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William F. Hosford
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AN OVERVIEW OF THE HISTORY OF PLASTICITY THEORY

INTRODUCTION

Plasticity theory deals with yielding of materials, often under complex states of stress. Plastic deformation, unlike elastic deformation, is permanent in the sense that after stresses are removed the shape change remains. Plastic deformation usually occurs almost instantaneously, but creep can be regarded as time-dependent deformation plastic deformation.

There are three approaches to plasticity theory. The approach most widely used is continuum theory. It depends on yield criteria, most of which are simply postulated without regard to how the deformation occurs. Continuum plasticity theory allows predictions of the stress states that cause yielding and the resulting strains. The amount of work hardening under different loading conditions can be compared.

A second approach focuses on the crystallographic mechanisms of slip (and twinning), and uses understanding of these to explain continuum behavior. This approach has been quite successful in predicting anisotropic behavior and how it depends on crystallographic texture. Ever since the 1930s, there has been increasing work bridging the connection between this crystallographic approach and continuum theory.

The third approach to plasticity has been concentrated on how slip and twinning occur. Dislocation theory, first postulated in the 1930s,

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has given insight and some understanding of how crystalline materials deform by slip. It explains strain hardening, but the connection to continuum theory has been difficult to bridge.

CONTINUUM THEORIES

The theoretical basis for yielding under complex stress states had its origins in the nineteenth century. The first systematic investigation of yielding can be attributed to Tresca [1] who conducted a series of experiments on extrusion and concluded that yielding occurred when the maximum shear stress reached a critical value. He was probably influenced by earlier work of Coulomb [2] on soil mechanics. In 1913, Von Mises [3] proposed his widely used yield criterion. Huber [4] had earlier published essentially the same criterion in Polish, but he may have been writing about fracture and his paper had attracted little attention. Von Mises work was also preceded by Maxwell [5] written in 1856 in an unpublished letter.

In 1937, Nadai [6] showed that the von Mises criterion corresponds to yielding when a critical shear stress is reached on the octahedral planes. It also was shown [7, 8] that the von Mises criterion can be derived, if one assumes that yielding occurs when the elastic distortional energy reaches a critical value. Although this has been taken as proof of the von Mises criterion, there is no fundamental reason for this assumption.

In 1948, Hill [9] proposed the first anisotropic yield criterion. However, it was not until the 1970s that non-quadratic yield criteria [10, 11] were proposed. A non-quadratic modification of Hill's 1948 criterion was proposed in 1979 [12].

CRYSTALLOGRAPHIC BASIS OF PLASTICITY

In 1900, Ewing and Rosenhain [13] showed that plastic deformation occurred by slip. This is the sliding of planes of atoms slide over one another. The planes on which slip occurs are called *slip planes* and the directions of the shear are the *slip directions*. These crystallographic planes and directions are characteristic of a material's crystal structure. The magnitude of the shear displacement is an integral number of inter-atomic distances, so that the lattice is left unaltered. In 1924, Schmid [14] proposed that slip occurs when the shear stress on the slip plane in the slip direction has to reach a critical value. Along with Boas, Schmid published *Kristallplastizität* [15], a classic book on slip.

Calculations of the critical stress to cause slip predicted strengths several orders of magnitude higher than those found experimentally. In 1934, dislocation theory was formulated by three independent scientists to explain this discrepancy [16, 17, 18]. In 1954, Frank and Read [19] showed how slip can generate dislocations. Since the introduction of dislocation theory, it has been realized dislocation climb and cross slip could overcome obstacles and that the intersection of dislocations on different planes is responsible for strain hardening.

In 1938, Taylor [20, 21] developed an upper bound model of the deformation of polycrystals based on the nature of slip. He assumed that every grain must undergo the same shape change. His analysis assumed that the shape change would occur with the minimum amount of slip. In 1951, Bishop and Hill [22, 23] proposed an alternate way of viewing the problem by finding the stress states that are capable of activating enough slip systems to allow every grain to undergo the same shape change. These theories allowed analysis of the deformation of polycrystalline metals.

General Treatments of Plasticity

In 1950, Hill wrote a classic book, *The Mathematical Theory of Plasticity* [24], which covered the basic theory of plasticity and applications to a number of problems. It also introduced a treatment of anisotropic plastic behavior. This was followed by Timoshenko's *History of the*

Strength of Materials in 1953 [25] and Calladine's *Engineering Plasticity* in 1969 [26]. However, since the preceding there have been no new general treatments of plasticity.

NOTE OF INTEREST

Although the book, *Kristallplasttizität*, by Schmid and Boas was first published in 1935, it was available only in German because the Nazis refused to allow it to be translated. Only after World War II, was translation undertaken. In 1950, an English edition was published by Chapman and Hall.

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YIELDING

Of concern in plasticity theory is the *yield strength,* which is the level of stress that causes appreciable plastic deformation. It is tempting to define yielding as occurring at an *elastic limit* (the stress that causes the first plastic deformation) or at a *proportional limit* (the first departure from linearity). However, neither definition is very useful because they both depend on accuracy of strain measurement. The more accurately the strain is measured, the lower is the stress at which plastic deformation and non-linearity can be detected.

To avoid this problem, the onset of plasticity is usually described by an *offset yield strength* that can be measured with more reproducibility. It is found by constructing a straight line parallel to the initial linear portion of the stress strain curve, but offset from it by a strain of $\Delta e = 0.002$ (0.2%). The yield strength is taken as the stress level at which this straight line intersects the stress strain curve (Figure 2.1). The rationale is that if the material had been loaded to this stress and then unloaded, the unloading path would have been along this offset line resulting in a plastic strain of $e = 0.002$ (0.2%). This method of defining yielding is easily reproduced.

If yielding in a tension test is defined by a 0.2% offset, for the purpose of assessing the anisotropy, yielding under any other form of loading must be defined by the plastic strain that involves the same amount of plastic work as the 0.2% offset in tension.

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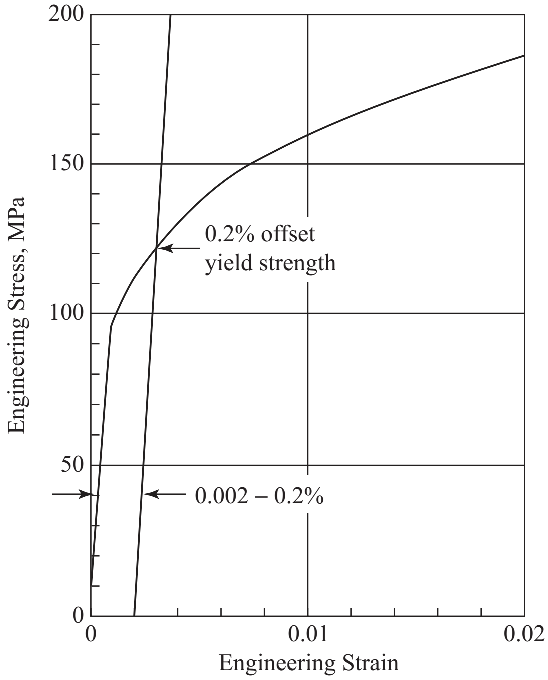

Figure 2.1. The low-strain region of the stress-strain curve for a ductile material. From W. F. Hosford, Mechanical Behavior of Materials, 2nd ed., Cambridge University Press (2010).

Yield points: The stress–strain curves of some materials (for example, low carbon steels and linear polymers), have an initial maximum followed by lower stress as shown in Figures 2.2a and 2.2b. After the initial maximum, at any given instant all of the deformation occurs within a relatively small region of the specimen. For steels, this deforming region is called a *Lüder's* band. Continued elongation occurs by propagation of the Lüder's band along the gauge section, rather than by continued deformation within it. Only after the band has traversed the entire gauge section, does the stress rise again. In the case of linear polymers, the yield strength is usually defined as the initial maximum stress. For steels, the subsequent lower yield strength is used to

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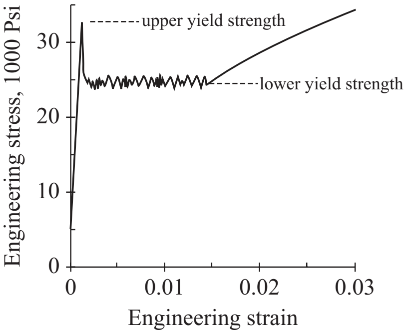

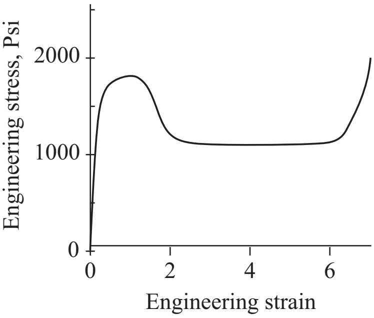

Figure 2.2. Inhomogeneous yielding of low carbon steel (left) and a linear polymer (right). After the initial stress maximum, the deformation in both materials occurs within a narrow band that propagates the length of the gauge section before the stress rises again. From W. F. Hosford and R. M. Caddell, *Metal Forming; Mechanics and Metallurgy*, 4th ed. Cambridge University Press (2007).

describe yielding because the initial maximum stress is too sensitive to specimen alignment to be a useful index. Even so, the lower yield strength is sensitive to the strain rate. The stress level during Lüder's band propagation fluctuates. Some laboratories report the minimum level as the yield strength and other use as the average level.

IDEALIZATION OF YIELDING BEHAVIOR

Typical tensile load-extension behavior with unloading and reloading is shown schematically in Figure 2.3a. Idealization of this behavior

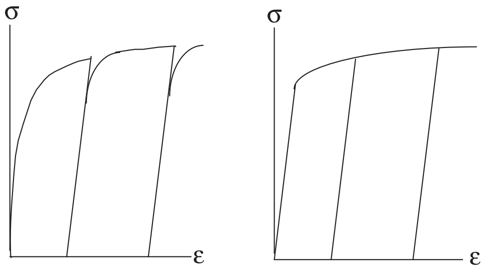

Figure 2.3. Idealization of yielding. Actual loading and unloading stress strain curves (A) are often idealized (B) by assuming sharp yielding on reloading after unloading.

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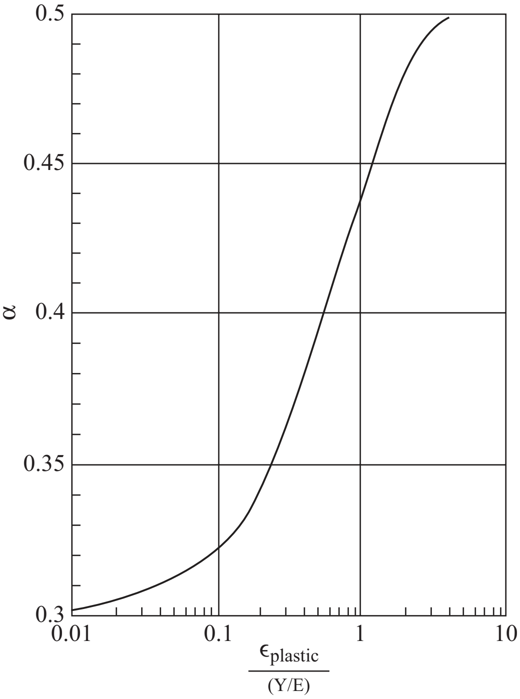

Figure 2.4 Change in the stress ratio, $\alpha = \sigma_y/\sigma_x$, for plane strain, $\varepsilon_y = 0$, as a function of strain. From W. F. Hosford and R. M. Caddell, *Metal Forming; Mechanics and Metallurgy*, 4th ed. Cambridge University Press (2007).

(Figure 2.3b) has a sharp initial yield stress (a) and assumes sharp yielding on reloading after unloading.

ELASTIC-PLASTIC TRANSITION

The transition from elastic to plastic flow is gradual as illustrated in Figure 2.4 for plane-strain deformation with $\varepsilon_y = 0$ and $\sigma_z = 0$. For elastic deformation, $\alpha = \nu$ and for fully plastic deformation $\alpha = 0.5$. In this figure, the ε_x is normalized by the ratio of the yield strength to the modulus. Note that 95% of the change from elastic to plastic deformation occurs when the plastic strain is three times the elastic strain.

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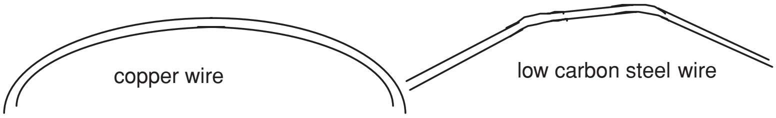

Figure 2.5 Bending of a low carbon steel wire will result in kinks because of the tendency to localize deformation rather than a continuous curve as with copper wire.

For a material that strain hardens, there is additional elastic deformation after yielding. The total strain is the sum of the elastic and plastic parts, $e = e_\mathrm{e} + e_\mathrm{p}$. Even though the elastic strain may be very small relative to the plastic strain, elastic recovery on unloading controls residual stresses and springback.

NOTE OF INTEREST

A simple experiment that demonstrates the yield point effect can be made with pieces of annealed florists wire, which is a low-carbon steel. When the wire is bent, it will form sharp kinks because once yielding occurs at one location, it takes less force to continue the bend at that location than to initiate bending somewhere else. On the other hand, copper wire that has no yield point will bend in a continuous arc (see Figure 2.5).

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