Foundations of Signal Processing

This comprehensive and engaging textbook introduces the basic principles and techniques of signal processing, from the fundamental ideas of signals and systems theory to real-world applications.

- Introduces students to the powerful foundations of modern signal processing, including the basic geometry of Hilbert space, the mathematics of Fourier transforms, and essentials of sampling, interpolation, approximation, and compression.
- Discusses issues in real-world use of these tools such as effects of truncation and quantization, limitations on localization, and computational costs.
- Includes over 160 homework problems and over 220 worked examples, specifically designed to test and expand students' understanding of the fundamentals of signal processing.
- Accompanied by extensive online materials designed to aid learning, including Mathematica® resources and interactive demonstrations.

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"*Foundations of Signal Processing* by Martin Vetterli, Jelena Kovacevic, and Vivek K Goyal lives up to its title by providing a thorough tour of the subject matter based on selected tools from real analysis which allow sufficient generality to develop the foundations of the classical Fourier methods along with modern wavelet approaches. A key distinction of the book is the use of Hilbert space ideas to provide a geometric interpretation and intuition that enhances both the classic and modern approaches by providing a unified view of their similarities and relative merits in the most important special cases. Many of the specific examples of signal processing considered can be viewed as examples of projections onto subspaces, which yield immediate properties and descriptions from the underlying fundamentals. The development is both pedagogically and theoretically sound, proceeding from the underlying mathematics through discrete-time systems to the more complicated continuous time systems into a wonderfully general and enlightening treatment of sampling and interpolation operations connecting discrete and continuous time. All of the important signal classes are considered and their basic properties and interrelations developed and summarized. The book then develops important topics not ordinarily found in signal processing texts – the accuracy of approximations involving the truncation of series expansions and the quantization of series coefficients, and the localization of signals in the time-frequency plane.

The completeness of the book results in a lengthy volume of roughly 800 pages, but it is easy to navigate to extract portions of interest while saving the many byways and special topics for future reference. The chapter introductions are particularly good at setting the stage in a simple but informative context and then sketching the details to come for the remainder of the chapter. Each chapter closes with a 'Chapter at a glance' section highlighting the primary ideas and results. The book will be a welcome addition to the library of students, practitioners, and researchers in signal processing for learning, reviewing, and referencing the broad array of tools and properties now available to analyze, synthesize, and understand signal processing systems."

Robert M. Gray, Stanford University and Boston University

"Finally a wonderful and accessible book for teaching modern signal processing to undergraduate students."

Stéphane Mallat, École Normale Supérieure

"This is a major book about a serious subject – the combination of engineering and mathematics that goes into modern signal processing: discrete time, continuous time, sampling, filtering, and compression. The theory is beautiful and the applications are so important and widespread."

Gil Strang, Massachusetts Institute of Technology

"This book (FSP) and its companion (FWSP) bring a refreshing new, and comprehensive approach to teaching the fundamentals of signal processing, from analysis and decompositions, to multiscale representations, approximations, and many other aspects that have a tremendous impact in modern information technology. Whereas classical texts were usually written for students in electrical or communication engineering programs, FSP and FWSP start from basic concepts in algebra and geometry, with the benefit of being easily accessible to a much broader set of readers, and also help those readers develop strong abstract reasoning and intuition about signals and processing operators. A must-read!"

Rico Malvar, Microsoft Research

"This is a wonderful book that connects together all the elements of modern signal processing. From functional analysis and probability theory, to linear algebra and computational methods, it's all here and seamlessly integrated, along with a summary of history and developments in the field. A real tour-de-force, and a must-have on every signal processor's shelf!"

Robert D. Nowak, University of Wisconsin–Madison

"Most introductory signal processing textbooks focus on classical transforms, and study how these can be used. Instead, Foundations of Signal Processing encourages readers to think of signals first. It develops a 'signal-centric' view, one that focuses on signals, their representation and approximation, through the introduction of signal spaces. Unlike most entry-level signal processing texts, this general view, which can be applied to many different signal classes, is introduced right at the beginning. From this, starting from basic concepts, and placing an emphasis on intuition, this book develops mathematical tools that give the readers a fresh perspective on classical results, while providing them with the tools to understand many state-of-the-art signal representation techniques."

Antonio Ortega, University of Southern California

"Foundations of Signal Processing by Vetterli, Kovačević, and Goyal, is a pleasure to read. Drawing on the authors' rich experience of research and teaching of signal processing and signal representations, it provides an intellectually cohesive and modern view of the subject from the geometric point of view of vector spaces. Emphasizing Hilbert spaces, where fine technicalities can be relegated to backstage, this textbook strikes an excellent balance between intuition and mathematical rigor, that will appeal to both undergraduate and graduate engineering students. The last two chapters, on sampling and interpolation, and on localization and uncertainty, take full advantage of the machinery developed in the previous chapters to present these two very important topics of modern signal processing, that previously were only found in specialized monographs. The explanations of advanced topics are exceptionally lucid, exposing the reader to the ideas and thought processes behind the results and their derivation. Students will learn not only a substantial body of knowledge and techniques, but also why things work, at a deep level, which will equip them for independent further reading and research. I look forward to using this text in my own teaching."

Yoram Bresler, University of Illinois at Urbana-Champaign

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To Marie-Laure, for her ∞ patience and many other qualities,
Thomas and Noémie, whom I might still convince of the beauty of this material,
and my parents, who gave me all the opportunities one can wish for.

— *MV*

To Danica and Giovanni, who make life beautiful.
To my parents, who made me who I am.

— *JK*

To Allie, Sundeep, and my family, who encourage me unceasingly, and
to the educators who made me want to be one of them.

— *VKG*

The cover illustration captures an experiment first described by Isaac Newton in *Opticks* in 1730, showing that white light can be split into its color components and then synthesized back into white light. It is a physical implementation of a decomposition of white light into its Fourier components – the colors of the rainbow – followed by a synthesis to recover the original.

Contents

Quick reference

Abbreviations

AR	Autoregressive
ARMA	Autoregressive moving average
AWGN	Additive white Gaussian noise
BIBO	Bounded input, bounded output
CDF	Cumulative distribution function
DCT	Discrete cosine transform
DFT	Discrete Fourier transform
DTFT	Discrete-time Fourier transform
DWT	Discrete wavelet transform
FFT	Fast Fourier transform
FIR	Finite impulse response
i.i.d.	Independent and identically distributed
IIR	Infinite impulse response
KLT	Karhunen–Loève transform
LMMSE	Linear minimum mean-squared error
LPSV	Linear periodically shift-varying
LSI	Linear shift-invariant
MA	Moving average
MAP	Maximum a posteriori probability
ML	Maximum likelihood
MMSE	Minimum mean-squared error
MSE	Mean-squared error
PDF	Probability density function
PMF	Probability mass function
POCS	Projection onto convex sets
rad	Radians
ROC	Region of convergence
SNR	Signal-to-noise ratio
SVD	Singular value decomposition
WSCS	Wide-sense cyclostationary
WSS	Wide-sense stationary

Abbreviations used in tables and captions but not in the text

FT	Fourier transform
FS	Fourier series
LT	Laplace transform

Sets

natural numbers	$\mathbb{N}$	$0, 1, \ldots$
integers	$\mathbb{Z}$	$\ldots, -1, 0, 1, \ldots$
positive integers	$\mathbb{Z}^+$	$1, 2, \ldots$
rational numbers	$\mathbb{Q}$	$p/q, \quad p, q \in \mathbb{Z}, \quad q \neq 0$
real numbers	$\mathbb{R}$	$(-\infty, \infty)$
positive real numbers	$\mathbb{R}^+$	$(0, \infty)$
complex numbers	$\mathbb{C}$	$a + jb$ or $re^{j\theta}$ with $a, b, r, \theta \in \mathbb{R}$
a generic index set	$\mathcal{I}$	
a generic vector space	V	
a generic Hilbert space	H	
closure of set S	$\overline{S}$	

Real and complex analysis

sequence	x_n	argument n is an integer, $n \in \mathbb{Z}$
function	$x(t)$	argument t is continuous-valued, $t \in \mathbb{R}$
ordered sequence	$(x_n)_n$	
set containing x_n	$\{x_n\}_n$	
vector x with x_n as elements	$[x_n]$	
Kronecker delta sequence	δ_n	$\delta_n = 1$ for $n = 0$; $\delta_n = 0$ otherwise
Dirac delta function	$\delta(t)$	$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$ for x continuous at 0
indicator function of interval I	1_I	$1_I(t) = 1$ for $t \in I$; $1_I(t) = 0$ otherwise
integration by parts		$\int u\,dv = uv - \int v\,du$
complex number	z	$a + jb$, $re^{j\theta}$, $a, b \in \mathbb{R}$, $r \in [0, \infty)$, $\theta \in [0, 2\pi)$
conjugation	z^*	$a - jb$, $re^{-j\theta}$
real part of	$\Re(\,\cdot\,)$	$\Re(a + jb) = a, \;\; a, b \in \mathbb{R}$
imaginary part of	$\Im(\,\cdot\,)$	$\Im(a + jb) = b, \;\; a, b \in \mathbb{R}$
conjugation of coefficients	$X_*(z)$	$X^*(z^*)$
principal root of unity	W_N	$e^{-j2\pi/N}$

Asymptotic notation

big O	$x \in O(y)$	$0 \leq x_n \leq \gamma y_n$ for all $n \geq n_0$; some n_0 and $\gamma > 0$
little o	$x \in o(y)$	$0 \leq x_n \leq \gamma y_n$ for all $n \geq n_0$; some n_0, any $\gamma > 0$
Omega	$x \in \Omega(y)$	$x_n \geq \gamma y_n$ for all $n \geq n_0$; some n_0 and $\gamma > 0$
Theta	$x \in \Theta(y)$	$x \in O(y)$ and $x \in \Omega(y)$
asymptotic equivalence	$x \asymp y$	$\lim_{n\to\infty} x_n/y_n = 1$

Standard vector spaces

Hilbert space of square-summable sequences	$\ell^2(\mathbb{Z})$	$\left\{ x : \mathbb{Z} \to \mathbb{C} \,\middle|\, \sum_n \lvert x_n \rvert^2 < \infty \right\}$ with inner product $\langle x, y \rangle = \sum_n x_n y_n^*$
Hilbert space of square-integrable functions	$\mathcal{L}^2(\mathbb{R})$	$\left\{ x : \mathbb{R} \to \mathbb{C} \,\middle|\, \int \lvert x(t) \rvert^2 \, dt < \infty \right\}$ with inner product $\langle x, y \rangle = \int x(t)\, y(t)^* \, dt$
normed vector space of sequences with finite ℓ^p norm, $1 \leq p < \infty$	$\ell^p(\mathbb{Z})$	$\left\{ x : \mathbb{Z} \to \mathbb{C} \,\middle|\, \sum_n \lvert x_n \rvert^p < \infty \right\}$ with norm $\lVert x \rVert_p = \left(\sum_n \lvert x_n \rvert^p \right)^{1/p}$
normed vector space of functions with finite $\mathcal{L}^p$ norm, $1 \leq p < \infty$	$\mathcal{L}^p(\mathbb{R})$	$\left\{ x : \mathbb{R} \to \mathbb{C} \,\middle|\, \int \lvert x(t) \rvert^p \, dt < \infty \right\}$ with norm $\lVert x \rVert_p = \left(\int \lvert x(t) \rvert^p \, dt \right)^{1/p}$
normed vector space of bounded sequences with supremum norm	$\ell^\infty(\mathbb{Z})$	$\left\{ x : \mathbb{Z} \to \mathbb{C} \,\middle|\, \sup_n \lvert x_n \rvert < \infty \right\}$ with norm $\lVert x \rVert_\infty = \sup_n \lvert x_n \rvert$
normed vector space of bounded functions with supremum norm	$\mathcal{L}^\infty(\mathbb{R})$	$\left\{ x : \mathbb{R} \to \mathbb{C} \,\middle|\, \operatorname{ess\,sup}_t \lvert x(t) \rvert < \infty \right\}$ with norm $\lVert x \rVert_\infty = \operatorname{ess\,sup}_t \lvert x(t) \rvert$

Bases and frames for sequences

standard basis	$\{e_k\}$	$e_{k,n} = 1$ for $n = k$; $e_{k,n} = 0$ otherwise
vector, element of basis or frame	φ	when applicable, a *column* vector
basis or frame	Φ	set of vectors $\{\varphi_k\}$
operator	Φ	concatenation of $\{\varphi_k\}$ in a linear operator: $[\varphi_0 \;\; \varphi_1 \;\; \ldots \;\; \varphi_{N-1}]$
vector, element of dual basis or frame	$\widetilde{\varphi}$	when applicable, a *column* vector
	$\widetilde{\Phi}$	set of vectors $\{\widetilde{\varphi}_k\}$
operator	$\widetilde{\Phi}$	concatenation of $\{\widetilde{\varphi}_k\}$ in a linear operator: $[\widetilde{\varphi}_0 \;\; \widetilde{\varphi}_1 \;\; \ldots \;\; \widetilde{\varphi}_{N-1}]$
expansion with a basis or frame	$x = \Phi\widetilde{\Phi}^* x$	

Discrete-time signal processing

Sequence	x_n	signal, vector
Convolution		
linear	$h * x$	$\sum_{k\in\mathbb{Z}} x_k h_{n-k}$
circular (N-periodic sequences)	$h \circledast x$	$\sum_{k=0}^{N-1} x_k h_{(n-k) \bmod N}$
	$(h * x)_n$	convolution result at n
Eigensequence	v_n	eigenvector
infinite time	$v_n = e^{j\omega n}$	$h * v = H(e^{j\omega})\, v$
finite time	$v_n = e^{j2\pi kn/N}$	$h \circledast v = H_k v$
Frequency response		eigenvalue corresponding to v_n
infinite time	$H(e^{j\omega})$	$\sum_{n\in\mathbb{Z}} h_n e^{-j\omega n}$
finite time	H_k	$\sum_{n=0}^{N-1} h_n e^{-j2\pi kn/N} = \sum_{n=0}^{N-1} h_n W_N^{kn}$

Continuous-time signal processing

Function	$x(t)$	signal
Convolution		
linear	$h * x$	$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$
circular (T-periodic functions)	$h \circledast x$	$\int_{-T/2}^{T/2} x(\tau)\, h(t-\tau)\, d\tau$
	$(h * x)(t)$	convolution result at t
Eigenfunction	$v(t)$	eigenvector
infinite time	$v(t) = e^{j\omega t}$	$h * v = H(\omega)\, v$
finite time	$v(t) = e^{j2\pi kt/T}$	$h \circledast v = H_k v$
Frequency response		eigenvalue corresponding to $v(t)$
infinite time	$H(\omega)$	$\int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$
finite time	H_k	$\int_{-T/2}^{T/2} h(\tau)\, e^{-j2\pi k\tau/T}\, d\tau$

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Spectral analysis

Fourier transform	$x(t) \stackrel{\text{FT}}{\longleftrightarrow} X(\omega)$	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$
inverse		$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$
Fourier series coefficients	$x(t) \stackrel{\text{FS}}{\longleftrightarrow} X_k$	$X_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-j(2\pi/T)kt}\, dt$
reconstruction		$x(t) = \sum_{k \in \mathbb{Z}} X_k e^{j(2\pi/T)kt}$
discrete-time Fourier transform	$x_n \stackrel{\text{DTFT}}{\longleftrightarrow} X(e^{j\omega})$	$X(e^{j\omega}) = \sum_{n \in \mathbb{Z}} x_n e^{-j\omega n}$
inverse		$x_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$
discrete Fourier transform	$x_n \stackrel{\text{DFT}}{\longleftrightarrow} X_k$	$X_k = \sum_{n=0}^{N-1} x_n W_N^{kn}$
inverse		$x_n = \frac{1}{N} \sum_{n=0}^{N-1} X_k W_N^{-kn}$
z-transform	$x_n \stackrel{\text{ZT}}{\longleftrightarrow} X(z)$	$X(z) = \sum_{n \in \mathbb{Z}} x_n z^{-n}$

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[1] http://www.fourierandwavelets.org

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Preface

Our main goals in this book and its companion volume, *Fourier and Wavelet Signal Processing (FWSP)* [57], are to enable an understanding of state-of-the-art signal processing methods and techniques, as well as to provide a solid foundation for those hoping to advance the theory and practice of signal processing. We believe that the best way to grasp and internalize the fundamental concepts in signal processing is through the geometry of Hilbert spaces, as this leverages the great innate human capacity for spatial reasoning. While using geometry should ultimately *simplify* the subject, the connection between signals and geometry is not innate. The reader will have to invest effort to see signals as vectors in Hilbert spaces before reaping the benefits of this view; we believe that effort to be well placed.

Many of the results and techniques presented in the two volumes, while rooted in classic Fourier techniques for signal representation, first appeared during a flurry of activity in the 1980s and 1990s. New constructions of local Fourier transforms and orthonormal wavelet bases during that period were motivated both by theoretical interest and by applications, multimedia communications in particular. New bases with specified time–frequency behavior were found, with impact well beyond the original fields of application. Areas as diverse as computer graphics and numerical analysis embraced some of the new constructions – no surprise given the pervasive role of Fourier analysis in science and engineering.

Many of these new tools for signal processing were developed in the applied harmonic analysis community. The resulting high level of mathematical sophistication was a barrier to entry for many signal processing practitioners. Now that the dust has settled, some of what was new and esoteric has become fundamental; we want to bring these new fundamentals to a broader audience. The Hilbert space formalism gives us a way to begin with the classical Fourier analysis of signals and systems and reach structured representations with time–frequency locality and their varied applications. Whenever possible, we use explanations rooted in elementary analysis over those that would require more advanced background (such as measure theory). We hope to have balanced the competing virtues of accessibility to the student, rigor, and adequate analytical power to reach important conclusions.

The book can be used as a self-contained text on the foundations of signal processing, where discrete and continuous time are treated on equal footing. All the necessary mathematical background is included, with examples illustrating the applicability of the results. In addition, the book serves as a precursor to *FWSP*, which relies on the framework built here; the two books are thus integrally related.

Foundations of Signal Processing This book covers the foundations for an in-depth understanding of modern signal processing. It contains material that many readers may have seen before scattered across multiple sources, but without the Hilbert space interpretations, which are essential in signal processing. Our aim is to teach signal processing *with geometry*, that is, to extend Euclidean geometric insights to abstract signals; we use Hilbert space geometry to accomplish that. With this approach, fundamental concepts – such as properties of bases, Fourier representations, sampling, interpolation, approximation, and compression – are often unified across finite dimensions, discrete time, and continuous time, thus making it easier to point out the few essential differences. Unifying results geometrically helps generalize beyond Fourier-domain insights, pushing the understanding farther, faster.

Chapter 2, *From Euclid to Hilbert*, is our main vehicle for drawing out unifying commonalities; it develops the basic geometric intuition central to Hilbert spaces, together with the necessary tools underlying the constructions of bases and frames.

The next two chapters cover signal processing on discrete-time and continuous-time signals, specializing general concepts from Chapter 2. Chapter 3, *Sequences and discrete-time systems*, is a crash course on processing signals in discrete time or discrete space together with spectral analysis with the discrete-time Fourier transform and discrete Fourier transform. Chapter 4, *Functions and continuous-time systems*, is its continuous-time counterpart, including spectral analysis with the Fourier transform and Fourier series.

Chapter 5, *Sampling and interpolation*, presents the critical link between discrete and continuous domains given by sampling and interpolation theorems. Chapter 6, *Approximation and compression*, veers from exact representations to approximate ones. The final chapter in the book, Chapter 7, *Localization and uncertainty*, considers time–frequency behavior of the abstract representation objects studied thus far. It also discusses issues arising in applications as well as ways of adapting the previously introduced tools for use in the real world.

Fourier and Wavelet Signal Processing The companion volume focuses on signal representations using local Fourier and wavelet bases and frames. It covers the two-channel filter bank in detail, and then uses it as the implementation vehicle for all sequence representations that follow. The local Fourier and wavelet methods are presented side-by-side, without favoring any one in particular; the truth is that each representation is a tool in the toolbox of the practitioner, and the problem or application at hand ultimately determines the appropriate one to use. We end with examples of state-of-the-art signal processing and communication problems, with sparsity as a guiding principle.

Teaching points Our aim is to present a synergistic view of signal representations and processing, starting from basic mathematical principles and going all the way to actual constructions of bases and frames, always with an eye on concrete applications. While the benefit is a self-contained presentation, the cost is a rather sizable manuscript. Referencing in the main text is sparse; pointers to the bibliography are given in *Further reading* at the end of each chapter.

The material grew out of teaching signal processing, wavelets, and applications in various settings. Two of us (MV and JK) authored a graduate textbook, *Wavelets and Subband Coding* (originally with Prentice Hall in 1995, now open access[2]), which we and others used to teach graduate courses at various US and European institutions. With the maturing of the field and the interest arising from and for these topics, the time was right for the three of us to write entirely new texts geared toward a broader audience. We and others have taught with these books, in their entirety or in parts, a number of times and to a number of different audiences: from senior undergraduate to graduate level, and from engineering to mixes that include life-science students.

The books and their website[3] provide a number of features for teaching and learning:

- *Exercises* are an integral part of the material and come in two forms: *solved exercises* with explicit solutions within the text, and *regular exercises* that allow students to test their knowledge. Regular exercises are marked with $^{(1)}$, $^{(2)}$, or $^{(3)}$ in increasing order of difficulty.
- Numerous *examples* illustrate concepts throughout the book.
- An *electronic version* of the text is provided with the printed copy. It includes PDF hyperlinks and an additional color to enhance interpretation of figures.
- A *free electronic version* of the text without PDF hyperlinks, exercises or solved exercises, and with figures in grayscale, is available at the book website.
- A *Mathematica® companion*, which contains the code to produce all numerical figures in the book, is provided with the printed version.
- Several interactive *Mathematica® demonstrations* using the free CDF player are available at the book website.
- Additional material, such as lecture slides, is available at the book website.
- To instructors, we provide a *Solutions Manual*, with solutions to all regular exercises in the book.

Notational points To traverse the book efficiently, it will help to know the various numbering conventions that we have employed. In each chapter, a single counter is used for definitions, theorems, and corollaries (which are all shaded) and another for examples (which are slightly indented). Equations, figures, and tables are also numbered within each chapter. A prefix E$n.m$– in the number of an equation, figure, or table indicates that it is part of Solved Exercise $n.m$ in Chapter n. The letter P is used similarly for statements of regular exercises.

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[2] http://waveletsandsubbandcoding.org/
[3] http://www.fourierandwavelets.org/