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Part One

Einstein's Triumph

Introduction

Recent media attention to the centenary of the outbreak of the First World War (WWI) reminds us that it was against this backdrop that Einstein, a Swiss citizen, announced the revolutionary theory of general relativity (GR). The war affected the theory's dissemination. Eddington's report introducing GR to the English-speaking world [1] relied on information from de Sitter in neutral Holland. Inevitably, the theory's adherents were caught up in the conflict, most notably Karl Schwarzschild, who died in 1916 while serving on the Russian front.

In 1915 Einstein was already a decade on from his *annus mirabilis* of 1905, in which he had announced the theory of special relativity, explained the already well-observed photoelectric effect as due to quantization of light (a vital step towards quantum theory), and explained Brownian motion assuming the reality of atoms, an explanation experimentally confirmed by Perrin in 1908. The second of these three great papers won him the 1921 Nobel prize – and they were not all he published that year! For example, he published the famous $E = mc^2$ equation, which later gave the basis of nuclear fusion and fission (whence Einstein's intervention in the development of atom bombs). Fusion in particular explained how stars could hold themselves up against gravity as long as they do. So Einstein had already triumphed well before 1915.

However, he was aware that his work left an awkwardly unresolved question – the need for a theory of gravity compatible with special relativity that agreed with Newton's theory in an appropriate limit. Here we will not recount Einstein's intellectual development of general relativity, which resolved that problem, nor describe the interactions with friends and colleagues which helped him find the right formulation. Those are covered by some good histories of science, and biographies of Einstein, as well as his own writings.

The theory's prediction of light-bending, confirmed to good accuracy [2] by the UK's 1919 eclipse expedition led by Eddington[1] and Crommelin, brought Einstein to the attention of the general public, in particular through the famous headline in the New York Times

[1] How Eddington, a Quaker, while preparing for this expedition, avoided being sent to work on the land as a conscientious objector, is itself an interesting WWI story.

of November 9th. From then on, he increasingly came to be seen as the personification of scientific genius.

Why then are we calling this first part of our centennial book "Einstein's triumph"? GR had already triumphed by 1919.

The triumph since 1919 lies in GR's ever increasing relevance and importance, shown in particular by the number and range of applications to real-world observations and applications, from terrestrial use in satellite navigation systems to considerations of cosmology on the largest scales. Moreover the different applications are now interwoven, for example in the relevance of black holes in cosmology and the use of pulsars, compact relativistic stars, in strong field tests of the theory. This part of the book outlines that progress.

As Ellis describes in Chapter 1, the starting points for many later confirmations were laid in the early years of the theory: the Schwarzschild solution, leading to solar system tests and black hole theory; light-bending, which grew into gravitational lensing; and the Friedmann(–Lemaître–Robertson–Walker: FLRW) solutions, basic in cosmology. Moreover, several confirmations relate to the three "classical tests": gravitational redshift, the anomaly in the perihelion advance of Mercury as computed from Newtonian theory,[2] and light-bending: for example, the analysis of GPS (the Global Positioning System), the study of the binary and double pulsars, and the use of microlensing to detect exoplanets. The theory remains the most nonlinear of the theories of physics, prompting development in analytic and numerical technique.

Classical differential geometry as studied in introductory courses (and as briefly outlined by Ellis) is adequate to discuss the starting points of those developments. But they soon require also the proper understanding of global structure and thus of singularities and asymptotics, for example in understanding the Schwarzschild solution, black holes and the energy carried away by gravitational radiation. This increasing sophistication was reflected in the best-selling text of Hawking and Ellis [3], and further developments are described in Part Three of this book.

Much of the development of GR has come in the last half century. For its first 50 years, a time when quantum theory was making big advances, one could argue that GR remained an intellectual ornament with only some limited applications in astronomy. Even its relevance to cosmology was debatable, because Hubble's erroneous distance scale led to a conflict between the geologically known age of the Earth and the age of the universe in an FLRW model, prompting the range of alternative explanations for this discrepancy described in Bondi's book [4]. While the notion of a stagnant phase is rather belied by the many significant papers from this time which have deservedly been included in the "Golden Oldies" series of the *General Relativity and Gravitation* journal, some of them cited by Ellis, it was certainly a less dynamic period than the following 50 years of GR.

[2] One may note that the anomalous part is $43''$ per century in a total of around $5000''$ per century.

The changes have been partly due to the already mentioned increasing mathematical sophistication among theoretical physicists. Taub's use of symmetry groups [5] and Petrov's algebraic classification of the Weyl tensor [6] were crucial steps forward made in the 1950s. The geometric concepts of connection and curvature have become fundamental in modern gauge theories. Progress in the theory of differential equations has given a firm basis to the idea that GR is like other physical theories in that initial configuration and motion determine the future evolution. The generating techniques for stationary axisymmetric systems used to obtain exact solutions[3] relate to modern work on integrable systems. Further developments in such areas are reflected in Chapter 1 and Part Three of this book.

Another important step was introducing the theory of the matter content within FLRW models. This enabled the understanding of the formation of the chemical elements, by combining the Big Bang and stellar nucleosyntheses, the provision of evidence that there were only three types of neutrino, and the prediction of the Cosmic Microwave Background (CMB).

Progress has depended even more on advances in technology and measurement technique. The first example was the revision of Hubble's distance scale in 1952 by Baade, using the 200 inch Palomar telescope commissioned in 1950. This led to increasing belief in the FLRW models, a belief eventually cemented by the 1965 observations of the CMB, which themselves arose from developments in microwave communications technology.

The 1957 launch of the first artificial satellite, Sputnik, intensified the need for detailed calculation of orbital effects in satellite motion, in order to very accurately plan satellite projects. Such work [9] was undertaken for both the US and USSR programs and was the first practical use of GR.

Radio astronomy, by showing source counts inconsistent with the alternative Steady State theory, had provided important evidence for FLRW models. It also, combined with optical observations, led to the discovery of quasars[4] which prompted Lynden-Bell to propose that they were powered by black holes [10]: the importance black holes have subsequently assumed in our understanding of astronomy and cosmology is described by Narayan and McClintock in Chapter 3. Radio astronomy also discovered the pulsars, announced in 1968, which gave extra impetus to the already developing study of relativistic stars, discussed by Friedman in Chapter 3.

The reality of gravitational waves in the theory, which had been debated earlier, was finally clarified in the work of Bondi *et al.* in 1959 [11]. The binary and double pulsar observations, described in Chapter 2, united the understanding of compact objects and gravitational waves to provide the first strong field tests of GR.

The exquisite precision now achieved in practical and observational areas of GR has made use of the development of very high precision atomic clocks and of the burgeoning

[3] The construction and interpretation of exact solutions are topics not covered by this book, as they are well covered by [7] and [8] and references therein. In particular we do not consider some important techniques used in those areas, such as computer algebra and the application of local spacetime invariants.

[4] 3C48 was identified in 1960 and 3C273's redshift was found in 1963.

of electronics since the invention of the transistor in 1947. Satellite-borne telescopes in several wavebands, computers of all scales from the largest (used in numerical relativity) to mobile devices (e.g. in GPS receivers), CCD devices (based of course on the photoelectric effect), and lasers (in terrestrial gravitational wave detectors – also used, for example, in determining the exact position of the moon) have all played major roles in the observations and experiments described in the following four chapters (and in the later parts of the book).

There were fundamental aspects of gravity (e.g. the Eötvös effect) which could be and were tested on Earth, but until the 1970s the focus was on the "classical tests", complemented by the time delay measurements for satellites. Dicke initiated a more systematic analysis of the equivalence principle and its tests, as described in Chapter 2. Thorne, Will and others then developed other frameworks, notably the PPN framework, which could encompass other types of test. While the application of these ideas still relied on solar system and terrestrial tests, these became much more precise and involved much new technology (e.g. laser ranging to the moon, superconducting gravimeters on the ground, use of atomic traps and atomic clocks in terrestrial and satellite experiments), and pinned the parameters of the PPN framework down with high precision.

Tests outside the solar system consisted of the understanding of compact stars such as white dwarfs, and supernova remnants, and of cosmology (for which there was only an incomplete understanding, for reasons described below), but did not lead to new precise constraints on the theory. That changed with the discovery and observations of the (first) binary pulsar, and still further with the several now known, including the double pulsar. These give some of the most precise measurements in physics (although, perhaps surprisingly, the Newtonian constant of gravitation, G, remains the least accurately known of the fundamental constants of nature).

It is notable that the understanding of pulsars not only required GR (because of the strong fields) but also entailed the simultaneous use of quantum theory and GR (because only by taking into account quantum theory could one have adequate equations of state to model white dwarfs and neutron stars). These types of compact object, and black holes, are now the starting points for the calculation of gravitational wave sources described in Part Two.

Relativistic astrophysics then developed in a number of directions (see Chapter 3). Numerical simulations gave much more detail on relativistic stars, their properties, stability and evolution. A whole new sub-discipline of black hole astrophysics came into being, concerned with the environments of black holes, especially (for stellar size black holes) accretion from neighbouring stars and (for supermassive black holes) accretion, nearby orbits and tidal capture of stars. The improved understanding enabled us to be rather certain not only that there really are black holes in the Universe, but that they are very common.

A further direction described in Chapter 3 came about with the discovery and increasingly detailed observations of gamma ray bursts. Both their long and short varieties turned out to require models of relativistic sources, as described by Mészáros and Rees. It is interesting that there is a link with the gravitational wave detectors described in Part Two, in that the absence of gravitational waves from GRB 070201 showed that, if it had a compact binary progenitor, then that progenitor had to be behind rather than in M31 [12].

While the standard FLRW models used up to 1980 or so did very well in describing the observed isotropy and homogeneity of the universe, and explaining the evolution of the matter content which led to formation of the chemical elements and the prediction of the CMB, they failed to explain the single most obvious fact about the Universe, namely that it had a highly non-uniform density. Naturally occurring thermal fluctuations and their evolution could not give large enough variations. The inflationary paradigm altered that radically by providing reasons for a nearly flat spectrum of density fluctuations at a time sufficiently early in the universe for the subsequent linear and nonlinear phases of evolution to produce the observed structures we see. The theory is described in detail by Sasaki in Chapter 4.

The resulting standard model has been compared with a range of very high precision observations, notably those of the CMB, the baryon acoustic oscillations (BAO) and the magnitude–redshift relation for supernovae (relating distances and expansion velocities in the Universe). These, especially the CMB observations, have generated the title “precision cosmology”, which, as Komatsu emphasizes in Chapter 4, required precision theory as well as precision observation. That precision in theory consists of very detailed consideration of perturbations of the FLRW models and of light propagation in perturbed models, enabling the link between the conditions produced by inflation (or some alternative to inflation providing suitable initial conditions) and the present-day observations. What is remarkable is the fine detail of those initial conditions that one can infer from observation.

To some degree, the role of GR has disappeared in the large volume of literature related to CMB, BAO and supernova, and other, observations, as almost all of it uses the FLRW models and their linearized perturbations, and may even make crucial steps using Newtonian analyses. Wands and Maartens remind us, in their introduction to Chapter 4, that GR in fact still has a crucial role to play, even in precision cosmology where its effects may be considerably larger than the very small error bars in the observations, and the correlations described in Chapter 3 imply it also has a role to play in structure formation below the scales tested by the CMB. Moreover it is essential in testing the robustness of the assumptions of the concordance model, a further topic discussed in Chapter 4.

What can we expect in the future? B-mode polarization[5] in the CMB could give evidence for primordial gravitational waves, as discussed in Chapter 4. A recent joint analysis of data [13] suggests the signal found by BICEP2 may have been due to polarized dust emission: it places only an upper limit on the gravitational wave contribution, while supporting the lensing contributions as seen by POLARBEAR. In 2015 Advanced LIGO will begin taking data (see Part Two). If such advanced gravitational wave detectors see the expected gravitational wave sources, we will have a new window for testing GR (but if no such sources are seen, that may be due only to poor astrophysical predictions). In the past, when

[5] There are two characteristic patterns of polarization alignments expected in the CMB. The E-mode is like that of the electric field round a charge and the B-mode like that of magnetic field round a current. Instances of these modes, with varying amplitudes and centred at random locations, will be superposed in the actual observations. For more details see Chapter 4.

new windows on the universe have opened, new and unforeseen phenomena have been found [14]; it would not be surprising if this happens again. Beyond that there are a plethora of new instruments being built or planned to study the sky in electromagnetic wavebands from low frequency radio to γ-rays: the chances of convincing funding agencies to support such work have probably been substantially enhanced by the spectacular results of recent past projects.

Gravitational lensing by galaxies seemed to surprise many when first found in 1979, even if it should not have. Now such lensing, and its stellar size counterpart, have become tools for astronomy, used for example to infer the distribution of mass within galaxies, the distribution of dark matter, the properties of distant galaxies, and the presence of new exoplanets. Recently, magnification due to microlensing was used to determine properties of a binary system containing a white dwarf and a Sun-like star [15].

We stress again that the galactic and intergalactic application is just one of the instances where different aspects of GR come together – here lensing and cosmological models.

Although the greatest challenge for GR may lie in finding and testing a good enough theory of quantum gravity, as discussed in Part Four, there are still challenges at the classical level. Cosmology provides the greatest of these, since its standard model requires three forms of matter – the inflaton, dark matter and dark energy – which have not been, and perhaps cannot be, observed in terrestrial laboratories, and whose properties are modeled only in simple and incomplete ways. It would of course be ironic if the triumph of GR in cosmology were to turn to disaster because the only way to deal with those apparently-required three forms of matter were to adopt a modified theory of gravity, but other explanations seem much more likely.

The inflaton is postulated as a way to produce the nearly flat spectrum of fluctuations required as initial data from which acoustic oscillations produce the observed CMB power spectrum. While the assumptions of inflation may be questionable, it is, as already mentioned, remarkably successful in producing the right distribution of fluctuations on present-day scales above 150 Mpc or so (a scale much larger than that of individual galaxies). Inflation theory predicts B-mode polarization due to gravitational waves, consistent with BICEP2's initial results. A definitive detection of such polarization would provide indirect evidence on quantum gravity and the quantum/classical correspondence, in that the theory assumes the quantum fluctuations of the inflationary era become classical.

The evidence for dark matter is rather securely based on observations at scales where a Newtonian approximation is good enough to show that not all the mass is visible, such as observations of galactic rotation curves and the distribution of X-ray emitting gas in clusters. It provides 25–30% of the critical energy density of the Universe, itself now known to be very close to the actual energy density (see Chapter 4). This was known before the more precise CMB and baryon acoustic oscillation (BAO) measurements [16]. Additional evidence has been provided by comparing the distribution of mass in colliding galaxies, as shown by its lensing effects, with the mass distribution of the visible gas. However, a change in the gravity theory might provide an explanation for these observations not requiring dark matter, though as yet no satisfactory such theory has been proposed.

The inference of the existence of dark energy is even more dependent on GR, in particular on the theory of perturbed FLRW models (see Chapter 4): it comes from the magnitude–redshift relation for supernovae (relating distance and expansion velocity of the Universe), CMB and BAO data. Attempted explanations within GR not requiring a new form of matter (in which we include the cosmological constant) have used both large and small scale inhomogeneities (see Chapter 4), or may arise from the astrophysics of supernovae and their environments. Or we may be able to pin down the properties of dark energy in some way independent of FLRW models, and thereby provide a further triumph for the predictions of GR.

Obtaining information about the three so far unobserved constituents of the standard model may not come from GR itself. But we would certainly like a better understanding of inhomogeneities and their back reaction and impact on light propagation. The evidence of correlations of galactic properties with central black hole masses suggests we also need to know much more about the messy nonlinear processes of galaxy and star formation and their interaction with the nonlinearities of GR.

Despite these lacunae, which may offer opportunities for future breakthroughs, when taken together the following four chapters illustrate very well the staggering extent of the triumph of Einstein's 1915 proposal of the theory of General Relativity.

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1

100 Years of General Relativity

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This chapter aims to provide a broad historical overview of the major developments in General Relativity Theory ('GR') after the theory had been developed in its final form. It will not relate the well-documented story of the discovery of the theory by Albert Einstein, but rather will consider the spectacular growth of the subject as it developed into a mainstream branch of physics, high-energy astrophysics, and cosmology. Literally hundreds of exact solutions of the full non-linear field equations are now known, despite their complexity [1]. The most important ones are the Schwarzschild and Kerr solutions, determining the geometry of the solar system and of black holes (Section 1.2), and the Friedmann–Lemaître–Robertson–Walker solutions, which are basic to cosmology (Section 1.4). Perturbations of these solutions make them the key to astrophysical applications.

Rather than tracing a historical story, this chapter is structured in terms of key themes in the study and application of GR:

1. The study of dynamic geometry (Section 1.1) through development of various technical tools, in particular the introduction of global methods, resulting in global existence and uniqueness theorems and singularity theorems.
2. The study of the vacuum Schwarzschild solution and its application to the Solar system (Section 1.2), giving very accurate tests of general relativity, and underlying the crucial role of GR in the accuracy of useful GPS systems.
3. The understanding of gravitational collapse and the nature of Black Holes (Section 1.3), with major applications in astrophysics, in particular as regards quasi-stellar objects and active galactic nuclei.
4. The development of cosmological models (Section 1.4), providing the basis for our understandings of both the origin and evolution of the universe as a whole, and of structure formation within it.
5. The study of gravitational lensing and its astronomical applications, including detection of dark matter (Section 1.5).
6. Theoretical studies of gravitational waves, in particular resulting in major developments in numerical relativity (Section 1.6), and with development of gravitational wave observatories that have the potential to become an essential tool in precision cosmology.