1

Introduction

1.1 Definition and examples of granular media

From sand to cereals, from rock avalanches to interplanetary aggregates like Saturn's rings and the asteroid belt (Fig. 1.1), granular media form an extremely vast family, composed of grains with very different shapes and materials, which can span several orders of magnitude in size. However, beyond this great diversity, all these particulate media share fundamental features. They are disordered at the grain level but behave like a solid or a fluid at the macroscopic level, exhibiting phenomena such as arching, avalanches and segregation.

In this book, we shall broadly define a granular medium as a collection of rigid[1] macroscopic particles, whose particle size is typically larger than 100 μm (Brown & Richards, 1970; Nedderman, 1992; Guyon & Troadec, 1994; Duran, 1997; Rao & Nott, 2008). As we shall see in Chapter 2, this limitation in size corresponds to a limitation in the type of interaction between the particles (Fig. 1.2). In this book, we will focus on non-Brownian particles that interact mainly by friction and collision. For smaller particles, of diameter between 1 μm and 100 μm, other interactions such as van der Waals forces, humidity and air drag start to play an important role as well. This is the domain of powders.[2] Finally, for even smaller particles, those of diameter below 1 μm, thermal agitation is no longer negligible. The world of colloids then begins (Russel *et al.*, 1989).

A central motivation for the study of granular media is their involvement in many industrial and natural phenomena. It has been estimated that more than 50% of sales in the world involve commodities produced using granular materials at some stage, which makes granular media the second most used type of material in

[1] Our definition a priori excludes very soft particles or those that fragment during the flow. However, an assembly of liquid drops, bubbles or soft beads can sometimes be treated as a granular medium, if the confining pressure is low enough for the particles not to deform.

[2] The terms 'granular material' and 'powder' usually apply to dry grains in the air. For particles immersed in a liquid, one speaks rather of 'wet granular media' or 'suspensions'.

Figure 1.1 Examples of granular media.

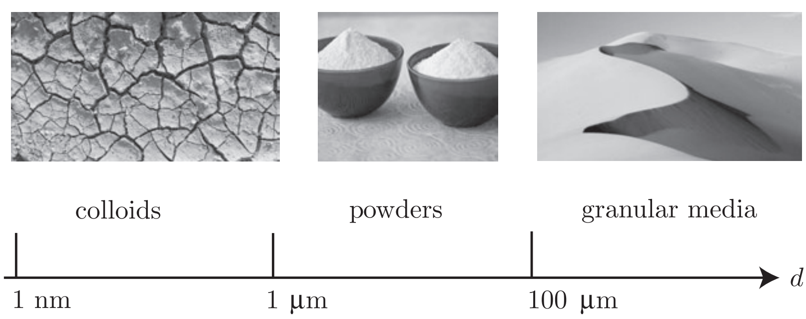

Figure 1.2 A classification of particulate matter as a function of the particle size: colloids (mud), powder (flour) and granular media (a sand dune).

industry after water (Duran, 1997; Bates, 2006). Major sectors handling granular materials include mining (extraction, transport, milling), civil engineering (concrete, bitumen, asphalt, embankments, ballast trains, soil stability), the chemical industry (fuel and catalysts are often deployed in the form of grains in order to maximize the surface of exchange), the pharmaceutical industry (from the handling

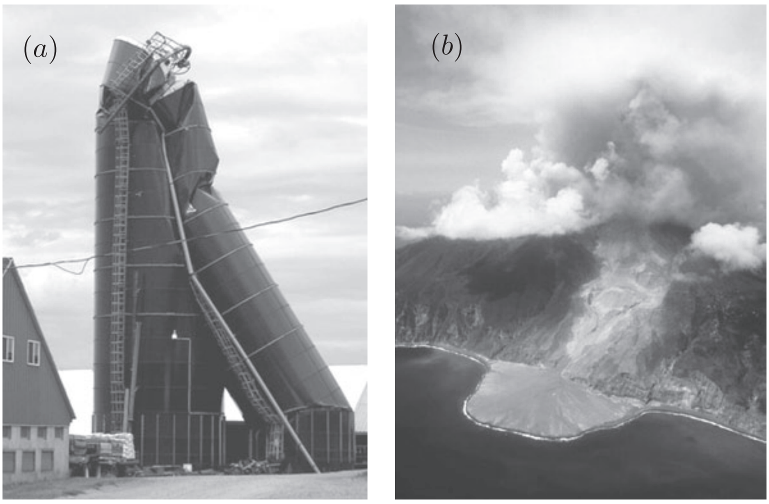

Figure 1.3 Granular media are involved in many industrial and geophysical applications. (*a*) The collapse of a silo. (*b*) Pyroclastic flow (Soufrière Hills volcano, Montserrat Island) (photograph from Steve O'Meara, Volcano Watch International).

of powders for the manufacture of medicine to the handling of drugs themselves), the food industry (cereals, animal food) and the glass industry (glass is made with sand), to name but a few. In all these areas, problems of storage (Fig. 1.3(*a*)), transportation, flow and mixing are often encountered, which are solved by engineers using empirical techniques.

The other major domain of application of granular materials concerns Earth science, our soil being mainly composed of grains. From sand dunes to landslides, from erosion patterns to pyroclastic flows (Fig. 1.3(*b*)), Nature offers some of the most spectacular examples of phenomena involving granular matter. Those phenomena are actually not limited to Earth. Dust and grains abound in space as well, as illustrated by Martian dunes, planetary rings and 'granular' asteroids (Fig. 1.1). Apart from their inherent beauty, all these natural phenomena play a key role in shaping our environment and therefore strongly interact with human activity. An important part of the research effort in granular media is thus devoted to the description and prediction of natural hazards such as avalanches, landslides, desertification and erosion of banks.

1.2 Between solid and liquid: what makes granular matter so difficult to describe?

Despite their industrial and geophysical applications, granular media still resist our understanding. No theoretical framework is available to describe the variety of behaviour observed, even in the ideal case of a medium consisting of identical spherical particles. This situation may appear paradoxical, more than a century after the great revolutions of modern physics! Actually, the behaviour of a single grain is governed by the laws of mechanics, which have not changed very much since Newton and Coulomb. Why then is the physics of a sand pile so complex? We can try to make a list of the main difficulties encountered when dealing with granular materials.

- *Granular media are composed of a large number of particles*. Let's consider a teaspoonfull of sugar. Assuming a grain diameter of 100 μm and a spoon volume of 1 cm^3, the number of sugar grains in the spoon can be estimated to be $(10^{-2}\ \text{m})^3/(10^{-4}\ \text{m})^3$, that is a million particles! This quantity is already close to the maximum number of particles that can be simulated with present-day computers. It therefore seems challenging to follow the movement of each individual grain in a much larger event such as a rock avalanche or the discharge of a silo. An alternative strategy is rather to define averaged quantities and to model the granular medium as a continuum. One of the main issues in the physics of granular matter is that of how to provide such a continuum description.
- *Thermal fluctuations are negligible*. Actually, a large number of particles is not necessarily a serious obstacle in physics. For example, gases and liquids are known to be well described on the macroscopic level by the Navier–Stokes equations, while the number of molecules in a glass of water or a gas bottle is much larger than the number of sand grains in an hourglass. However, the key difference is thermal agitation. In liquids or gases, thermal agitation enables molecules to rapidly reach a local equilibrium state where all possible configurations are visited according to their statistical weight, enabling the derivation of macroscopic quantities from microscopic ones. In contrast, in a granular medium, the particles are too large to experience Brownian motion and statistical averaging over different configurations is not possible. Granular media are thus athermal systems.[3] To illustrate this, let's compare the thermal energy and gravitational energy of a glass bead of diameter $d = 1$ mm and density $\rho_p = 2500$ kg m^{-3}, at room temperature, $T = 300$ K, and under gravity, $g = 9.8\ \text{m s}^{-2}$. The thermal energy is $E_{\text{th}} \sim k_B T = 4 \times 10^{-21}$ J, where $k_B = 1.38 \times 10^{-23}\ \text{J K}^{-1}$ is the Boltzmann constant. The gravitational energy corresponding to a vertical displacement of d is $E_p \sim mgd = 10^{-10}$ J. As expected, the thermal energy is completely negligible compared with the gravitational energy. One

[3] This does not mean that temperature is always irrelevant for granular media. At the contact scale between grains, ageing phenomena activated by temperature may occur (creep, capillary condensation, oxidation), which can affect the overall properties of the pile, such as the angle of avalanche or the electrical properties of the medium (see the box 'Electrical contact between grains and the Branly effect' in Chapter 2)

can estimate the size d_c below which thermal fluctuations play a role. Taking $T = 300$ K gives $d_c \sim [k_B T/(\rho_p g)]^{1/4} \simeq 1$ μm. This corresponds to the frontier between colloids and powders given in Fig. 1.2.

- *Lack of scale separation*. The continuum description of granular media is also made difficult by the lack of clear scale separation between the microscopic scale, i.e. the grain size, and the macroscopic scale, i.e. the size of the flow. Typically, when sand flows down on a pile, the flow thickness is about 10–20 particle diameters. Similarly, the breakdown of a granular soil is often localized in faults, or shear bands, of extent a few tens of grain widths. This lack of scale separation raises questions about the validity of the continuum approach and the definition of an elementary volume for the averaging process. In this sense, the physics of granular media shares similarities with nanofluidics and nanomechanics, where the effects of the size of the molecule start to play a role.
- *Interactions between grains are complex*. At the grain level, the laws of solid contact between two particles involve non-trivial and highly non-linear phenomena such as friction and inelastic shocks. When grains are further immersed in a viscous fluid, hydrodynamic interactions must also be taken into account. Those also exhibit peculiar features such as divergence at contact, due to lubrication forces, and long-range interaction between particles in viscous flows.
- *Granular media easily dissipate energy*. A bowling ball dropped into a sandbox does not bounce. All the kinetic energy is almost instantly dissipated by collision and friction between the grains of sand. This dissipation at the microscopic level is an important difference from the classical systems studied in statistical physics.
- *Granular media can exhibit different states of matter.* Depending on the way it is handled, a granular material can behave like a solid, a liquid or a gas (Fig. 1.4) (Jaeger *et al.*, 1996). Grains can sustain stresses and create a static pile, but can also flow like a liquid in an hourglass, or can create a gas when they are strongly agitated. These different flow regimes can also coexist in a single configuration, as illustrated by the flow of beads on a pile (Fig. 1.4).

This behaviour intermediate between that of a solid and that of a liquid is a fundamental characteristic of granular media, and is shared by other disordered materials such as foams, emulsions and pastes (Coussot & Ancey, 1999; Larson, 1999). In all these systems, the medium is composed of mesoscopic elements (bubbles, drops, particles), such that thermal fluctuation is negligible compared with the particles' interaction. Above a critical density, these elements jam and the medium behaves like a solid. To make these systems flow, one has to apply a yield stress or decrease the density. The attempt to unify within a single

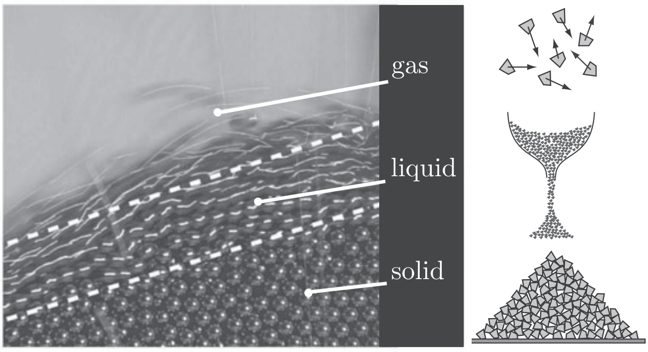

Figure 1.4 Granular media can behave like a solid, a liquid or a gas, depending on the situation.

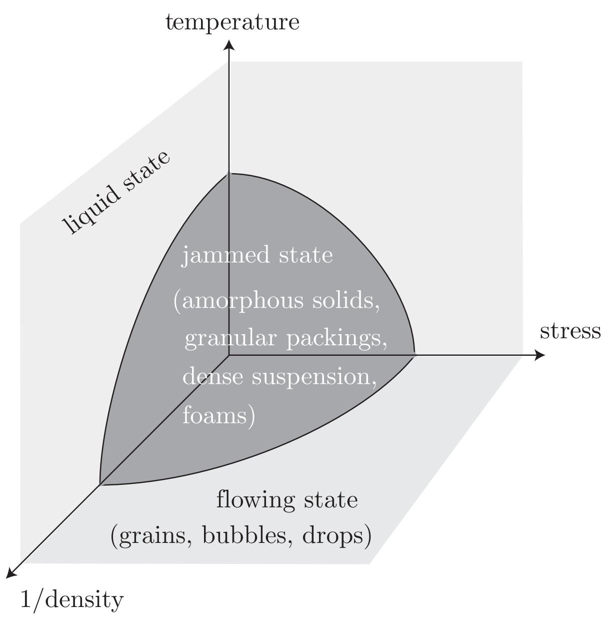

Figure 1.5 A hypothetical phase diagram for the jamming transition in disordered media proposed by Liu and Nagel (1998). The solid or 'jammed' state arises at low temperature (molecular glasses), low external stresses (foams, pastes) and high density (granular media, emulsion).

framework the physics of these jammed systems is the subject of much research. Some authors go further and notice that there are some similarities between jammed soft media and amorphous molecular solids (Liu & Nagel, 1998) (Fig. 1.5). By definition, amorphous solids such as glass and rubber contrast with crystalline solids in that they do not exhibit long-range translational order. As the temperature decreases, they do not crystallize but remain frozen in a disordered out-of-equilibrium state, called the glassy state, which does not correspond to a global energy minimum. This sharp slowing down of the dynamics and increase of viscosity is called the glass transition, and can be interpreted in terms of an energy landscape. For an amorphous system, the landscape is random and has many local minima. As the temperature decreases, temperature-activated jumps between the different potential wells become increasingly difficult and the system takes more and more time to change configuration. The system can then be trapped in a jammed state, pretty much like grains in a pile. This nice analogy between soft disordered media and glassy solids is the subject of much research (Berthier & Biroli, 2009; Ikeda *et al.*, 2012). However, this general issue is beyond the scope of this book.

1.3 A sketch of the book

The objective of this book is to provide an introduction to the different aspects of the physics of granular media, ranging from the solid behaviour of a sandpile to the flow of an avalanche and to very dilute media. In doing so, we could not present a fully comprehensive and detailed treatment of every aspect of each topic. This implies choices and a certain amount of subjectivity, which in the case of granular media is especially true in that research in this area is still very active and no unifying description yet exists. We hope that the many references throughout the book will enable the interested reader to familiarize himself or herself with the most recent work and deepen her or his understanding of specific topics.

In this book, we mainly focus on dry granular media, for which interactions between grains are dominated by solid contact – typically sand in air. We do not address the broad field of dilute or semi-dilute suspensions, for which interactions between particles mainly occur through hydrodynamic forces (Jackson, 2000; Guazzelli & Morris, 2012). However, the role of the surrounding fluid is crucial and will be discussed in the later chapters of the book, when dealing with natural phenomena such as saturated soils, submarine avalanches, sediment transport and erosion.

The book is organized as follows. We begin with a discussion of the interaction forces at the grain level, giving a brief overview of contact physics and hydrodynamic forces around solid bodies (Chapter 2). We then discuss the solid regime of granular media. Chapter 3 describes the statics of granular media (packing, force chains, stresses) and the elastic regime of small reversible deformations (elastic moduli, acoustics). Chapter 4 is devoted to the plasticity of granular media, when deformations are no longer reversible. This field is historically closely related to soil mechanics and is also central to our understanding of the solid–liquid transition in disordered athermal media. The second part of the book is devoted to granular flows. We begin with the case of rapid and dilute flows (the gas regime), which benefits from the most advanced framework with the kinetic theory of granular media (Chapter 5). We then shift to the flows of dense granular media, when particles flow like a liquid (Chapter 6). This regime is the most commonly observed in industry and geophysics but also the least well understood, despite significant advances. Chapter 7 discusses the role of the presence of an interstitial fluid between grains and is devoted to immersed granular media and very dense suspensions. These media occur in many geotechnical and geophysical applications, since soils are often saturated with water. The last part of the book is devoted to geophysical applications of the physics of granular media. Chapter 8 investigates sediment transport and erosion. Chapter 9 gives an introduction to dynamical geomorphology, describing gravity-driven flows and the formation of dunes and rivers.

The natural origin of granular media

A large part of the sediments comes from the breakdown and weathering of rocks by physical, chemical and biological action. The physical weathering is mainly due to the formation of fractures in rocks due to dilatation, related either to temperature variations or simply to the relaxation of the pre-stresses under which the rocks were formed (decompression). In humid climates, cyclic freezing and thawing expand fractures due to the variation of the volume of water. Plant roots can play the same role. Finally, rocks can be directly eroded by the flow of water or ice, or by the collisions of grains carried by the wind. Chemical weathering acts mainly in the presence of water and air. Some minerals (halite, calcite) dissolve completely, and ions are removed in solution. Other minerals such as micas and feldspars are transformed into other mineral species, which are often of finer size (clay) and more easily entrained by erosion. These reactions can be accelerated by biological action. Fermentation and respiration induce oxidation of organic matter that produces water and carbon dioxide, the latter being crucial for the reactions of dissolution (e.g. for calcite). Furthermore, microorganisms are able to dissolve minerals by acid reaction, releasing ions (especially metallic ions).

In addition to the production by weathering, sediments are produced by volcanic activity. Magma contains dissolved volatiles (predominantly water and carbon dioxide). Upon ascent-driven decompression, the solubility of these volatiles decreases, the magma becomes supersaturated in volatiles, and gas bubbles nucleate and grow. A magmatic foam is formed, which expands rapidly. During explosive eruptions, fragmentation then takes place, and the foam transforms into a suspension of ash, lava and coarser debris. These processes give rise to soil and rock debris.

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Excerpt
More information

Table 1.1 *The size classification of Wentworth (1922) used in geology*

Name		Size (mm)
	Boulders	≥ 256
	Cobbles	64–256
	Gravels	32–64
	Pebbles	4–32
	Granules	2–4
Sand	Very coarse sand	1–2
	Coarse sand	0.5–1
	Medium sand	0.25–0.5
	Fine sand	0.125–0.25
	Very fine sand	0.0625–0.125
Mud	Coarse silt	0.0312–0.0625
	Medium silt	0.0156–0.0312
	Fine silt	0.0078–0.0156
	Very fine silt	0.00390625–0.0078
	Clay	0.0001–0.00390625
	Colloid	<0.0001

Geologists use a different terminology for each class of grain sizes (see Table 1.1). This classification is more detailed than the physical classification given in the introduction, and reflects the degree of polydispersity of natural sediments. This polydispersity depends on whether or not a sorting process occurs during the sediment transport from active weathering zones to the sedimentation basins where they are deposited. Glacial moraines constituted by sediments eroded and transported by glaciers are highly polydisperse. Fluvial deposits of pebbles, sand and silt are in comparison slightly more sorted, and lacustrine and marine deposits even more so. Finally, aeolian deposits and especially sand dunes are constituted by nearly monodisperse grains (due to the so-called aeolian sieving).

The grain shape depends also on its origin. The grains of fluvial and glacial deposits are very sharp and reflect light. During transport, each impact leaves an asymmetric crescent-shaped trace. The grains of aeolian sand deposits are rounded, frosted and matte. They show traces of abrasion induced by collisions with the ground, which are characteristic of saltation.

The colour of the sand grains can be acquired during diagenesis. Reactions with iron oxides bring about a red colour. During the expulsion of fluids, the presence of organic matter depletes the environment in oxygen and brings about the formation of Fe^{2+} ions of green colour. The quartz grains of aeolian deposits can be coated with a red haematitic pigment. Since grain impacts erode this rust coating, a dune is more red when it is static and more white when it is mobile.

Granulometry

In this book, we will often model granular media as a collection of spherical particles, all having the same size to within a few per cent. However, materials found in industry and geophysics are in general much more complex. Granular media containing particles of different sizes and shapes are called polydisperse, in contrast with monodisperse media that contain only one type of grain. Granulometry is the measurement of the shape and size distribution of a large collection of polydisperse grains (Allen, 1996).

Particle size and shape

For a particle of simple geometry such as a glass bead, size is well defined and characterized by a single parameter, namely the diameter d of the sphere. However, for more complex particle shapes like that of a sand grain, the notion of size is less obvious. A common practice consists of defining an equivalent diameter of the object, corresponding to the diameter of an ideal object that has the same geometrical properties as the real one (e.g. volume, area). Additional parameters called shape factors are often defined in order to improve the description of the shape, such as the ellipticity (the length-to-width ratio) and the roundness (the ratio of the square of the perimeter to the projected area). In practice, the choice of these parameters is highly dependent on the method of measurement.

Distribution of particles

The particle size distribution of an assembly of N_{tot} grains is characterized by giving the number ΔN of particles having a diameter d within a range Δd. The probability of finding a particle with a diameter between d and $d + \Delta d$ is therefore $\Delta N/N_{\mathrm{tot}} = f(d)\Delta d$, where $f(d) = (1/N_{\mathrm{tot}})(\Delta N/\Delta d)$ is the (normalized) particle density distribution in number. Another useful representation is the cumulative frequency distribution in number given by $F(d) = \sum_{d'<d} f(d')\Delta d'$, which is the probability of finding a particle with a diameter smaller than d. Figure 1.6 shows these two kinds of representation in the case of sieved beach sand. From the distribution law, one can compute various quantities such as the most probable diameter, the mean diameter and the distribution width.

Measurement methods

Sieving is probably the oldest technique to sort grain, and is still in use today. The size distribution in mass is obtained by weighing the grains retained in a stack of sieves of different mesh sizes. The sieving technique is simple but not very precise, since small