

## Non-Hausdorff Topology and Domain Theory

This unique book on modern topology looks well beyond traditional treatises, and explores spaces that may, but need not, be Hausdorff. This is essential for domain theory, the cornerstone of semantics of computer languages, where the Scott topology is almost never Hausdorff. For the first time in a single volume, this book covers basic material on metric and topological spaces, advanced material on complete partial orders, Stone duality, stable compactness, quasi-metric spaces, and much more. An early chapter on metric spaces serves as an invitation to the topic (continuity, limits, compactness, completeness) and forms a complete introductory course by itself.

Graduate students and researchers alike will enjoy exploring this treasure trove of results. Full proofs are given, as well as motivating ideas, clear explanations, illuminating examples, application exercises, and some more challenging problems for advanced readers.

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# Non-Hausdorff Topology and Domain Theory

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## Contents

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<b>1</b>	<b>Introduction</b>	1
<b>2</b>	<b>Elements of set theory</b>	4
2.1	Foundations	4
2.2	Finiteness, countability	6
2.3	Order theory	8
2.4	The Axiom of Choice	15
<b>3</b>	<b>A first tour of topology: metric spaces</b>	18
3.1	Metric spaces	18
3.2	Convergence, limits	20
3.3	Compact subsets	25
3.4	Complete metric spaces	33
3.5	Continuous functions	37
<b>4</b>	<b>Topology</b>	46
4.1	Topology, topological spaces	46
4.2	Order and topology	53
4.3	Continuity	62
4.4	Compactness	67
4.5	Products	74
4.6	Coproducts	80
4.7	Convergence and limits	82
4.8	Local compactness	91
4.9	Subspaces	95
4.10	Homeomorphisms, embeddings, quotients, retracts	97
4.11	Connectedness	106
4.12	A bit of category theory I	111
<b>5</b>	<b>Approximation and function spaces</b>	120
5.1	The way-below relation	120
5.2	The lattice of open subsets of a space	140

5.3	Spaces of continuous maps	149
5.4	The exponential topology	151
5.5	A bit of category theory II	169
5.6	$\mathcal{C}$ -generated spaces	180
5.7	bc-domains	194
<b>6</b>	<b>Metrics, quasi-metrics, hemi-metrics</b>	<b>203</b>
6.1	Metrics, hemi-metrics, and open balls	203
6.2	Continuous and Lipschitz maps	209
6.3	Topological equivalence, hemi-metrizability, metrizability	214
6.4	Coproducts, quotients	236
6.5	Products, subspaces	239
6.6	Function spaces	241
6.7	Compactness and symcompactness	251
<b>7</b>	<b>Completeness</b>	<b>260</b>
7.1	Limits, $d$ -limits, and Cauchy nets	260
7.2	A strong form of completeness: Smyth-completeness	267
7.3	Formal balls	279
7.4	A weak form of completeness: Yoneda-completeness	288
7.5	The formal ball completion	311
7.6	Choquet-completeness	323
7.7	Polish spaces	334
<b>8</b>	<b>Sober spaces</b>	<b>341</b>
8.1	Frames and Stone duality	341
8.2	Sober spaces and sobrification	351
8.3	The Hofmann–Mislove Theorem	364
8.4	Colimits and limits of sober spaces	383
<b>9</b>	<b>Stably compact spaces and compact pospaces</b>	<b>397</b>
9.1	Stably locally compact spaces, stably compact spaces	397
9.2	Coproducts and retracts of stably compact spaces	411
9.3	Products and subspaces of stably compact spaces	413
9.4	Patch-continuous, perfect maps	419
9.5	Spectral spaces	423
9.6	Bifinite domains	440
9.7	Noetherian spaces	453
	<i>References</i>	475
	<i>Notation index</i>	480
	<i>Index</i>	484