

Cambridge University Press
978-1-107-03534-8 - Singularities of the Minimal Model Program
János Kollár
Frontmatter
[More information](#)

CAMBRIDGE TRACTS IN MATHEMATICS

General Editors

B. BOLLOBÁS, W. FULTON, A. KATOK, F. KIRWAN,
P. SARNAK, B. SIMON, B. TOTARO

200 Singularities of the Minimal Model Program

Cambridge University Press

978-1-107-03534-8 - Singularities of the Minimal Model Program

János Kollár

Frontmatter

[More information](#)

CAMBRIDGE TRACTS IN MATHEMATICS

GENERAL EDITORS

B. BOLLOBÁS, W. FULTON, A. KATOK, F. KIRWAN, P. SARNAK,
B. SIMON, B. TOTAROA complete list of books in the series can be found at www.cambridge.org/mathematics.

Recent titles include the following:

167. Poincaré Duality Algebras, Macaulay's Dual Systems, and Steenrod Operations. By D. MEYER and L. SMITH
168. The Cube-A Window to Convex and Discrete Geometry. By C. ZONG
169. Quantum Stochastic Processes and Noncommutative Geometry. By K. B. SINHA and D. GOSWAMI
170. Polynomials and Vanishing Cycles. By M. TIBĀR
171. Orbifolds and Stringy Topology. By A. ADEM, J. LEIDA, and Y. RUAN
172. Rigid Cohomology. By B. LE STUM
173. Enumeration of Finite Groups. By S. R. BLACKBURN, P. M. NEUMANN, and G. VENKATARAMAN
174. Forcing Idealized. By J. ZAPLETAL
175. The Large Sieve and its Applications. By E. KOWALSKI
176. The Monster Group and Majorana Involutions. By A. A. IVANOV
177. A Higher-Dimensional Sieve Method. By H. G. DIAMOND, H. HALBERSTAM, and W. F. GALWAY
178. Analysis in Positive Characteristic. By A. N. KOCHUBEI
179. Dynamics of Linear Operators. By F. BAYART and É. MATHERON
180. Synthetic Geometry of Manifolds. By A. KOCK
181. Totally Positive Matrices. By A. PINKUS
182. Nonlinear Markov Processes and Kinetic Equations. By V. N. KOLOKOLTSOV
183. Period Domains over Finite and p -adic Fields. By J.-F. DAT, S. ORLIK, and M. RAPOPORT
184. Algebraic Theories. By J. ADÁMEK, J. ROSICKÝ, and E. M. VITALE
185. Rigidity in Higher Rank Abelian Group Actions I: Introduction and Cocycle Problem. By A. KATOK and V. NIȚIȚĂ
186. Dimensions, Embeddings, and Attractors. By J. C. ROBINSON
187. Convexity: An Analytic Viewpoint. By B. SIMON
188. Modern Approaches to the Invariant Subspace Problem. By I. CHALENDAR and J. R. PARTINGTON
189. Nonlinear Perron–Frobenius Theory. By B. LEMMENS and R. NUSSBAUM
190. Jordan Structures in Geometry and Analysis. By C.-H. CHU
191. Malliavin Calculus for Lévy Processes and Infinite-Dimensional Brownian Motion. By H. OSSWALD
192. Normal Approximations with Malliavin Calculus. By I. NOURDIN and G. PECCATI
193. Distribution Modulo One and Diophantine Approximation. By Y. BUGEAUD
194. Mathematics of Two-Dimensional Turbulence. By S. KUKSIN and A. SHIRIKYAN
195. A Universal Construction for R -free Groups. By I. CHISWELL and T. MÜLLER
196. The Theory of Hardy's Z -Function. By A. IVIĆ
197. Induced Representations of Locally Compact Groups. By E. KANIUTH and K. F. TAYLOR
198. Topics in Critical Point Theory. By K. PERERA and M. SCHECHTER
199. Combinatorics of Minuscule Representations. By R. M. GREEN
200. Singularities of the Minimal Model Program. By J. KOLLÁR
201. Coherence in Three-Dimensional Category Theory. By N. GURSKI

Cambridge University Press
978-1-107-03534-8 - Singularities of the Minimal Model Program
János Kollár
Frontmatter
[More information](#)

Singularities of the Minimal Model Program

JÁNOS KOLLÁR
Princeton University

with the collaboration of
SÁNDOR KOVÁCS
University of Washington



Cambridge University Press
 978-1-107-03534-8 - Singularities of the Minimal Model Program
 János Kollár
 Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
 Cambridge, New York, Melbourne, Madrid, Cape Town,
 Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
 The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
 Information on this title: www.cambridge.org/9781107035348

© János Kollár 2013

This publication is in copyright. Subject to statutory exception
 and to the provisions of relevant collective licensing agreements,
 no reproduction of any part may take place without the written
 permission of Cambridge University Press.

First published 2013

Printed and bound in the United Kingdom by the MPG Books Group

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
 Kollár, János.

Singularities of the minimal model program / János Kollár, Princeton University ; with the
 collaboration of Sándor Kovács, University of Washington.
 pages cm. – (Cambridge tracts in mathematics ; 200)

Includes bibliographical references and index.

ISBN 978-1-107-03534-8

1. Singularities (Mathematics) 2. Algebraic spaces. I. Kovács, Sándor J. (Sándor József)
 II. Title.

QA614.58.K685 2013

516.3'5 – dc23 2012043204

ISBN 978-1-107-03534-8 Hardback

Cambridge University Press has no responsibility for the persistence or
 accuracy of URLs for external or third-party internet websites referred to
 in this publication, and does not guarantee that any content on such
 websites is, or will remain, accurate or appropriate.

Contents

<i>Preface</i>	<i>page ix</i>
Introduction	1
1 Preliminaries	4
1.1 Notation and conventions	4
1.2 Minimal and canonical models	13
1.3 Canonical models of pairs	16
1.4 Canonical models as partial resolutions	27
1.5 Some special singularities	33
2 Canonical and log canonical singularities	37
2.1 (Log) canonical and (log) terminal singularities	38
2.2 Log canonical surface singularities	53
2.3 Ramified covers	63
2.4 Log terminal 3-fold singularities	72
2.5 Rational pairs	77
3 Examples	93
3.1 First examples: cones	94
3.2 Quotient singularities	101
3.3 Classification of log canonical surface singularities	108
3.4 More examples	129
3.5 Perturbations and deformations	143
4 Adjunction and residues	150
4.1 Adjunction for divisors	152
4.2 Log canonical centers on dlt pairs	163
4.3 Log canonical centers on lc pairs	167

4.4	Crepanant log structures	172
4.5	Sources and springs of log canonical centers	182
5	Semi-log canonical pairs	187
5.1	Demi-normal schemes	188
5.2	Statement of the main theorems	193
5.3	Semi-log canonical surfaces	196
5.4	Semi-divisorial log terminal pairs	200
5.5	Log canonical stratifications	205
5.6	Gluing relations and sources	209
5.7	Descending the canonical bundle	211
6	Du Bois property	214
6.1	Du Bois singularities	215
6.2	Semi-log canonical singularities are Du Bois	229
7	Log centers and depth	232
7.1	Log centers	232
7.2	Minimal log discrepancy functions	239
7.3	Depth of sheaves on slc pairs	242
8	Survey of further results and applications	248
8.1	Ideal sheaves and plurisubharmonic functions	248
8.2	Log canonical thresholds and the ACC conjecture	251
8.3	Arc spaces of log canonical singularities	253
8.4	F -regular and F -pure singularities	254
8.5	Differential forms on log canonical pairs	256
8.6	The topology of log canonical singularities	259
8.7	Abundance conjecture	261
8.8	Moduli spaces for varieties	262
8.9	Applications of log canonical pairs	263
9	Finite equivalence relations	266
9.1	Quotients by finite equivalence relations	266
9.2	Descending seminormality of subschemes	285
9.3	Descending line bundles to geometric quotients	287
9.4	Pro-finite equivalence relations	292
10	Ancillary results	297
10.1	Birational maps of 2-dimensional schemes	297
10.2	Seminormality	306
10.3	Vanishing theorems	317

Cambridge University Press
978-1-107-03534-8 - Singularities of the Minimal Model Program
János Kollár
Frontmatter
[More information](#)

	<i>Contents</i>	vii
10.4	Semi-log resolutions	324
10.5	Pluricanonical representations	334
10.6	Cubic hyperresolutions	340
	<i>References</i>	348
	<i>Index</i>	363

Cambridge University Press

978-1-107-03534-8 - Singularities of the Minimal Model Program

János Kollár

Frontmatter

[More information](#)

Preface

In 1982 Shigefumi Mori outlined a plan – now called *Mori's program* or the *minimal model program* – whose aim is to investigate geometric and cohomological questions on algebraic varieties by constructing a birational model especially suited to the study of the particular question at hand.

The theory of minimal models of surfaces, developed by Castelnuovo and Enriques around 1900, is a special case of the 2-dimensional version of this plan. One reason that the higher dimensional theory took so long in coming is that, while the minimal model of a smooth surface is another smooth surface, a minimal model of a smooth higher dimensional variety is usually a *singular* variety. It took about a decade for algebraic geometers to understand the singularities that appear and their basic properties. Rather complete descriptions were developed in dimension 3 by Mori and Reid and some fundamental questions were solved in all dimensions.

While studying the compactification of the moduli space of smooth surfaces, Kollár and Shepherd-Barron were also led to the same classes of singularities.

At the same time, Demailly and Siu were exploring the role of singular metrics in complex differential geometry, and identified essentially the same types of singularities as the optimal setting.

The aim of this book is to give a detailed treatment of the singularities that appear in these theories.

We started writing this book in 1993, during the 3rd Salt Lake City summer school on Higher Dimensional Birational Geometry. The school was devoted to moduli problems, but it soon became clear that the existing literature did not adequately cover many properties of these singularities that are necessary for a good theory of moduli for varieties of general type. A few sections were written and have been in limited circulation, but the project ended up in limbo.

The main results on terminal, canonical and log terminal singularities were treated in Kollár and Mori (1998) and for many purposes of Mori's original program these are the important ones.

There have been attempts to revive the project, most notably an AIM conference in 2004, but real progress did not restart until 2008. At that time several long-standing problems were solved and it also became evident that for many problems, including the abundance conjecture, a detailed understanding of log canonical and semi-log canonical singularities and pairs is necessary. In retrospect we see that many of the necessary techniques have not been developed until recently, so the earlier efforts were rather premature.

Although the study of these singularities started only 30 years ago, the theory has already outgrown the confines of a single monograph. Thus many of the important developments could not be covered in detail. Our aim is to focus on the topics that are important for moduli theory. Many other areas are developing rapidly and deserve a treatment of their own.

Sections 6.1, 8.4, 8.5 and 10.6 were written by SK. Sections 2.5, 6.2 and the final editing were done collaboratively.

Acknowledgments Throughout the years many of our colleagues and students listened to our lectures or read early versions of the manuscript; we received especially useful comments from A. Chiecchio, L. Erickson, O. Fujino, K. Fujita, S. Grushevsky, C. Hacon, A.-S. Kaloghiros, D. Kim, M. Lieblich, W. Liu, Y.-H. Liu, S. Rollenske, B. Totaro, C. Xu, R. Zong and M. Zowislok.

Much of the basic theory we learned from Y. Kawamata, Y. Miyaoka, S. Mori, M. Reid and V. Shokurov.

Conversations with our colleagues D. Abramovich, V. Alexeev, F. Ambro, F. Bogomolov, S. Casalaina-Martin, H. Clemens, A. Corti, J.-P. Demailly, T. de Fernex, O. Fujino, Y. Gongyo, D. Greb, R. Guralnick, L. Ein, P. Hacking, C. Hacon, B. Hassett, S. Ishii, M. Kapovich, N. Katz, M. Kawakita, J. McKernan, R. Lazarsfeld, J. Lipman, S. Mukai, M. Mustașă, M. Olsson, Zs. Patakfalvi, M. Popa, C. Raicu, K. Schwede, S. Sierra, Y.-T. Siu, C. Skinner, K. Smith, M. Temkin, Y. Tschinkel, K. Tucker, C. Voisin, J. Wahl and J. Włodarczyk helped to clarify many of the issues.

Partial financial support to JK was provided by the NSF under grant number DMS-07-58275. Partial financial support for SK was provided by the NSF under grant number DMS-0856185 and the Craig McKibben and Sarah Merner Endowed Professorship in Mathematics at the University of Washington. Portions of this book were written while the authors enjoyed the hospitality of RIMS, Kyoto University.