PRINCIPLES OF DISCRETE TIME MECHANICS

Could time be discrete on some unimaginably small scale? Exploring the idea in depth, this unique introduction to discrete time mechanics systematically builds the theory up from scratch, beginning with the historical, physical and mathematical background to the chronon hypothesis.

Covering classical and quantum discrete time mechanics, this book presents all the tools needed to formulate and develop applications of discrete time mechanics in a number of areas, including spreadsheet mechanics, classical and quantum register mechanics, and classical and quantum mechanics and field theories. A consistent emphasis on contextuality and the observer–system relationship is maintained throughout.

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Preface

Long ago, great minds speculated on the nature of time. The following question was asked: could time be divided into ever smaller and smaller pieces, just like a length of wood? We know this for a historical fact, because some of Zeno's paradoxes have survived the ravages of time and these paradoxes discuss precisely this question.

Contrary to what might be believed, interest in Zeno's paradoxes has not been extinguished by the rigours of modern mathematics, although we are taught that it has. Yes, the paradox of Achilles and the tortoise can be explained away in terms of a convergent infinite sum. But the concept of an infinitesimal has not been killed off: far from it, for mathematicians have developed a rigorous, consistent mathematical framework called non-standard analysis that allows for such things.

What I believe this debate about time highlights is how conditioned humans can be. We learn from an early age to think in certain terms and, if we are not careful, we end up regarding them as the only possible framework for our thoughts. So it is with time, which has been regarded as continuous throughout the history of mathematics and physics. It is hard to imagine any physical theory without the concept of a time derivative, and that requires continuity in time.

However, it is the obligation of theorists not only to explore current theories to their natural horizons, but to look beyond those horizons and to step outside of them if that is possible. That's really what theorists are paid for, not for the confirmation of established paradigms. I started to be concerned about standard physics when I first encountered wavefunction renormalization, that notorious method of dealing with the divergences of quantum field theory. Now, many years later, I can see that this concern was a portent of what was to come, for a very large quantity can be regarded as the reciprocal of a very small quantity. Very large energies and momenta are related to very small timescales and intervals of space, as I shall discuss.

Can we resolve these problems? Is it possible to understand Zeno's paradoxes about the vanishingly small and understand the divergences in quantum field theory within the same framework?

I think the answer is *possibly*, but it will require a deeper examination of the role of the observer. The observer has long taken a back seat in scientific theory, because the focus in science has generally been on the systems under observation. It is my belief that the balance has to be redressed, particularly when it comes to time.

This book is not about approximations to continuous time models but an exploration of discrete time as a model in its own right. I am not interested in finding good discrete time approximations to continuous time equations or their solutions. So do not look here for advice about the latest and best convergent lattice discretizations in fluid mechanics, non-abelian gauge theory or gravitation. There are plenty of sources on those topics. I am exploring the following question: what would be the consequences of the conjecture that time is really discrete?

This book will necessarily be centred on my own experiences: what I have read, what I have written, and what has come to me through talking to others. So I will inevitably have missed some important topics and papers written by others, for which I apologize profusely in advance.

A preface is generally the place where an author expresses their unbounded gratitude to others. I do so now. I am indebted to all my teachers and lecturers, colleagues and students down the years who have given me far more of value than I have given them. In particular, I benefited from the wisdom and inspiration of Professors Nicholas Kemmer, Julian Schwinger and Peter Landshoff at various points in my career. I am indebted also to Andy Walker and Anne Lomax, and to Volodia and Rumy Nikolaev.

I thank also all my research students who took the risk of working with me on discrete time: Keith Norton, Jon Eakins, Jason Ridgway-Taylor and Fernando Aguayo.

My deepest thanks go to all members of my family, past and present: my wife Małgorzata and daughter Joanna for their endless patience and support, and to my parents for the priceless values they gave me.

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