Quantum Field Theory and the Standard Model

Providing a comprehensive introduction to quantum field theory, this textbook covers the development of particle physics from its foundations to the discovery of the Higgs boson. Its combination of clear physical explanations, with direct connections to experimental data, and mathematical rigor make the subject accessible to students with a wide variety of backgrounds and interests. Assuming only an undergraduate-level understanding of quantum mechanics, the book steadily develops the Standard Model and state-of-the art calculation techniques. It includes multiple derivations of many important results, with modern methods such as effective field theory and the renormalization group playing a prominent role. Numerous worked examples and end-of-chapter problems enable students to reproduce classic results and to master quantum field theory as it is used today. Based on a course taught by the author over many years, this book is ideal for an introductory to advanced quantum field theory sequence or for independent study.

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To my mother,
and to Carolyn, Eve and Alec

Contents

Preface

Quantum field theory (QFT) provides an extremely powerful set of computational methods that have yet to find any fundamental limitations. It has led to the most fantastic agreement between theoretical predictions and experimental data in the history of science. It provides deep and profound insights into the nature of our universe, and into the nature of other possible self-consistent universes. On the other hand, the subject is a mess. Its foundations are flimsy, it can be absurdly complicated, and it is most likely incomplete. There are often many ways to solve the same problem and sometimes none of them are particularly satisfying. This leaves a formidable challenge for the design and presentation of an introduction to the subject.

This book is based on a course I have been teaching at Harvard for a number of years. I like to start my first class by flipping the light switch and pointing out to the students that, despite their comprehensive understanding of classical and quantum physics, they still cannot explain what is happening. Where does the light come from? The emission and absorption of photons is a quantum process for which particle number is not conserved; it is an everyday phenomenon which cannot be explained without quantum field theory. I then proceed to explain (with fewer theatrics) what is essentially Chapter 1 of this book. As the course progresses, I continue to build up QFT, as it was built up historically, as the logical generalization of the quantum theory of creation and annihilation of photons to the quantum theory of creation and annihilation of any particle. This book is based on lecture notes for that class, plus additional material.

The main guiding principle of this book is that QFT is primarily a theory of physics, not of mathematics, and not of philosophy. QFT provides, first and foremost, a set of tools for performing practical calculations. These calculations take as input measured numbers and predict, sometimes to absurdly high accuracy, numbers that can be measured in other experiments. Whenever possible, I motivate and validate the methods we develop as explaining natural (or at least in principle observable) phenomena. Partly, this is because I think having tangible goals, such as explaining measured numbers, makes it easier for students to understand the material. Partly, it is because the connection to data has been critical in the historical development of QFT.

The historical connection between theory and experiment weaves through this entire book. The great success of the Dirac equation from 1928 was that it explained the magnetic dipole moment of the electron (Chapter 10). Measurements of the Lamb shift in the late 1940s helped vindicate the program of renormalization (Chapters 15 to 21). Measurements of inelastic electron–proton scattering experiments in the 1960s (Chapter 32) showed that QFT could also address the strong force. Ironically, this last triumph occurred only a few years after Geoffrey Chew famously wrote that QFT "is sterile with respect to strong interactions and that, like an old soldier, it is destined not to die but just to fade away." [Chew, 1961, p. 2]. Once asymptotic freedom (Chapter 26) and the renormalizability of the Standard Model (Chapter 21 and Part IV) were understood in the 1970s, it was clear that QFT was capable of precision calculations to match the precision experiments that were being performed. Our ability to perform such calculations has been steadily improving ever since, for example through increasingly sophisticated effective field theories (Chapters 22, 28, 31, 33, 35 and 36), renormalization group methods (Chapter 23 and onward), and on-shell approaches (Chapters 24 and 27). The agreement of QFT and the Standard Model with data over the past half century has been truly astounding.

Beyond the connection to experiment, I have tried to present QFT as a set of related tools guided by certain symmetry principles. For example, Lorentz invariance, the symmetry group associated with special relativity, plays an essential role. QFT is the theory of the creation and destruction of particles, which is possible due to the most famous equation of special relativity $E = mc^2$. Lorentz invariance guides the definition of particle (Chapter 8), is critical to the spin-statistics theorem (Chapter 12), and strongly constrains properties of the main objects of interest in this book: scattering or S-matrix elements (Chapter 6 and onward). On the other hand, QFT is useful in space-times for which Lorentz invariance is not an exact symmetry (such as our own universe, which since 1998 has been known to have a positive cosmological constant), and in non-relativistic settings, where Lorentz invariance is irrelevant. Thus, I am reluctant to present Lorentz invariance as an axiom of QFT (I personally feel that as QFT is a work in progress, an axiomatic approach is premature). Another important symmetry is unitarity, which implies that probabilities should add up to 1. Chapter 24 is entirely dedicated to the implications of unitarity, with reverberations throughout Parts IV and V. Unitarity is closely related to other appealing features of our description of fundamental physics, such as causality, locality, analyticity and the cluster decomposition principle. While unitarity and its avatars are persistent themes within the book, I am cautious of giving them too much of a primary role. For example, it is not clear how well cluster decomposition has been tested experimentally.

I very much believe that QFT is not a finished product, but rather a work in progress. It has developed historically, it continues to be simplified, clarified, expanded and applied through the hard work of physicists who see QFT from different angles. While I do present QFT in a more or less linear fashion, I attempt to provide multiple viewpoints whenever possible. For example, I derive the Feynman rules in five different ways: in classical field theory (Chapter 3), in old-fashioned perturbation theory (Chapter 4), through a Lagrangian approach (Chapter 7), through a Hamiltonian approach (also Chapter 7), and through the Feynman path integral (Chapter 14). While the path-integral derivation is the quickest, it is also the furthest removed from the type of perturbation theory to which the reader might already be familiar. The Lagrangian approach illustrates in a transparent way how tree-level diagrams are just classical field theory. The old-fashioned perturbation theory derivation connects immediately to perturbation theory in quantum mechanics, and motivates the distinct advantage of thinking off-shell, so that Lorentz invariance can be kept manifest at all stages of the calculation. On the other hand, there are some instances where an on-shell approach is advantageous (see Chapters 24 and 27).

Other examples of multiple derivations include the four explanations of the spin-statistics theorem I give in Chapter 12 (direct calculation, causality, stability and Lorentz invariance of the S-matrix), the three ways I prove the path integral and canonical formulations of quantum field theory equivalent in Chapter 14 (through the traditional Hamiltonian derivation, perturbatively through the Feynman rules, and non-perturbatively through the Schwinger–Dyson equations), and the three ways in which I derive effective actions in Chapter 33 (matching, with Schwinger proper time, and with Feynman path integrals). As different students learn in different ways, providing multiple derivations is one way in which I have tried to make QFT accessible to a wide audience.

This textbook is written assuming that the reader has a solid understanding of quantum mechanics, such as what would be covered in a year-long undergraduate class. I have found that students coming in generally do not know much classical field theory, and must relearn special relativity, so these topics are covered in Chapters 2 and 3. At Harvard, much of the material in this book is covered in three semesters. The first semester covers Chapters 1 to 22. Including both QED and renormalization in a single semester makes the coursework rather intense. On the other hand, from surveying the students, especially the ones who only have space for a single semester of QFT, I have found that they are universally glad that renormalization is covered. Chapter 22, on non-renormalizable theories, is a great place to end a semester. It provides a qualitative overview of the four forces in the Standard Model through the lens of renormalization and predictivity.

The course on which this textbook is based has a venerable history, dominated by the thirty or so years it was taught by the great physicist Sidney Coleman. Sidney provides an evocative description of the period from 1966 to 1979 when theory and experiment collaborated to firmly establish the Standard Model [Coleman, 1985, p. xiii]:

> This was a great time to be a high-energy theorist, the period of the famous triumph of quantum field theory. And what a triumph it was, in the old sense of the word: a glorious victory parade, full of wonderful things brought back from far places to make the spectator gasp with awe and laugh with joy.

Sidney was able to capture some of that awe and joy in his course, and in his famous Erice Lectures from which this quote is taken. Over the past 35 years, the parade has continued. I hope that this book may give you a sense of what all the fuss is about.

Acknowledgements

The book would not have been possible without the perpetual encouragement and enthusiasm of the many students who took the course on which this book is based. Without these students, this book would not have been written. The presentation of most of the material in this book, particularly the foundational material in Parts I to III, arose from an iterative process. These iterations were promoted in no small part by the excellent questions the students posed to me both in and out of class. In ruminating on those questions, and discussing them with my colleagues, the notes steadily improved.

I have to thank in particular the various unbelievable teaching assistants I had for the course, particularly David Simmons-Duffin, Clay Cordova, Ilya Feige and Prahar Mitra for their essential contributions to improving the course material. The material in this book was refined and improved due to critical conversations that I had with many people. In particular, I would like to thank Frederik Denef, Ami Katz, Aneesh Manohar, Yasunori Nomura, Michael Peskin, Logan Ramalingam, Matthew Reece, Subir Sachdev, Iain Stewart, Matthew Strassler, and Xi Yin for valuable conversations. More generally, my approach to physics, by which this book is organized, has been influenced by three people most of all: Lisa Randall, Nima Arkani-Hamed and Howard Georgi. From them I learned to respect non-renormalizable field theories and to beware of smoke and mirrors in theoretical physics.

I am indebted to Anders Andreassen, David Farhi, William Frost, Andrew Marantan and Prahar Mitra for helping me convert a set of decent lecture notes into a coherent and comprehensive textbook. I also thank Ilya Feige, Yang-Ting Chien, Yale Fan, Thomas Becher, Zoltan Ligeti and Marat Freytsis for critical comments on the advanced chapters of the book.

Some of the material in the book is original, and some comes from primary literature. However, the vast majority of what I write is a rephrasing of results presented in the existing vast library of fantastic quantum field theory texts. The textbooks by Peskin and Schroeder and by Weinberg were especially influential. For example, Peskin and Schroeder's nearly perfect Chapter 5 guides my Chapter 13. Weinberg's comprehensive two volumes of *The Quantum Theory of Fields* are unequalled in their rigor and generality. Less general versions of many of Weinberg's explanations have been incorporated into my Chapters 8, 9, 14 and 24.

I have also taken some material from Srednicki's book (such as the derivation of the LSZ reduction formula in my Chapter 6), from Muta's book on quantum chromodynamics (parts of my Chapters 13, 26 and 32), from Banks' dense and deep *Concise Introduction* (particularly his emphasis on the Schwinger–Dyson equations which affected my Chapters 7 and 14), from Halzen and Martin's very physical book *Quarks and Leptons* (my Chapter 32), Rick Field's book *Applications of Perturbative QCD* (my Chapter 20). I have always found Manohar and Wise's monograph *Heavy Quark Physics* (on which my Chapter 35 is based) to be a valuable reference, in particular its spectacularly efficient first chapter. Zee's *Quantum Field Theory in a Nutshell* also had much influence on me (and on my Chapter 15). In addition, a few historical accounts come from Pais' *Inward Bound*, which I recommend any serious student of quantum field theory to devour.

Finally, I would like to thank my wife Carolyn, for her patience, love and support as this book was being written, as well as for some editorial assistance.

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Cambridge, Massachusetts
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