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Part I

General concepts

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1 Transmission lines and scattering parameters

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1.1 Introduction

This chapter introduces the reader to the topics presented in the rest of the book, and serves as a quick guide to the basic concepts of wave propagation and scattering parameters.

Understanding these concepts becomes very important when dealing with RF and microwave frequencies, as is shown in Section 1.2, where a simplified formulation for the transmission line theory is given.

Section 1.3 provides the definition of the scattering matrix or S-matrix, the key element to describe networks at RF, microwaves, and higher frequencies.

Section 1.4 deals with the most important component in microwave measurements, the directional coupler, while Section 1.5 revises a common way to represent quantities in the RF domain, the Smith Chart.

Finally, in Appendix A signal flow graphs, a typical way to represent simple linear algebra operations, are presented, while Appendix B summarizes the various types of transmission lines cited in this book.

1.2 Fundamentals of transmission lines, models and equations

1.2.1 Introduction

Electromagnetic waves travel at about the speed of light ($c = 299\ 792\ 458$ m/s) in air. Using the relationship

$$\nu = f\lambda, \tag{1.1}$$

where ν is velocity ($= c$ in air), f is frequency and λ is wavelength, the wavelength of a 100 GHz wave is about 3 mm. If a simple connection on a circuit is of the order of magnitude of a wavelength, it is then necessary to consider its behavior as distributed and regard it as a transmission line. In fact, propagation phenomena already appear for lengths of 1/10$^{\text{th}}$ of a wavelength.

Let's clarify this concept by a simple example. When a source of electrical power is connected to a load, as shown in Figure 1.1, the voltage appears at the load instantaneously over a short distance.

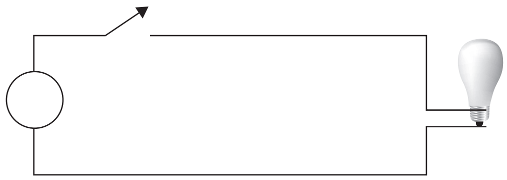

Fig. 1.1 Connection of a light bulb close to the source of electrical power.

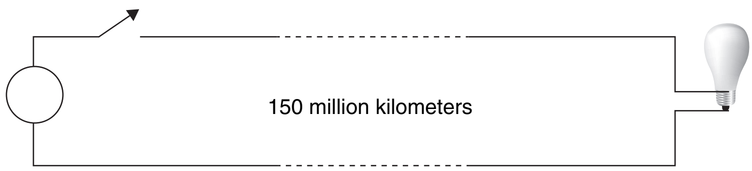

Fig. 1.2 Connection of a light bulb at 150 million km from the source of electrical power.

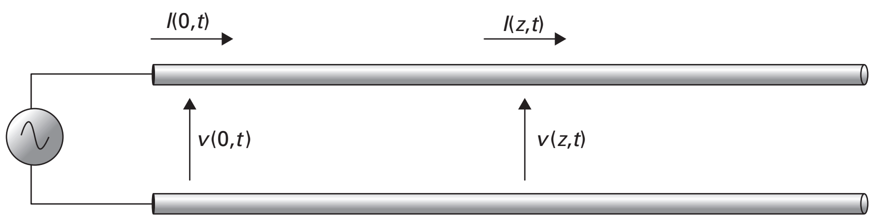

Fig. 1.3 Two-wire line.

However, if the connection wiring is very long, as shown in Figure 1.2, it takes time for the signal to propagate to the load. In this example, using the approximate distance from the sun, the bulb would light some 8 minutes after the switch is closed.

This means that the connection cannot be modeled with a short circuit anymore, since the voltage and current (or electric and magnetic fields) are now functions of both time and position.

Let us consider a two-wire line, as shown in Figure 1.3.

Here both the voltage and current are functions of position and time. Now, if we model the line as an infinite number of very short sections, each element can be considered as a series inductance and shunt capacitance with associated losses, as shown in Figure 1.4. This model can actually be applied to any kind of transmission line (waveguide, coaxial, microstrip, etc.; see Appendix B for a brief description of the most common types of transmission lines referred to in this book).

1.2.2 Propagation and characteristic impedance

In a two-conductor line, the model may be explained physically. The wire properties and skin effect generate the inductance, the two conductors the capacitance, and leakage

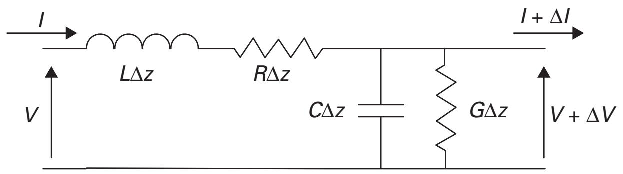

Fig. 1.4 Lumped-element model of a section of the two-wire line of Fig. 1.3.

and losses produce the parasitic resistances. These model elements are also functions of frequency.

Solving the model circuit for the voltage and current, yields

$$\Delta V(z,t) = (R\Delta z + j\omega L\Delta z)I(z,t) \tag{1.2}$$

and

$$\Delta I(z,t) = (G\Delta z + j\omega C\Delta z)V(z,t). \tag{1.3}$$

Taking Δz as infinitely short, the partial derivatives of voltage and current with respect to the z coordinate appear as:

$$\frac{\partial V(z,t)}{\partial z} = -(R + j\omega L)\, I(z,t) \tag{1.4}$$

$$\frac{\partial I(z,t)}{\partial z} = -(G + j\omega C)\, V(z,t). \tag{1.5}$$

Then, by differentiating (1.4) again with respect to z and substituting (1.5) in the obtained equation (and vice versa) one gets:

$$\frac{\partial^2 V(z,t)}{dz^2} = \gamma^2 V(z,t) \quad \text{and} \quad \frac{\partial^2 I(z,t)}{dz^2} = \gamma^2(z,t)I(z,t), \tag{1.6}$$

where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$ is the ***propagation constant***.

The equations have exponential solutions of the form

$$V = V_1 e^{-j\gamma z} + V_2 e^{+j\gamma z}, \tag{1.7}$$

where the first part of the solution ($V^+ = V_1 e^{-j\gamma z}$) is referred to as an ***incident wave***, and the second part ($V^- = V_2 e^{+j\gamma z}$) as a ***reflected wave***.

In the same way, one can write the solution for the current as

$$I = I_1 e^{-j\gamma z} + I_2 e^{+j\gamma z}. \tag{1.8}$$

By substituting (1.7) and (1.8) inside (1.4) and (1.5) one can find the relationship between I_1-V_1 and I_2-V_2, which are:

$$V_1 = Z_0 I_1 \tag{1.9}$$

and

$$V_2 = -Z_0 I_2 \tag{1.10}$$

with

$$Z_0 = \frac{\sqrt{R + j\omega L}}{\sqrt{G + j\omega C}}, \tag{1.11}$$

where Z_0 is referred to as the *characteristic impedance* of the transmission line. Note that the ***wave number*** β can be expressed as a function of v_p, the so-called ***phase velocity***, or of the wavelength (λ):

$$\beta = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}. \tag{1.12}$$

The time dependence of the voltage and current can be made explicit in this way

$$V(z,t) = V(z)e^{j\omega t} \quad I(z,t) = I(z)e^{j\omega t} \tag{1.13}$$

and the circuit equations rewritten as

$$\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t}\right) \quad \text{and} \quad \frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t}\right). \tag{1.14}$$

Again, differentiating gives

$$\frac{\partial^2 V}{\partial z^2} = R\left(GV + C\frac{\partial V}{\partial t}\right) + L\left(G\frac{\partial V}{\partial t} + C\frac{\partial^2 V}{\partial t^2}\right) \tag{1.15}$$

or

$$\frac{\partial^2 V}{dz^2} = -(RC + LG)\frac{\partial V}{\partial t} - LC\frac{\partial^2 V}{dt^2} - RGV = 0. \tag{1.16}$$

Note that the current I satisfies an identical equation.

In the case of lossless transmissions lines with $R = G = 0$, the propagation constant and the characteristic impedance simplify to the trivial

$$\gamma = j\beta = j\omega\sqrt{LC} \quad \text{and} \quad Z_0 = \sqrt{\frac{L}{C}}. \tag{1.17}$$

For most practical purposes, however, especially in a hollow pipe waveguide, the low-loss case ($R = \omega L,\ G = \omega C$) provides accurate values:

$$\gamma \approx \alpha + j\beta = j\omega\sqrt{LC} + \frac{1}{2}\sqrt{LC}\left(\frac{R}{L} + \frac{G}{C}\right) \tag{1.18}$$

with

$$\alpha = \frac{1}{2}\sqrt{LC}\left(\frac{R}{L} + \frac{G}{C}\right) = \frac{1}{2}(RY_0 + GZ_0) \tag{1.19}$$

1.2.3 Terminations, reflection coefficient, SWR, return loss

We have seen how the total voltage on a transmission line is the vector sum of the incident and reflected voltages and the phase relationship between the waves depends on the position along the line. The nature of a discontinuity determines the phase relationship of the incident and reflected waves at that point on the line and that phase relationship is repeated at points that are multiples of a half-wavelength (180°).

The classical example is when the line is terminated with a load impedance Z_L that is not the characteristic impedance. Some of the incident energy may be absorbed by the load and the rest is reflected. The maximum and minimum values of the standing wave voltage and the positions of these maxima and minima are related to Z_L. The maximum occurs where the incident and reflected voltages are in phase, the minimum where they are 180° out of phase.

$$E_{\max} = |V_{incident}| + \left|V_{reflected}\right| \quad \text{and} \quad E_{\min} = |V_{incident}| - \left|V_{reflected}\right| \tag{1.20}$$

with $V_{incident}$ a constant and $V_{reflected}$ a function of Z_L, $\frac{E_{\max}}{E_{\min}}$ is the **Voltage Standing Wave Ratio**, abbreviated VSWR or SWR and is a way of describing the discontinuity at the plane of the load. The SWR is 1 when the load termination is equal to the characteristic impedance of the line, since $V_{reflected} = 0$, and infinite when a lossless reflective termination (short circuit, open circuit, capacitance, etc.) is connected, since $V_{reflected} = V_{incident}$ in that case. SWR is commonly used as a specification for components, most commonly loads and attenuators.

For a finite Z_L, the magnitude and phase of the reflected signal depends on the ratio of Z_L/Z_0. Since the total voltage (and current) across Z_L is the vector sum of the incident and reflected voltages (and currents) we have

$$Z_L = \frac{V_L}{I_L} = \frac{V_{incident} + V_{reflected}}{I_{incident} + I_{reflected}}. \tag{1.21}$$

The voltage and current in each of the waves on the transmission line are related by the characteristic impedance, as already shown in (1.9) and (1.10)

$$\frac{V_{incident}}{I_{incident}} = Z_0 \quad \text{and} \quad \frac{V_{reflected}}{I_{reflected}} = -Z_0 \tag{1.22}$$

so

$$Z_L = \frac{V_{incident} + V_{reflected}}{\frac{V_{incident}}{Z_0} - \frac{V_{refelected}}{Z_0}} = Z_0 \frac{1 + \frac{V_{reflected}}{V_{incident}}}{1 - \frac{V_{reflected}}{V_{incident}}} = Z_0 \frac{1+\Gamma}{1-\Gamma} \tag{1.23}$$

where Γ is the **reflection coefficient**, a complex value with magnitude and phase. The magnitude of Γ is usually denoted by the symbol ρ and its phase by θ. The values of ρ vary from zero to one. It is common practice to refer to the magnitude of the reflection coefficient as the **return loss** ($20\log_{10}\rho$).

Note that ρ, the magnitude of Γ, remains constant as the observation point is moved along a lossless transmission line. In this case, the phase θ changes and thus the complex value of Γ rotates around a circle on a polar plot. Since, at the plane of the load

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{1.24}$$

the value of the impedance seen looking into the transmission line at any point is readily calculated by rotating Γ by the electrical length (a function of the signal frequency, $360° = \lambda/2$) between the plane of the load and the point of observation. Thus, for example, at a quarter-wavelength distance (180° electrical length) from the plane of a short circuit, the impedance appears as an open circuit. The same impedance repeats at multiples of a half-wavelength.

1.2.4 Power transfer to load

The maximum power transfer from sources with source impedance of R_s to load impedance of R_L occurs at the value of R_s equal to R_L. For complex impedances, the maximum power transfer occurs when $\mathrm{Z}_L = \mathrm{R}_L + \mathrm{jX}_L$, $\mathrm{Z}_s = \mathrm{R}_s - \mathrm{jX}_s$ and $\mathrm{R}_s = \mathrm{R}_L$, and $\mathrm{X}_L = \mathrm{X}_s$, otherwise there will be a mismatch and standing wave ratio.

1.3 Scattering parameters

A key assumption when making measurements is that networks can be completely characterized by quantities measured at the network terminals (ports) regardless of the contents of the networks. Once the parameters of a (linear) n-port network have been determined, its behavior in any external environment can be predicted.

At low frequencies, typical choices of network parameters to be measured and handled are Z-parameters or Y-parameters, i.e. the impedance or admittance matrix, respectively. In microwave design, S-parameters are the natural choice because they are easier to measure and work with at high frequencies than other kinds of parameters. They are conceptually simple, analytically convenient, and capable of providing a great insight into a measurement or design problem.

Similarly to when light interacts with a lens, and a part of the light incident is reflected while the rest is transmitted, scattering parameters are measures of reflection and transmission of voltage waves through an electrical network.

Let us now focus on the generic n-port network, shown in Figure 1.5

To characterize the performance of such a network, as we said, any of several parameter sets can be used, each of which has certain advantages. Each parameter set is related to a set of $2n$ variables associated with the n-port model. Of these variables, n represents the excitation of the network (independent variables), and the remaining n represents the response of the network to the excitation (dependent variables). The network of Figure 1.5, assuming it has a linear behavior, can be represented by its

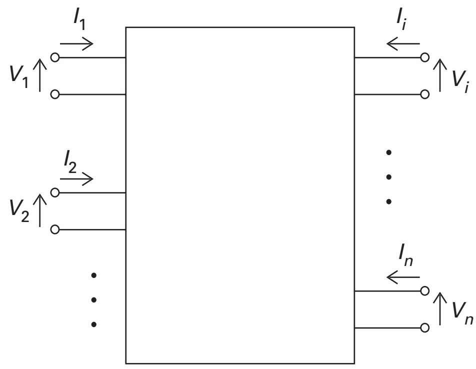

Fig. 1.5 Generic n-port network.

Z-matrix (impedance matrix):

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}, \tag{1.25}$$

where V_1-V_n are the node voltages and I_1-I_n are the node currents. Alternatively, one can use the dual representation:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}. \tag{1.26}$$

Here, port voltages are the independent variables and port currents are the dependent variables; the relating parameters are the short-circuit admittance parameters, or Y-parameters. In the absence of additional information, n^2 measurements are required to determine the n^2 Y-parameter. Each measurement is made with one port of the network excited by a voltage source while all the other ports are short-circuited. For example, Y_{21}, the forward trans-admittance, is the ratio of the current at port 2 to the voltage at port 1, when all other ports are short-circuited:

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=\ldots=V_n=0}. \tag{1.27}$$

If other independent and dependent variables had been chosen, the network would have been described, as before, by n linear equations similar to (1.24), except that the variables and the parameters describing their relationships would be different. However, all parameter sets contain the same information about a network, and it is always possible to calculate any set in terms of any other set [1,2].

"Scattering parameters," which are commonly referred to as S-parameters, are a parameter set that relates to the traveling waves that are scattered or reflected when an n-port network is inserted into a transmission line.

Scattering parameters were first defined by Kurokawa [3], where the assumption was to have real and positive reference impedances Z_i. For complex reference impedances, Marks and Williams [4] addressed the general case in 1992 and gave a comprehensive solution to it. They describe the interrelationships of a new set of variables, the ***pseudo-waves*** a_i, b_i, which are the normalized complex voltage waves incident on and reflected from the ith port of the network, defined as:

$$a_i = \alpha\sqrt{\Re\{Z_i\}}\frac{V_i + Z_i I_i}{2|Z_i|}.$$

$$b_i = \alpha\sqrt{\Re\{Z_i\}}\frac{V_i - Z_i I_i}{2|Z_i|}. \tag{1.28}$$

where voltage V_i and I_i are the terminal voltages and currents, Z_i are arbitrary (complex) reference impedances and α is a free parameter whose only constraint is to have unitary modulus, from now on assumed to be 1.

The linear equations describing the n-port network are therefore:

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \tag{1.29}$$

where by definition

$$S_{ij} = \left.\frac{b_j}{a_i}\right|_{a_2=\ldots=a_n=0}. \tag{1.30}$$

Note that in principle each port can use a different reference Z_i, and they need not be related to any physical characteristic impedance.

The ease with which scattering parameters can be measured makes them especially well suited for describing transistors and other active devices. Measuring most other parameters calls for the input and output of the device to be successively opened and short-circuited. This can be hard to do, especially at RF frequencies where lead inductance and capacitance make short and open circuits difficult to obtain. At higher frequencies these measurements typically require tuning stubs, separately adjusted at each measurement frequency, to reflect short or open circuit conditions to the device terminals. Not only is this inconvenient and tedious, but a tuning stub shunting the input or output may cause a transistor to oscillate, making the measurement invalid.

S-parameters, on the other hand, are usually measured with the device embedded between a matched load and source, and there is very little chance for oscillations to occur. Another important advantage of S-parameters stems from the fact that traveling waves, unlike terminal voltages and currents, do not vary in magnitude at points along a lossless transmission line. This means that scattering parameters can be measured on