Cambridge University Press
978-1-107-03534-8 - Singularities of the Minimal Model Program
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# Singularities of the Minimal Model Program

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107035348



First published 2013

Printed and bound in the United Kingdom by the MPG Books Group

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*
Kollár, János.
Singularities of the minimal model program / János Kollár, Princeton University ; with the
collaboration of Sándor Kovács, University of Washington.
pages cm. – (Cambridge tracts in mathematics ; 200)
Includes bibliographical references and index.
ISBN 978-1-107-03534-8
1. Singularities (Mathematics) 2. Algebraic spaces. I. Kovács, Sándor J. (Sándor József)
II. Title.
QA614.58.K685 2013
516.3′5 – dc23 2012043204

ISBN 978-1-107-03534-8 Hardback

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# Preface

In 1982 Shigefumi Mori outlined a plan – now called *Mori's program* or the *minimal model program* – whose aim is to investigate geometric and cohomological questions on algebraic varieties by constructing a birational model especially suited to the study of the particular question at hand.

The theory of minimal models of surfaces, developed by Castelnuovo and Enriques around 1900, is a special case of the 2-dimensional version of this plan. One reason that the higher dimensional theory took so long in coming is that, while the minimal model of a smooth surface is another smooth surface, a minimal model of a smooth higher dimensional variety is usually a *singular* variety. It took about a decade for algebraic geometers to understand the singularities that appear and their basic properties. Rather complete descriptions were developed in dimension 3 by Mori and Reid and some fundamental questions were solved in all dimensions.

While studying the compactification of the moduli space of smooth surfaces, Kollár and Shepherd-Barron were also led to the same classes of singularities.

At the same time, Demailly and Siu were exploring the role of singular metrics in complex differential geometry, and identified essentially the same types of singularities as the optimal setting.

The aim of this book is to give a detailed treatment of the singularities that appear in these theories.

We started writing this book in 1993, during the 3rd Salt Lake City summer school on Higher Dimensional Birational Geometry. The school was devoted to moduli problems, but it soon became clear that the existing literature did not adequately cover many properties of these singularities that are necessary for a good theory of moduli for varieties of general type. A few sections were written and have been in limited circulation, but the project ended up in limbo.

The main results on terminal, canonical and log terminal singularities were treated in Kollár and Mori (1998) and for many purposes of Mori's original program these are the important ones.

There have been attempts to revive the project, most notably an AIM conference in 2004, but real progress did not restart until 2008. At that time several long-standing problems were solved and it also became evident that for many problems, including the abundance conjecture, a detailed understanding of log canonical and semi-log canonical singularities and pairs is necessary. In retrospect we see that many of the necessary techniques have not been developed until recently, so the earlier efforts were rather premature.

Although the study of these singularities started only 30 years ago, the theory has already outgrown the confines of a single monograph. Thus many of the important developments could not be covered in detail. Our aim is to focus on the topics that are important for moduli theory. Many other areas are developing rapidly and deserve a treatment of their own.

Sections 6.1, 8.4, 8.5 and 10.6 were written by SK. Sections 2.5, 6.2 and the final editing were done collaboratively.

*Acknowledgments* Throughout the years many of our colleagues and students listened to our lectures or read early versions of the manuscript; we received especially useful comments from A. Chiecchio, L. Erickson, O. Fujino, K. Fujita, S. Grushevsky, C. Hacon, A.-S. Kaloghiros, D. Kim, M. Lieblich, W. Liu, Y.-H. Liu, S. Rollenske, B. Totaro, C. Xu, R. Zong and M. Zowislok.

Much of the basic theory we learned from Y. Kawamata, Y. Miyaoka, S. Mori, M. Reid and V. Shokurov.

Conversations with our colleagues D. Abramovich, V. Alexeev, F. Ambro, F. Bogomolov, S. Casalaina-Martin, H. Clemens, A. Corti, J.-P. Demailly, T. de Fernex, O. Fujino, Y. Gongyo, D. Greb, R. Guralnick, L. Ein, P. Hacking, C. Hacon, B. Hassett, S. Ishii, M. Kapovich, N. Katz, M. Kawakita, J. McKernan, R. Lazarsfeld, J. Lipman, S. Mukai, M. Mustaţă, M. Olsson, Zs. Patakfalvi, M. Popa, C. Raicu, K. Schwede, S. Sierra, Y.-T. Siu, C. Skinner, K. Smith, M. Temkin, Y. Tschinkel, K. Tucker, C. Voisin, J. Wahl and J. Włodarczyk helped to clarify many of the issues.

Partial financial support to JK was provided by the NSF under grant number DMS-07-58275. Partial financial support for SK was provided by the NSF under grant number DMS-0856185 and the Craig McKibben and Sarah Merner Endowed Professorship in Mathematics at the University of Washington. Portions of this book were written while the authors enjoyed the hospitality of RIMS, Kyoto University.