33  
 Taylor series, 178  
   error of, 521, 597  
   expansion, 509, 520–522, 597, 607  
 Textbook on  
   bases, 162  
   compression, 599  
   discrete-time signal processing, 328, 403, 599  
   fast algorithms, 328  
   filter design, 599  
   finite elements, 599  
   Fourier representations, 48, 403  
   frames, 162  
   harmonic analysis, 403  
   information theory, 599  
   interpolation and approximation, 599  
   linear algebra, 10, 141, 162  
   multidimensional signal processing, 328  
   multirate processing, 328  
   numerical analysis, 599  
   probability, 10, 162  
   pseudocirculant polynomial matrix, 328  
   real analysis, 135  
   sampling, 498  
   signal processing, 403  
   signal processing for communications, 328  
   statistical signal processing, 328, 403  
   vector spaces, 10, 162  
   wavelet representations, 48, 403, 599  
   z-transform, 328  
 Thao, Nguyen T., 600  
 Theodoric of Freiberg, 1  
   *De iride*, 1  
 Thomas, Joy A., 599  
 Tight frame, 104, 104–107  
   as projection, 338  
   for  $\mathbb{R}^2$ , 16, 104  
   for affine functions, 176  
   normalization, 104  
   Parseval, 106  
   Parseval equality for, 105  
   redundancy of, 106  
   with nonequal-norm vectors, 176  
 Time  
   center and spread  
     for box function, 621  
     for box sequence, 629  
     for functions, 621, 666, 669  
     for Gaussian function, 621  
     for sequences, 629, 666, 672  
     for sinc function, 621  
     for sinc sequence, 629  
   division, 663  
   localization, 616  
     for functions, 620–621  
     for sequences, 629–630  
   reversal, 277  
     DTFT pair, 223  
     Fourier series pair, 385  
     Fourier transform pair, 367  
     z-transform pair, 241  
   series, 286  
 Time–frequency  
   actual plot, 638  
   analysis of music, 659  
   division, 663  
   idealized tiling, 638  
   localization  
     local Fourier, 647, 652  
     wavelet, 649, 656  
   plane, 624, 637–643  
   tile, 624  
   tiling, 637–643  
     dyadic, 640, 643  
 Toeplitz  
   matrix, 150, 341  
     representation of LSI system, 197  
     vector product, 341  
   system, iterative solution of, 132



- Tonelli's theorem, 138  
 Topology, 135  
 Trace, 146  
 Transfer function  
   rational, BIBO stability, 250  
   z-transform, 249  
 Transform  
   block, 652  
   coding, 585, 584–591, 598  
     bit allocation, 587–588, 612  
     distortion, 585  
     transform optimization, 588–591  
     visualizing effect of transform, 588  
   coefficient, 70, 161, 585  
   decoding, 585  
   domain, 161  
   encoding, 585  
   Walsh–Hadamard, 334, 335  
 Transition band, 442  
 Triangle  
   function, 94  
     dual to basis of periodic, 94  
     Fourier transform pair, 362, 366  
     in frequency, 458, 459, 506  
     spectral decay, 377  
   inequality, 27  
     norm axiom, 27  
     proof of, 171  
   spectrum  
     Fourier series, 506  
   wave  
     (non)linear approximation of, 564, 611  
     Fourier series pair, 386, 389, 409  
     sampling, 485  
 Truncation, 608  
   series, 610  
     approximation by, 560–576, 598  
 Trushkin, Alexander V., 599  
 Tsitsiklis, John N., 162  
 Tukey, John W., 328  
 Two-dimensional sequence, 193–194  
 Two-sided filter, 208  
 Unbiased estimation, 157  
 Unbounded operator, 41  
 Uncertainty principle  
   for B-splines, 668  
   for box sequence, 636  
   for discrete splines, 672  
   for finite-length sequences, 636–637, 666, 672  
   for functions, 624–627, 666  
     complex, 671  
     Heisenberg box, 625  
   for sequences, 633–636, 666, 672  
     Heisenberg box, 634  
 Unconditional basis, 70, 160, 162  
 Undersampling, 459  
 Uniform  
   convergence, 137  
   distribution, 154  
   quantization, 580–581  
     step size, 580  
   random variable, 154  
   splines, 537, 538, 546–547  
 Uniformly Lipschitz function, 378  
 Unimodular polynomial matrix, 321  
   Laurent, 322  
 Unique expansion, 105  
 Unit  
   antidiagonal matrix, 150  
   vector, 10  
 Unit-step  
   function, 348  
   sequence, 187  
 Unit-width box function, 376  
 Unitary  
   analysis, 79  
   matrix, 147, 161, 162  
   operator, 48, 47–48  
   synthesis, 79  
 Unser, Michael, 498, 599  
 Upper bound, 135  
 Upsampling, 268–270, 628, 630, 631, 634, 641  
   and downsampling, 270–271, 340  
   and filtering, 275, 272–278  
   by 2, 265, 268  
   by  $N$ , 270  
   commutativity with downsampling, 271, 340  
   DTFT pair, 223, 269, 270  
   followed by downsampling, 270, 340  
   followed by filtering, 273  
   cost of computing, 312  
   of discrete-time stochastic process, 299  
   matrix representation of, 268  
   of discrete-time stochastic process, 298  
   operator, 268  
   preceded by downsampling, 270  
   z-transform pair, 241, 269, 270  
 Vaidyanathan, P. P., 162, 328  
 Vandermonde matrix, 150–151  
   DFT, 254  
 Variable-rate quantization, 582  
 Variance, 152, 152–153  
   of continuous-time stochastic process, 395  
   of discrete-time stochastic process, 286  
   of white noise, 288  
 Vector  
   adjoint of polynomial, 322  
   finite-dimensional  
     bandwidth of, 503

- full-band, 503
  - interpolation for, 420–429
  - sampling for, 420–429
- of Laurent polynomials, 322
- of polynomials, 321–324
- of ratios of polynomials, 322
- Vector space, xviii, xix, 10, 18, 18–35, 162
  - additive identity property, 18
  - associativity property, 18
  - commutativity property, 18
  - dimension of, 22
  - distributivity property, 18
  - multiplicative identity property, 18
  - of bounded
    - functions, 35
    - sequences, 34
  - of complex-valued
    - finite-dimensional vectors, 31
    - functions over  $\mathbb{R}$ , 19
    - sequences over  $\mathbb{Z}$ , 19
  - of continuous functions with  $q$  continuous derivatives, 32
  - of functions with finite  $\mathcal{L}^p$  norm, 35
  - of polynomials, 19
  - of sequences with finite  $\ell^p$  norm, 33
  - of square-integrable functions, 31
  - of square-summable sequences, 31
  - properties of, 18
- Vetterli, Martin, 185, 232, 328, 498, 600, 667, 668
- Vilbé, Pierre, 667
- von Neumann, John, 162
- Walsh–Hadamard
  - matrix, 334, 335
  - transform, 334, 335
- Wave
  - sawtooth
    - Fourier series pair, 386, 409
    - Gibbs phenomenon, 409
  - square
    - (non)linear approximation of, 564
    - Fourier series pair, 386, 388
  - triangle
    - (non)linear approximation of, 564, 611
    - Fourier series pair, 386, 389, 409
    - sampling, 485
- Wavelet
  - basis, 644–656
    - approximation with, 649, 655
    - for functions, 648–650
    - for sequences, 653–656
    - Haar, 3, 648, 654
  - Battle–Lemarié, 610
  - representations, 3, 48
  - time–frequency localization
    - for functions, 649
    - for sequences, 656
- Weierstrass approximation theorem, 524
- Weierstrass, Karl, 524
- Weight function, 517
- Weighted
  - Dirac comb, 415
  - Fourier transform, 456
  - error, 533
  - minimax approximation
    - filter design as, 533–537
- Weyl, Hermann, 162
- White noise
  - autocorrelation of, 288, 397
  - continuous-time stochastic process, 397
  - discrete-time stochastic process, 288
  - mean of, 288, 397
  - power spectral density, 294, 400
  - standard deviation of, 288
  - variance of, 288
- Whitening, 288
- Whittaker, Edmund T., 457, 498
- Whittaker, John M., 457, 498
- Wide-sense stationary process
  - autocorrelation of, 287, 396
  - continuous-time, 396
  - cyclostationary, 294
  - discrete-time, 287
  - mean of, 287, 396
  - power, 225, 292, 370, 399
  - power spectral density, 225, 292, 370, 399
  - white noise, 288, 397
- Wiener filtering, 300, 334
- Window sequence, 188
  - raised cosine, 189
  - rectangular, 188
- Winograd FFT, 307
- Young, Nicholas, 162
- $z$ -transform, xxi, 235, 233–252, 328
  - convergence of, 235–240
  - convolution in time, 242
    - failure of, 244
  - definition, 234–235
  - deterministic
    - autocorrelation, 245
    - autocorrelation of vector sequence, 246
    - crosscorrelation, 246
  - differentiation, 241
  - downsampling
    - by 2, 267
    - by  $N$ , 268
    - preceded by filtering, 272
  - downsampling followed by upsampling
    - by 2, 270
    - by  $N$ , 271
  - existence of, 235–240

- inversion, 238–251
  - by inspection, 238
  - using partial fraction expansion, 238–240
  - using power-series expansion, 240
- linearity, 241
- moments, 241
- of difference equation, 249
- of filter, 249–252
- pole of rational, 237
- properties of, 243, 240–249
- rational, 237
  - autocorrelation, 245
  - transfer function, 249, 250
- relation to DTFT, 235
- ROC of, 235, 236, 337
- scaling
  - in  $z$ , 241
  - in time, 241
- shift in time, 241
- spectral factorization, 247
- time reversal, 241
- upsampling
  - by 2, 269, 270
  - followed by filtering, 274
- zero of rational, 237
- Zero-phase filter
  - continuous-time, 373
  - discrete-time, 227