

Contents

Preface [page ix]

PART I FIRST STEPS IN LOGICAL REASONING

- 1 Starting points [3]
 - 1.1 Origins [3]
 - 1.2 Demonstrative arguments [4]
 - 1.3 Propositions and assertions [7]
 - 1.4 The connectives [8]
 - 1.5 Grammatical variation, unique readability [10]
 - 1.6 A grammar for propositional logic [12]
 - 1.7 Idealization [14]
- 2 Rules of proof [15]
 - 2.1 Steps in proofs [15]
 - 2.2 Negation [18]
 - 2.3 Natural deduction in a linear form [19]
 - 2.4 The notion of a derivation [21]
 - 2.5 How to construct derivations? [23]
 - 2.6 Schematic formulas, schematic derivations [25]
 - 2.7 The structure of derivations [27]
 - Notes and exercises to Chapter 2 [29]
- 3 Natural deduction [31]
 - 3.1 From linear derivations to derivation trees [31]
 - 3.2 Gentzen's rules of natural deduction [34]
 - 3.3 Derivations with cases [38]
 - 3.4 A final modification [42]
 - 3.5 The role of falsity and negation [46]
 - 3.6 Axiomatic logic [49]
 - 3.7 Proofs of unprovability [58]
 - 3.8 Meaning explanations [61]
 - Notes and exercises to Chapter 3 [62]
- 4 Proof search [64]
 - 4.1 Naturally growing trees [64]
 - 4.2 Invertibility [68]
 - 4.3 Translation to sequent calculus [70]
 - 4.4 Unprovability through failed proof search [74]

4.5	Termination of proof search	[76]
	Notes and exercises to Chapter 4	[78]
5	Classical natural deduction	[80]
5.1	Indirect proof	[80]
5.2	Normal derivations and the subformula property	[85]
	Notes and exercises to Chapter 5	[88]
6	Proof search in classical logic	[89]
6.1	Assumptions and cases	[89]
6.2	An invertible classical calculus	[90]
	Notes and exercises to Chapter 6	[95]
7	The semantics of propositional logic	[96]
7.1	Logical truth	[96]
7.2	The semantics of intuitionistic propositional logic	[101]
7.3	Empty tautologies?	[108]
7.4	The completeness of classical propositional logic	[110]
	Notes and exercises to Chapter 7	[112]
PART II LOGICAL REASONING WITH THE QUANTIFIERS		
8	The quantifiers	[115]
8.1	The grammar of predicate logic	[115]
8.2	The meaning of the quantifiers	[120]
	Notes to Chapter 8	[128]
9	Derivations in predicate logic	[129]
9.1	Natural deduction for predicate logic	[129]
9.2	Proof search	[133]
9.3	Classical predicate logic	[143]
	Notes and exercises to Chapter 9	[150]
10	The semantics of predicate logic	[152]
10.1	Interpretations	[152]
10.2	Completeness	[155]
10.3	Interpretation of classical logic in intuitionistic logic	[156]
PART III BEYOND PURE LOGIC		
11	Equality and axiomatic theories	[161]
11.1	Equality relations	[161]
11.2	Sense and denotation	[169]
11.3	Axiomatic theories	[172]
12	Elements of the proof theory of arithmetic	[177]
12.1	The Peano axioms	[177]

12.2	Heyting arithmetic	[179]
12.3	The existence property	[185]
12.4	A simple-minded consistency proof	[188]
PART IV COMPLEMENTARY TOPICS		
13	Normalization and cut elimination	[195]
13.1	Proofs by structural induction	[195]
13.2	A proof of normalization	[198]
13.3	The Curry–Howard correspondence	[205]
13.4	Cuts, their elimination and interpretation	[210]
14	Deductive machinery from Aristotle to Heyting	[220]
14.1	Aristotle’s deductive logic	[220]
14.2	The algebraic tradition of logic	[228]
14.3	The logic of Frege, Peano, and Russell	[239]
14.4	Axiomatic logic in the 1920s	[248]
	<i>Suggestions for the use of this book</i>	[253]
	<i>Further reading</i>	[256]
	<i>Bibliography</i>	[258]
	<i>Index of names</i>	[261]
	<i>Index of subjects</i>	[262]