

## **Modern Particle Physics**

Unique in its coverage of all aspects of modern particle physics, this textbook provides a clear connection between the theory and recent experimental results, including the discovery of the Higgs boson at CERN. It provides a comprehensive and self-contained description of the Standard Model of particle physics suitable for upper-level undergraduate students and graduate students studying experimental particle physics.

Physical theory is introduced in a straightforward manner with step-by-step mathematical derivations throughout. Fully worked examples enable students to link the mathematical theory to results from modern particle physics experiments. End-of-chapter exercises, graded by difficulty, provide students with a deeper understanding of the subject

Online resources available at www.cambridge.org/MPP feature password-protected fully worked solutions to problems for instructors, numerical solutions and hints to the problems for students and PowerPoint slides and JPEGs of figures from the book.

**Mark Thomson** is Professor in Experimental Particle Physics at the University of Cambridge. He is an experienced teacher and has lectured particle physics at introductory and advanced levels. His research interests include studies of the electroweak sector of the Standard Model and the properties of neutrinos.





# **Modern Particle Physics**

MARK THOMSON

University of Cambridge





# **CAMBRIDGE**UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org Information on this title: www.cambridge.org/MPP

© M. Thomson 2013

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2013 4th printing 2015

Printed in the United Kingdom by TJ International Ltd, Padstow, Cornwall

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing-in-Publication data
Thomson, Mark, 1966–
Modern particle physics / Mark Thomson.
pages cm
ISBN 978-1-107-03426-6 (Hardback)

1. Particles (Nuclear physics)—Textbooks. I. Title.
QC793.2.T46 2013
539.7'2–dc23 2013002757

Additional resources for this publication at www.cambridge.org/MPP

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

ISBN 978-1-107-03426-6 Hardback



> To Sophie, Robert and Isabelle for their love, support and endless patience





## Contents

| P | reface                         | page x111                                 |      |  |
|---|--------------------------------|---|------|--|
| A | cknowled                       | gements                                   | XV   |  |
| 1 | Introduct                      | ion                                       | 1    |  |
| ٠ |                                |   |      |  |
|   |                                | teractions of particles with matter       | 1 13 |  |
|   |                                | ollider experiments                       | 22   |  |
|   |                                | easurements at particle accelerators      | 25   |  |
|   | Summa                          | -   | 27   |  |
|   | Problem                        | •   | 28   |  |
| 2 | Underlying concepts            |   | 30   |  |
|   | 2.1 U                          | nits in particle physics                  | 30   |  |
|   | 2.2 Sp                         | pecial relativity                         | 33   |  |
|   | 2.3 No                         | on-relativistic quantum mechanics         | 40   |  |
|   | Summa                          | ry  | 54   |  |
|   | Problen                        | ns  | 55   |  |
| 3 | Decay rates and cross sections |   | 58   |  |
|   | 3.1 Fe                         | ermi's golden rule                        | 58   |  |
|   | 3.2 Pł                         | hase space and wavefunction normalisation | 59   |  |
|   | 3.3 Pa                         | article decays                            | 66   |  |
|   | 3.4 In                         | teraction cross sections                  | 69   |  |
|   | 3.5 D                          | ifferential cross sections                | 72   |  |
|   | Summary                        |   | 77   |  |
|   | Problem                        | ns  | 78   |  |
| 4 | The Dirac equation             |   | 80   |  |
|   | 4.1 Tl                         | ne Klein–Gordon equation                  | 80   |  |
|   | 4.2 Tl                         | ne Dirac equation                         | 82   |  |
|   | 4.3 Pr                         | obability density and probability current | 85   |  |
|   | 4.4 *5                         | Spin and the Dirac equation               | 86   |  |
|   | 4.5 Co                         | ovariant form of the Dirac equation       | 89   |  |

۷ij



| viii | ontents |
|------|---------|
|      |         |

|   | 4.6 Solutions to the Dirac equation                 | 92  |
|---|---|-----|
|   | 4.7 Antiparticles                                   | 96  |
|   | 4.8 Spin and helicity states                        | 104 |
|   | 4.9 Intrinsic parity of Dirac fermions              | 108 |
|   | Summary   | 111 |
|   | Problems  | 112 |
| 5 | Interaction by particle exchange                    | 114 |
|   | 5.1 First- and second-order perturbation theory     | 114 |
|   | 5.2 Feynman diagrams and virtual particles          | 118 |
|   | 5.3 Introduction to QED                             | 121 |
|   | 5.4 Feynman rules for QED                           | 124 |
|   | Summary   | 127 |
|   | Problems  | 127 |
| 6 | Electron—positron annihilation                      | 128 |
|   | 6.1 Calculations in perturbation theory             | 128 |
|   | 6.2 Electron–positron annihilation                  | 130 |
|   | 6.3 Spin in electron–positron annihilation          | 139 |
|   | 6.4 Chirality                                       | 140 |
|   | 6.5 *Trace techniques                               | 144 |
|   | Summary   | 157 |
|   | Problems  | 158 |
| 7 | Electron—proton elastic scattering                  | 160 |
|   | 7.1 Probing the structure of the proton             | 160 |
|   | 7.2 Rutherford and Mott scattering                  | 161 |
|   | 7.3 Form factors                                    | 166 |
|   | 7.4 Relativistic electron–proton elastic scattering | 168 |
|   | 7.5 The Rosenbluth formula                          | 171 |
|   | Summary   | 176 |
|   | Problems  | 176 |
| 8 | Deep inelastic scattering                           | 178 |
|   | 8.1 Electron–proton inelastic scattering            | 178 |
|   | 8.2 Deep inelastic scattering                       | 183 |
|   | 8.3 Electron–quark scattering                       | 186 |
|   | 8.4 The quark–parton model                          | 189 |
|   | 8.5 Electron–proton scattering at the HERA collider | 199 |
|   | 8.6 Parton distribution function measurements       | 202 |
|   | Summary   | 203 |
|   | Problems  | 204 |



ix Contents

| 9  | Symmetries and the quark model                    | 207 |
|----|---|-----|
|    | 9.1 Symmetries in quantum mechanics               | 207 |
|    | 9.2 Flavour symmetry                              | 211 |
|    | 9.3 Combining quarks into baryons                 | 215 |
|    | 9.4 Ground state baryon wavefunctions             | 219 |
|    | 9.5 Isospin representation of antiquarks          | 221 |
|    | 9.6 SU(3) flavour symmetry                        | 223 |
|    | Summary   | 238 |
|    | 9.7 *Addendum: Flavour symmetry revisited         | 239 |
|    | Problems  | 240 |
| 10 | Quantum Chromodynamics (QCD)                      | 242 |
|    | 10.1 The local gauge principle                    | 242 |
|    | 10.2 Colour and QCD                               | 245 |
|    | 10.3 Gluons                                       | 247 |
|    | 10.4 Colour confinement                           | 248 |
|    | 10.5 Running of $\alpha_S$ and asymptotic freedom | 253 |
|    | 10.6 QCD in electron–positron annihilation        | 259 |
|    | 10.7 Colour factors                               | 264 |
|    | 10.8 Heavy mesons and the QCD colour potential    | 271 |
|    | 10.9 Hadron–hadron collisions                     | 274 |
|    | Summary   | 282 |
|    | Problems  | 283 |
| 11 | The weak interaction                              | 285 |
|    | 11.1 The weak charged-current interaction         | 285 |
|    | 11.2 Parity                                       | 285 |
|    | 11.3 $V - A$ structure of the weak interaction    | 290 |
|    | 11.4 Chiral structure of the weak interaction     | 293 |
|    | 11.5 The W-boson propagator                       | 295 |
|    | 11.6 Helicity in pion decay                       | 298 |
|    | 11.7 Experimental evidence for $V - A$            | 303 |
|    | Summary   | 304 |
|    | Problems  | 304 |
| 12 | The weak interactions of leptons                  | 307 |
|    | 12.1 Lepton universality                          | 307 |
|    | 12.2 Neutrino scattering                          | 309 |
|    | 12.3 Neutrino scattering experiments              | 319 |
|    | 12.4 Structure functions in neutrino interactions | 322 |
|    | 12.5 Charged-current electron–proton scattering   | 324 |



Contents

|    | Summary                                      | 327 |
|----|--|-----|
|    | Problems                                     | 327 |
| 13 | Neutrinos and neutrino oscillations          | 329 |
|    | 13.1 Neutrino flavours                       | 329 |
|    | 13.2 Solar neutrinos                         | 330 |
|    | 13.3 Mass and weak eigenstates               | 336 |
|    | 13.4 Neutrino oscillations of two flavours   | 338 |
|    | 13.5 Neutrino oscillations of three flavours | 342 |
|    | 13.6 Neutrino oscillation experiments        | 351 |
|    | 13.7 Reactor experiments                     | 353 |
|    | 13.8 Long-baseline neutrino experiments      | 357 |
|    | 13.9 The global picture                      | 360 |
|    | Summary                                      | 361 |
|    | Problems                                     | 362 |
| 14 | CP violation and weak hadronic interactions  | 364 |
|    | 14.1 CP violation in the early Universe      | 364 |
|    | 14.2 The weak interactions of quarks         | 365 |
|    | 14.3 The CKM matrix                          | 368 |
|    | 14.4 The neutral kaon system                 | 371 |
|    | 14.5 Strangeness oscillations                | 384 |
|    | 14.6 B-meson physics                         | 394 |
|    | 14.7 CP violation in the Standard Model      | 402 |
|    | Summary                                      | 405 |
|    | Problems                                     | 405 |
| 15 | Electroweak unification                      | 408 |
|    | 15.1 Properties of the W bosons              | 408 |
|    | 15.2 The weak interaction gauge group        | 415 |
|    | 15.3 Electroweak unification                 | 418 |
|    | 15.4 Decays of the Z                         | 424 |
|    | Summary                                      | 426 |
|    | Problems                                     | 426 |
| 16 | Tests of the Standard Model                  | 428 |
|    | 16.1 The Z resonance                         | 428 |
|    | 16.2 The Large Electron–Positron collider    | 434 |
|    | 16.3 Properties of the W boson               | 442 |
|    | 16.4 Quantum loop corrections                | 448 |
|    | 16.5 The top quark                           | 450 |



xi Contents

| Sum      | mary  | 456 |  |
|----------|---|-----|--|
| Prob     | lems  | 457 |  |
| 17 The F | iggs boson                                      | 460 |  |
| 17.1     | The need for the Higgs boson                    | 460 |  |
| 17.2     | Lagrangians in Quantum Field Theory             | 461 |  |
| 17.3     | Local gauge invariance                          | 467 |  |
| 17.4     | Particle masses                                 | 469 |  |
| 17.5     | The Higgs mechanism                             | 470 |  |
| 17.6     | Properties of the Higgs boson                   | 487 |  |
| 17.7     | The discovery of the Higgs boson                | 490 |  |
| Sum      | mary  | 493 |  |
| 17.8     | 17.8 *Addendum: Neutrino masses                 |     |  |
| Prob     | lems  | 497 |  |
| 18 The S | tandard Model and beyond                        | 499 |  |
| 18.1     | The Standard Model                              | 499 |  |
| 18.2     | Open questions in particle physics              | 501 |  |
| 18.3     | Closing words                                   | 510 |  |
| Appendi  | A The Dirac delta-function                      | 512 |  |
| A.1      | Definition of the Dirac delta-function          | 512 |  |
|          | Fourier transform of a delta-function           | 513 |  |
| A.3      | Delta-function of a function                    | 513 |  |
| Appendi  | B Dirac equation                                | 515 |  |
| B.1      | Magnetic moment of a Dirac fermion              | 515 |  |
| B.2      | Covariance of the Dirac equation                | 517 |  |
|          | Four-vector current                             | 520 |  |
| Prob     | lems  | 521 |  |
| Appendi  | C The low-mass hadrons                          | 523 |  |
| Appendi  | D Gauge boson polarisation states               | 525 |  |
| D.1      | Classical electromagnetism                      | 525 |  |
| D.2      | Photon polarisation states                      | 527 |  |
| D.3      | Polarisation states of massive spin-1 particles | 528 |  |
| D.4      | Polarisation sums                               | 530 |  |
| Appendi  | E Noether's theorem                             | 535 |  |
| Prob     | lem   | 536 |  |



| xii | Contents        |                            |     |
|-----|-----------------|----------------------------|-----|
|     | Appendix F      | Non-Abelian gauge theories | 537 |
|     | References      |                            | 543 |
|     | Further reading |                            | 545 |
|     | Index           |                            | 546 |



## **Preface**

The Standard Model of particle physics represents one of the triumphs of modern physics. With the discovery of the Higgs boson at the LHC, all of the particles in the Standard Model have now been observed. The main aim of this book is to provide a broad overview of our current understanding of particle physics. It is intended to be suitable for final-year undergraduate physics students and also can serve as an introductory graduate-level text. The emphasis is very much on the modern view of particle physics with the aim of providing a solid grounding in a wide range of topics.

Our current understanding of the sub-atomic Universe is based on a number of profound theoretical ideas that are embodied in the Standard Model of particle physics. However, the development of the Standard Model would not have been possible without a close interplay between theory and experiment, and the structure of this book tries to reflects this. In most chapters, theoretical concepts are developed and then are related to the current experimental results. Because particle physics is mostly concerned with fundamental objects, it is (in some sense) a relatively straightforward subject. Consequently, even at the undergraduate level, it is quite possible to perform calculations that can be related directly to the recent experiments at the forefront of the subject.

### Pedagogical approach

In writing this textbook I have tried to develop the subject matter in a clear and accessible manner and thought long and hard about what material to include. Whilst the historical development of particle physics is an interesting topic in its own right, it does not necessarily provide the best pedagogical introduction to the subject. For this reason, the focus of this book is on the contemporary view of particle physics and earlier experimental results are discussed only to develop specific points. Similarly, no attempt is made to provide a comprehensive review of the many experiments, instead a selection of key measurements is used to illustrate the theoretical concepts; the choice of which experimental measurements to include is primarily motivated by the pedagogical aims of this book.

This textbook is intended to be self-contained, and only a basic knowledge of quantum mechanics and special relativity is assumed. As far as possible, I have tried to derive everything from first principles. Since this is an introductory textbook, the

xiii



xiv Preface

mathematical material is kept as simple as possible, and the derivations show all the main steps. I believe that this approach enables students relatively new to the subject to develop a clear understanding of the underlying physical principles; the more sophisticated mathematical trickery can come later. Calculations are mostly performed using helicity amplitudes based on the explicit Dirac—Pauli representation of the particle spinors. I believe this treatment provides a better connection to the underlying physics, compared to the more abstract trace formalism (which is also described). Some of the more-challenging material is included in optional *starred* sections. When reading these sections, the main aim should be to understand the central concepts, rather than the details.

The general structure of this book is as follows: Chapters 1–5 introduce the underlying concepts of relativistic quantum mechanics and interaction by particle exchange; Chapters 6–12 describe the electromagnetic, strong and weak interactions; and Chapters 13–18 cover major topics in modern particle physics. This textbook includes an extensive set of problems. Each problem is graded according to the *relative* time it is likely to take. This does not always reflect the difficulty of the problem and is meant to provide a guide to students, where for example a shorter graded problem should require relatively little algebra. Hints and outline solutions to many of the problems are available at www.cambridge.org/MPP.

#### For instructors

This book covers a wide range of topics and can form the basis of a long course in particle physics. For a shorter course, it may not be possible to fit all of the material into a single semester and certain sections can be omitted. In this case, I would recommend that students read the introductory material in Chapters 1–3 as preparation for a lecture course. Chapters 4–8, covering the calculations of the  $e^+e^- \to \mu^+\mu^-$  annihilation and  $e^-p$  scattering cross sections, should be considered essential. Some of the material in Chapter 9 on the quark model can be omitted, although not the discussion of symmetries. The material in Chapter 14 stands alone and could be omitted or covered only partially. The material on electroweak unification and the tests of the Standard Model, presented in Chapters 15 and 16, represents one of the highlights of modern particle physics and should be considered as core. The chapter describing the Higgs mechanism is (necessarily) quite involved and it would be possible to focus solely on the properties of the Higgs boson and its discovery, rather than the detailed derivations.

Fully worked solutions to all problems are available to instructors, and these can be found at www.cambridge.org/MPP. In addition, to aid the preparation of new courses, PowerPoint slides covering most of the material in this book are available at the same location, as are all of the images in this book.



# Acknowledgements

I would like to thank colleagues in the High Energy Physics group at the Cavendish Laboratory for their comments on early drafts of this book. This book is based on my final-year undergraduate lecture course in the Physics Department at the University of Cambridge and as such it represents an evolution of earlier courses; for this reason I am indebted to R. Batley and M. A. Parker who taught the previous incarnations. For their specific comments on a number of the more technical chapters, I am particularly grateful to A. Bevan, B. Webber and J. Wells.

For the permissions to reproduce figures and to use experimental data I am indebted to the following authors and experimental collaborations:

- R. Felst and the JADE Collaboration for Figure 6.7;
- S. Wojcicki and the DELCO Collaboration for Figure 6.12;
- M. Breidenbach for Figure 8.3;
- S. Schmitt and the H1 Collaboration for Figures 8.13 and 12.14;
- D. Plane and the OPAL Collaboration for Figures 10.12, 10.19, 16.8 and 16.9;
- S. Bethke for Figure 10.14;
- C. Kiesling and the CELLO Collaboration for Figure 10.16;
- C. Vellidis, L. Ristori and the CDF Collaboration for Figures 10.29, 16.14 and 16.21:
- J. Incandela and the CMS Collaboration for Figures 10.32, 17.18 and 17.19;
- F. Gianotti and the ATLAS Collaboration for Figures 10.30, 17.18 and 17.19;
- J. Steinberger, F. Dydak and the CDHS Collaboration for Figure 12.10;
- R. Bernstein and the NuTeV Collaboration for Figure 12.12;
- M. Pinsonneault for Figure 13.3;
- Y. Suzuki and the Super-Kamiokande Collaboration for Figures 13.4 and 13.6;
- N. Jelley, R.G.H. Robertson and the SNO Collaboration for Figure 13.8;
- K-B. Luk, Y. Wang and the Daya Bay Collaboration for Figure 13.19;
- K. Inoue and the KamLAND Collaboration for Figure 13.20;
- R. Patterson for Figure 13.21;
- R. Plunkett, J. Thomas and the MINOS Collaboration for Figure 13.22;
- P. Bloch, N. Pavlopoulos and the CPLEAR Collaboration for Figures 14.14–14.16;
- L. Piilonen, H. Hayashii, Y. Sakai and the Belle Collaboration for Figure 14.21;
- M. Roney and the BaBar Collaboration for Figure 14.24;

χV



xvi

#### Acknowledgements

The LEP Electroweak Working Group for Figures 16.2, 16.5, 16.6 and 16.10; S. Mele and the L3 Collaboration for Figure 16.13.

I am grateful to the Durham HepData project, which is funded by the UK Science and Technologies Facilities Council, for providing the online resources for access to high energy physics data that greatly simplifying the production of a number of the figures in this book.

Every effort has been made to obtain the necessary permissions to reproduce or adapt copyrighted material and I acknowledge:

Annual Reviews for Figure 8.4;

The American Physical Society for Figure 6.12 from Bacino *et al.* (1978), Figure 7.7 from Hughes *et al.* (1965), Figure 7.8 from Sill *et al.* (1993) and Walker *et al.* (1994), Figure 8.3 from Breidenbach *et al.* (1969), Figure 8.4 from Friedman and Kendall (1972) and Bodek *et al.* (1979), Figure 8.14 from Beringer *et al.* (2012), Figure 10.29 from Abe *et al.* (1999), Figure 12.12 from Tzanov *et al.* (2006), Figure 13.3 from Bahcall and Pinsonneault (2004), Figure 13.6 from Fukada *et al.* (2001), Figure 13.8 from Ahmad *et al.* (2002), Figure 13.19 from An *et al.* (2012), Figure 13.20 from Abe *et al.* (2008), Figure 13.22 from Adamson *et al.* (2011), Figure 14.21 from Abe *et al.* (2005), Figure 14.24 from Aubert *et al.* (2007), Figure 16.14 from Aaltonen *et al.* (2012) and Figure 16.21 from Aaltonen *et al.* (2011);

Elsevier for Figure 8.2 from Bartel *et al.* (1968), Figures 8.11, 8.12 and 8.18 from Whitlow *et al.* (1992), Figure 10.16 from Behrend *et al.* (1987), Figures 16.2, 16.5 and 16.6 from LEP and SLD Collaborations (2006);

Springer for Figure 6.7 from Bartel *et al.* (1985), Figure 10.12 from Abbiendi *et al.* (2004), Figure 10.14 from Bethke (2009), Figure 12.10 from de Groot *et al.* (1979), Figure 12.14 from Aaron *et al.* (2012), Figure 14.15 from Angelopoulos *et al.* (2001), Figure 14.16 from Angelopoulos *et al.* (2000), Figure 16.8 from Abbiendi *et al.* (2001) and Figure 16.13 from Achard *et al.* (2006);

CERN Information Services for Figures 10.32, 17.18 and 17.19.