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978-1-107-03577-5 - Stochastic Networks
Frank Kelly and Elena Yudovina
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Stochastic Networks

Communication networks underpin our modern world, and provide fascinating and challenging examples of large-scale stochastic systems. Randomness arises in communication systems at many levels: for example, the initiation and termination times of calls in a telephone network, or the statistical structure of the arrival streams of packets at routers in the Internet. How can routing, flow control and connection acceptance algorithms be designed to work well in uncertain and random environments?

This compact introduction illustrates how stochastic models can be used to shed light on important issues in the design and control of communication networks. It will appeal to readers with a mathematical background wishing to understand this important area of application, and to those with an engineering background who want to grasp the underlying mathematical theory. Each chapter ends with exercises and suggestions for further reading.

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Preface

Communication networks underpin our modern world, and provide fascinating and challenging examples of large-scale stochastic systems. Randomness arises in communication systems at many levels: for example, the initiation and termination times of calls in a telephone network, or the statistical structure of the arrival streams of packets at routers in the Internet. How can routing, flow control, and connection acceptance algorithms be designed to work well in uncertain and random environments? And can we design these algorithms using simple local rules so that they produce coherent and purposeful behaviour at the macroscopic level?

The first two parts of the book will describe a variety of classical models that can be used to help understand the behaviour of large-scale stochastic networks. Queueing and loss networks will be studied, as well as random access schemes and the concept of an effective bandwidth. Parallels will be drawn with models from physics, and with models of traffic in road networks.

The third part of the book will study more recently developed models of packet traffic and of congestion control algorithms in the Internet. This is an area of some practical importance, with network operators, content providers, hardware and software vendors, and regulators actively seeking ways of delivering new services reliably and effectively. The complex interplay between end-systems and the network has attracted the attention of economists as well as mathematicians and engineers.

We describe enough of the technological background to communication networks to motivate our models, but no more. Some of the ideas described in the book are finding application in financial, energy, and economic networks as computing and communication technologies transform these areas. But communication networks currently provide the richest and best-developed area of application within which to present a connected account of the ideas.

The lecture notes that have become this book were used for a Mas-

ters course (“Part III”) in the Faculty of Mathematics at the University of Cambridge. This is a one-year postgraduate course which assumes a mathematically mature audience, albeit from a variety of different mathematical backgrounds. Familiarity with undergraduate courses on Optimization and Markov Chains is helpful, but not absolutely necessary. Appendices are provided on continuous time Markov processes, Little’s law, Lagrange multipliers, and Foster–Lyapunov criteria, reviewing the material needed. At Cambridge, students may attend other courses where topics touched on in these notes, for example Poisson point processes, large deviations, or game theory, are treated in more depth, and this course can serve as additional motivation.

Suggestions on further reading are given at the end of each chapter; these include reviews where the historical development and attribution of results can be found – we have not attempted this here – as well as some recent papers which give current research directions.

The authors are grateful to students of the course at Cambridge (and, for one year, Stanford) for their questions and insights, and especially to Joe Hurd, Damon Wischik, Gaurav Raina, and Neil Walton, who produced earlier versions of these notes.

Frank Kelly and Elena Yudovina