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978-1-107-03540-9 - Stability Regions of Nonlinear Dynamical Systems: Theory, Estimation, and Applications

Hsiao-Dong Chiang and Luís F. C. Alberto

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Stability Regions of Nonlinear Dynamical Systems

Theory, Estimation, and Applications

This authoritative treatment covers theory, optimal estimation, and a range of practical applications.

The first book on the subject, written by leading researchers, this clear and rigorous work presents a comprehensive theory for both the stability boundary and the stability regions of a range of nonlinear dynamical systems, including continuous, discrete, complex, two-time-scale, and non-hyperbolic systems, illustrated with numerical examples. The authors also propose the new concepts of quasi-stability regions and relevant stability regions and their complete characterizations.

Optimal schemes for estimating stability regions of general nonlinear dynamical systems are also covered, and finally the authors describe and explain how the theory is applied in areas including direct methods for power system transient stability analysis, nonlinear optimization for finding a set of high-quality optimal solutions, stabilization of nonlinear systems, ecosystem dynamics, and immunization problems.

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Preface

There are several books available dedicated to the broad subject of stability of nonlinear dynamical systems. There are, however, just a few books available covering the important subject of stability regions (or regions of attraction or domains of attraction) of nonlinear systems. This subject is usually covered in books on nonlinear systems at an introductory level and in a fragmented manner. The motivation for writing this book was prompted by the critical need to treat the subject of stability regions in a clear, concise and comprehensive way for a wide range of audiences.

Knowledge of stability regions is fundamental in general nonlinear dynamical systems. It is of equal, if not more, importance to the notion of stability in nonlinear systems. The problem of determining stability regions of nonlinear dynamical systems plays an important role in many disciplines in engineering and the sciences. For instance, this problem appears in the areas of direct methods for power system stability analysis, stabilization of nonlinear systems, design of manipulators in robotics, design of nonlinear controllers, analysis and synthesis of power electronics, associative memory in artificial neural networks, solution methods for nonlinear optimization problems, ecosystem dynamics, immunization problems, economics and so forth. Hence, there is a broad range of professionals who can benefit from knowledge of current developments in the area of stability regions.

Currently, the practice of studying the stability of an equilibrium point or operating state is insufficient for the characterization of nonlinear dynamical systems since stable operating points of physical systems are rarely globally stable. In other words, the stability region is usually only a subset of the state space, and determination of this region is essential in many applications. In fact, the problem of determining the stability region is old and yet so difficult that breakthroughs in the complete characterization of stability regions of nonlinear dynamical systems were achieved only in the last twenty-five years. The development of the theory of stability boundaries and its applications has reached a level of sufficient maturity that the publication of a book exclusively dedicated to this important subject is justified.

The main objective of this book is to provide a comprehensive treatment of the theory of stability regions of nonlinear dynamical systems and effective and yet scalable methods for estimating stability regions. The book is divided into four parts. The first part devotes nine chapters to the theoretical treatment of stability regions. The theory of the stability region is developed for several classes of nonlinear dynamical systems, including continuous, discrete-time, complex and constrained nonlinear dynamical

systems. The second part is devoted to the development of effective, yet scalable, numerical methods for estimating stability regions. This part is composed of six chapters, which contain methods for optimally estimating stability regions for continuous, discrete-time, complex and constrained nonlinear dynamical systems. Methods for estimating relevant stability boundaries are also presented. It is also shown that by exploring the special structure of the study system model, improved and more powerful results can be obtained. Some developments in this direction are presented in this part, in which the structure of second-order systems is explored to obtain more powerful results on the structure of the stability boundary and to develop more efficient methods for estimating stability regions.

It is our belief that sound theoretical development should be combined with practical implementation and application to solve real-world problems. The fourth part of this book presents two practical applications of stability region theory and estimation methods to real-world problems. The on-line direct stability analysis of large-scale power grids is presented in Chapter 20. The theory of stability regions plays a key role in this practical application. The development of stability-regions-based methodologies to overcome the drawbacks of iterative local search methods and that of modern heuristics in the search for the global optimum is another application presented in Chapter 21. The new paradigm for solving nonlinear optimization problems has several distinguishing features achieved by transforming the search space into a state space composed of the union of stability regions.

In summary, this book offers, in a rigorous manner, a comprehensive theory for both the stability boundary and the stability regions of general nonlinear dynamical systems. Several topological and dynamical characterizations of the stability boundaries for large classes of nonlinear dynamical systems, including both continuous and discrete-time dynamical systems, are presented. Several effective and scalable numerical methods for optimally estimating stability regions, along with their practical applications, are presented to illustrate the theoretical developments presented in the book.

We hope that this book can contribute to inspire others to apply the theory and estimation methods of stability regions to relevant problems arising in the sciences and in engineering and to promote further developments in this fascinating area of research.

Acknowledgements

We started to work on the general subject of stability regions of nonlinear dynamical systems as graduate students at the University of California, Berkeley, in the mid 1980s and at the University of São Paulo, São Carlos, Brazil, in the mid 1990s, respectively. The need to develop fast on-line direct methods for power system stability analysis re-triggered the development of the theory of stability regions in the 1980s, and we were lucky to work on this exciting subject and to contribute to this area with theoretical and numerical developments, and practical applications.

We acknowledge our research advisors, Professor Felix Wu (EECS) and Professor Morris Hirsch (Mathematics), from University of California at Berkeley, and Professor Newton G. Bretas, from University of São Paulo, for introducing us to the world of stability regions. We carry the importance of their advice with us to this day. We also acknowledge those who patiently taught us about nonlinear systems, nonlinear circuits and dynamical systems in the beginning of our careers as researchers. Our thanks go to Professor Morris Hirsch, Professor Shankar Sastry, Professor Pravin Varaiya and Professor Leon Chua of University of California at Berkeley and Professor Hildebrando M. Rodrigues of University of São Paulo.

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