Introduction

Grand View of the Standard Model

What are the fundamental building blocks of matter? This question is easy to understand, but it took centuries and required tremendous theoretical and experimental efforts to approach an answer. The desire to know what everything is made of led to big scientific achievements, new technologies, and a better understanding of our universe. Since the 1860s we have known that atoms are basic building blocks of matter. But opposed to what was originally thought, they are neither indestructible nor fundamental as they are composed of protons, neutrons, and electrons. The finding that the whole periodic table is made of only three basic ingredients was already a big step forward toward answering the question of what everything is made of. However protons and neutrons are again not fundamental particles but rather complicated objects. Simplified one can say that they consist of three quarks: two up quarks and one down quark for the proton and vice versa for the neutron. It is amazing to know at first sight that everyday objects are made of only up and down quarks and electrons. Nevertheless this is not the end of the story as these quarks have to be bound in protons and neutrons, and this happens with the help of strong interactions that bring in new particles: gluons. Moreover pairs of quarks and antiquarks, not only of up and down quarks but also heavier quarks can also be present deep in protons and neutrons. This shows that the true picture of protons and neutrons is not as simple as often presented to pedestrians. What is amazing is the fact that this picture has been developed only in the last fifty years. This is now where the Standard Model of particle physics (SM) enters the stage.

The SM describes the properties of all elementary particles and three out of the four fundamental forces in nature: the weak force, electromagnetic force, and strong force. From a quantum field theoretical point of view the latter two are called, respectively, quantum electrodynamics (QED) and quantum chromodynamics (QCD). These forces are mediated by the exchange of gauge bosons. The weak force is mediated by the exchange of $W^\pm$ and Z^0 boson, the photon γ is the mediator of electromagnetic interactions, and gluons G^a are mediators of strong interactions. Gravity is not included in the SM. It is by far the weakest of the fundamental forces and not relevant for particle physics at the energy scales presently explored in experiments.

The SM sorts the elementary particles by their properties: there are the matter particles with spin-$\frac{1}{2}$ (fermions), the gauge bosons with spin-1, and one scalar particle with spin-0, the so-called Higgs boson. The fermions can be divided into quarks and leptons. The six

Table I.1 Particle content of the SM.

Fermions: three generations			Gauge Bosons
ν_e	ν_μ	ν_τ	γ
e	μ	τ	G^a
u	c	t	$W^\pm$
d	s	b	Z^0
	Higgs		

quarks – up u, down d, charm c, strange s, top t, and bottom b – feel all three interactions, as they have a weak charge (called weak isospin), an electric charge, and a color charge related to strong interactions. Leptons do not participate in the strong interactions as they are color neutral. The three charged leptons – electron e, muon μ, and tau τ – participate in the electromagnetic and weak interactions, whereas the neutral leptons – the three neutrinos ν_e, ν_μ, and ν_τ – only feel the weak force.

Similar to the periodic table of elements where you have both a horizontal and a vertical order, the SM collects leptons and quarks into three generations as seen in Table I.1. The first generation consists of electron neutrino ν_e, electron e, up u, and down quark d. It is the first generation of particles that all the matter around us is made of. Nothing else at first sight is needed. However there exist also a second (ν_μ, μ, c, s) and a third generation (ν_τ, τ, t, b), which are in principle exact copies of the first generation (same charges and properties as seen in Table 2.1) except that they are heavier and thus unstable. At the end they decay into particles of the first generation. That is why the muon is sometimes called the heavy brother of the electron: They have the same quantum numbers, but differ in mass. Flavor physics studies such transitions between different generations that are mediated by the weak interaction. We cannot answer yet why there are exactly three generations, but so far we could already learn a lot about their properties in studying weak decays.

The SM is a theoretical model based on a (gauge) symmetry principle and very successful in describing and predicting experimental results. From this symmetry principle it follows that all fermions and gauge bosons must be massless, which is obviously not the case. Only photons and gluons are massless. This is now where the Higgs boson (or more precisely the Higgs field together with spontaneous symmetry breaking) enters the scene. It is responsible within the SM for the masses of quarks, charged leptons, and in particular gauge bosons of weak interactions. Even if the discovered nonvanishing neutrino masses around the turn of the millennium were not predicted within this model, its success in describing most of the data made the majority of particle physicists believe in the existence of the Higgs boson, and a lot of effort was put into finding this last missing piece of the SM. Its discovery in 2012 was a great scientific achievement – a true milestone.

The development of the SM was a big step forward to answer our initial question. Nevertheless, we still do not think that the story ends here. Despite its great success the SM does not answer several important questions about the nature surrounding us, and most physicists think that it is only an effective theory of a more fundamental theory. We will now address this important topic. Before continuing we would like to stress that this Grand View and the following one has been written by Jennifer Girrbach-Noe.

Grand View of New Physics

The ultimate question of elementary particle physics is: What is the fundamental Lagrangian of nature surrounding us? The Lagrangian of the SM is very successful in describing nature at the currently available energy range. The discovery of the Higgs boson completed the particle spectrum of the SM and it is another proof of how well the SM, works. Nevertheless, the SM cannot be the end of the story, and it is for sure not the fundamental Lagrangian of nature. The Lagrangian of the SM loses its validity at the latest at the Planck scale where gravitational effects become noticeable. Most physicists think of the SM as an *effective theory* that has to be replaced by a more fundamental theory above the TeV scale. What the word *effective* really means will hopefully be clear at later stages of our book. For the time being we will list some problems and open questions of the SM.

What Is Dark Matter Made Of?

We know that dark matter-energy makes up around 27 percent of the universe. Ordinary matter that is built of particles of the SM account for only 5 percent of the mass-energy of the universe. The remaining part is dark energy about which we know even less than about dark matter. Consequently we do not understand 95 percent of the universe's energy budget. Dark matter is called dark because it does not interact with photons and thus is electrically neutral and cannot be seen directly with telescopes. However, dark matter has gravitational effects that can be observed. This additional matter is needed to explain the rotational speeds of galaxies, gravitational lensing, and the anisotropies in the cosmic microwave background. Consequently the existence of dark matter is generally accepted, but we have no idea what dark matter is really made of. No particle of the SM can explain it. Some models beyond the SM have appropriate particle candidates for dark matter, e.g., the lightest neutralino in Supersymmetry with R-parity, and the search for this particle is one of major efforts in particle physics.

Why Is There More Matter Than Antimatter in the Universe?

Everything around us is made of matter, from the smallest life forms on Earth to the largest stellar objects. Antimatter, produced, for example, in particle collisions in experiments or in the atmosphere, can only exist for a very short time because matter and antimatter annihilate each other, and only pure energy in the form of photons is left over. According to Einstein's famous equation $E = mc^2$, mass and energy can be transformed into each other. In the big bang the same amount of matter and antimatter must have been created out of energy in the early universe. Matter and antimatter particles are produced in pairs out of energy, and if they come in contact, they annihilate each other. To create the observed matter abundance and not a universe made of pure energy, a mechanism that creates a tiny asymmetry between matter and antimatter is needed: Only about one particle per billion must have managed to survive; all others particles annihilated with their antiparticle. In the SM there actually is a mechanism that treats matter and antimatter differently, called CP violation. However, its strength is much too small to explain the observed matter-antimatter asymmetry, which is essential for our existence. Combined with the puzzling aspects of dark matter and dark energy, in principle we do not understand the 5 percent of ordinary matter either!

Why Are Atoms Electrically Neutral?

The simplest atom is a hydrogen atom: It consists of a proton and an electron, and both have exactly the same charge but with opposite sign such that hydrogen is electrically neutral. Although this might sound plausible, it is highly nontrivial. A proton consists of three quarks: two up quarks and one down quark with charges $+\frac{2}{3}e$ and $-\frac{1}{3}e$, respectively. The electron has charge $-e$. Now in the SM there is at least at first sight no connection or relation between quarks and electrons although the self-consistency of the theory at the quantum level requires some relations between quarks and leptons in order to remove the so-called *gauge anomalies* to be discussed briefly at later stages of our book. In any case, the fact that the charges of the proton and electron sum up to zero exactly is remarkable. Is it just an accident or is a deeper reason behind this? In principle the electric charge of a particle could be anything.[1] So why are the electric charges quantized? This problem can be solved if the SM is embedded in a grand unified theory.[2]

Why Are the Forces of Nature of Such Different Strengths?

The four forces of nature are of very different strengths. Gravity is by far the weakest of all forces. Is it possible that all forces have the same origin and thus the same strength at a certain energy scale. Did they develop from the same force present in the early universe? The idea to unify different forces or interactions goes back very far in the history of physics. Already Isaac Newton (1642–1727) linked the planetary motions with the falling of the stone on earth. James C. Maxwell (1831–1879) accomplished unifying electricity and magnetism to electromagnetism. Nearly 100 years later Sheldon Glashow, Abdus Salam, and Steven Weinberg achieved unifying QED with the weak interactions to the electroweak interactions. Together with QCD they build the SM of particle physics. Is it now also possible to unify strong and electroweak force at a higher energy? The energy dependence of the strength of the forces in a quantum field theory suggests that this might be possible, although an exact unification cannot be achieved without modifying the theory along all the energy range. Embedding the SM into a Grand Unified Theory (GUT) would also solve the problem of why atoms are neutral as discussed earlier. Such GUT models further predict correlations between quantum numbers and masses of quarks and leptons not present in the SM. Theoretically many GUT models were studied and their predictions compared with experimental measurements. But so far no clear hints for the existence of such a unified interaction are found.

[1] This is due to the fact that the electromagnetic force is based on an abelian U(1) symmetry in which the normalization of the coupling and charges are arbitrary. However, in the case of nonabelian symmetries like SU(5) or SO(10) the electric charges are fixed.

[2] As soon as the U(1) force is embedded in a larger nonabelian symmetry group also the U(1) charges are automatically quantized.

Why Are Neutrino Masses Nonzero and So Small?

Neutrino masses are special in two ways. First of all, it is because neutrinos are exactly massless in the SM. Consequently we already found physics beyond the SM! It is possible to incorporate neutrino masses in the Lagrangian of the SM but there are different ways to do this. Either new particles as right-handed neutrinos or Higgs-triplets have to be added to the particle spectrum, and an additional heavy mass scale for Majorana masses is needed or a symmetry that forbids this mass term. Either way, neutrino masses require a nontrivial extension of the SM. The second thing is that neutrino masses are much smaller than the masses of other fermions. There is already a large mass hierarchy between the electron and the top quark of roughly six orders of magnitude, and neutrino masses extend this hierarchy to another six orders. This is a part of the so-called *flavor problem.*

Why Are There Three Generations of Particles?

The objects around us are all made up of particles of the first generation, namely electron e, up u, and down d. Out of up and down quarks we can build protons p and neutrons n and that is all we need for the atoms of the periodic table of elements. The electron neutrino ν_e is emitted in radioactive decays of those atoms. If that's all we need, why are there more matter particles? The muon μ, the heavy brother of the electron was first found in cosmic rays and belongs to the second particle generation. It has exactly the same properties as the electron but it is roughly 200 times heavier and consequently unstable. It decays and finally ends up in particles of the first generations. The same applies to the other particles of the second and third generation. The strange quark s and the bottom quark b are heavier versions of the down quark d. All these particles were not discovered at once. When "half" of a generation was found, it was expected that the other half must also exist due to symmetry principles. For example the charm quark was predicted at times when the strange quark was already found and similarly for the top quark when the bottom quark was already known. We do not know why there are exactly three generations and not two, four, five . . . or only one. But it is very satisfying that one could exclude additional generations through the study of Higgs decays. On the other hand, in a SM with only two or one generation there would be no CP violation,[3] thus no matter-antimatter asymmetry. This is only possible with more than two generations. However as already mentioned earlier, the CP violation present in a SM with three generations is still way too small to explain the matter abundance of the universe. On the other hand, it is sufficiently large to accommodate other phenomena observed in nature, and we will discuss them in detail in this book.

How Can We Stabilize the Electroweak Scale?

This is also called the hierarchy problem and is often seen as one of the biggest (theoretical) problems of the SM. If the SM is considered as an effective theory we get an additional higher energy scale Λ, which characterizes the valid energy range of the SM. Λ corresponds to the mass of new heavy particles. Under the hierarchy problem one understands the

[3] We put here aside, the so-called strong CP violation to be briefly discussed in Section 17.3.

difficulty to stabilize – under the presence of this scale Λ – the electroweak scale in every order of perturbation theory and to explain the large gap between the electroweak scale $v = 246$ GeV and the GUT scale $M_{\mathrm{GUT}} = 10^{16}$ GeV or Planck scale $M_{\mathrm{Pl}} = 10^{19}$ GeV. Fermion masses m_f are proportional to the only scale in the SM, $m_f \propto v$. They are protected by the chiral symmetry, which is only broken by the mass parameter such that radiative corrections stay small: $\delta m_f \propto m_f \log \frac{\Lambda}{m_f}$. These corrections vanish for $m_f \to 0$, that is in the limit of exact chiral symmetry. The mass of the photon on the other hand is also protected, namely by the gauge symmetry, and stays zero in all orders of perturbation theory. This relation between symmetry and a small parameter is summarized in the naturalness principle by 't Hooft [4]: A small parameter of a theory is natural if and only if the symmetry of the system enlarges when this parameter is set to zero. Fine-tuning between different large contributions to a given quantity by hand with the goal to obtain a much smaller total contribution than the individual ones is regarded as unnatural. The problem here is of course how to quantify this. How much fine-tuning is still natural? In the SM this problem arises in connection with the Higgs mass that is not protected by any symmetry. The Higgs mass gets quadratic corrections proportional to the highest present scale, e.g., $\delta m_h^2/m_h^2 \propto \Lambda^2$ and thus destabilizes the electroweak scale.

Naturalness is of course not a necessary condition for a theory to be internally consistent, but it is also not only an aesthetic requirement. It assumes the lack of certain conspiracies between phenomena occurring at very different energy scales. Naturalness might be – and often was in the past – a helpful guiding principle to infer the energy scale at which an effective theory breaks down. The separation of scales is inherent in effective theories, and applying the naturalness principle could help to find the energy scale where new physics (NP) enters, e.g., the problem of the electromagnetic energy of a classical electron is cured by the positron.

There are different possibilities to end up with a Higgs mass at the electroweak scale and not at a much higher scale:

- Fine-tuning: It could be that the parameters of the Lagrangian are fine-tuned such that the physical Higgs mass is at 125 GeV. However, this accidental cancellations up to 32 significant digits ($M_{\mathrm{Pl}}^2/M_W^2 \approx 10^{32}$) must happen in every order of perturbation theory (and not only once).
- Symmetry: There might be an additional symmetry that protects the Higgs mass from large radiative corrections. Supersymmetry, which connects bosons and fermions, is the most prominent example for this.
- The Higgs is not a fundamental scalar particle, but composed of further particles.
- A softly broken scale invariance: In the SM only the mass parameter μ of the Higgs potential breaks scale invariance. If this is the only symmetry breaking parameter all corrections would be proportional to μ^2.

These are some of the reasons why one can think that the SM should be extended to a more fundamental theory. However, it is not clear in which way NP will manifest itself. In principle only the hierarchy problem requires new phenomena not too far away from the electroweak scale and thus reachable by the LHC. Also dark matter studies suggest that the new dark matter particles should be weakly interacting and around the electroweak scale (so called WIMP, weakly interacting massive particles). It is also possible that some of the other problems might be solved at a much higher energy scale not reachable within the near future with the help of a high energy collider.

After this list of problems and open questions of the SM, we shortly want to indicate how to look for new particles and new interactions beyond the SM. In order to discover NP there are in principle two strategies:

- **Direct Detection**

  In collider experiments new particles could be produced and directly detected. The limiting factor here is the center of mass energy of the experiment. According to $E = mc^2$, if this energy is too small or the new particle too heavy, it cannot be produced as a real particle but only exists as a virtual particle. The advantage of a direct detection is, of course, that one really sees the new particle. The direct detection of the Higgs boson was the final proof that it really exists. Prior to its detection some indirect hints and limits where known, and from the theory side it was predicted to exist somewhere around the electroweak scale. With the LHC it is possible to reach distance scales of $5 \cdot 10^{-20}$ m or equivalent energy scales in the ballpark of 5–7 TeV if sufficient statistics are available.
- **Indirect Detection**

  The existence of new particles can become noticeable indirectly through quantum fluctuations, which results in deviation (often a very small one) between SM prediction and measurement. To trace such small effects a very high precision both from theory and experiment is required. This way is followed by weak decay experiments LHCb, SuperKEKB and NA62 and as we will demonstrate in this book these decays are in principle sensitive to distance scales as small as 10^{-21} m and even smaller scales. The influence of new particles as a sign of physics beyond the SM, is for example, possible in some weak decays that are suppressed in the SM such that NP effects can compete with the SM contributions. A historic example not related to particle physics is the discovery of the Neptune by Le Verrier in 1846. He studied the deviations of the orbit of the Uranus from the predicted one and concluded that it must be due to gravitational effects of a further planet whose position he predicted with an accuracy of 1°. Similarly one can try to track new particles, and it was often done in the past.

  Indeed, already in 1987 a heavy top quark was predicted at the B factory DORIS at DESY with the ARGUS experiment, where for the first time $B_d - \overline{B}_d$ oscillations were discovered. Similarly, the existence of charm quark was predicted by measuring and calculating the rate for $K_L \to \mu^+\mu^-$. Without the charm quark the theoretical prediction for the rate of the decay $K_L \to \mu^+\mu^-$ was predicted to be much larger than experimentally measured. Including the charm quark, which together with the strange quark builds a doublet under the electroweak gauge symmetry, the branching ratio for $K_L \to \mu^+\mu^-$ could be suppressed and made consistent with data. Even more, Gaillard and Lee [5], calculating the $K_L - K_S$ mass difference ΔM_K and comparing it with its measured value, could predict in 1974 the mass of the charm quark to be in the ballpark of 2 GeV before its discovery. But even in 2019 we do not know whether at some level NP enters both $B_d - \overline{B}_d$ oscillations and ΔM_K because of theoretical and parametric uncertainties.

In this context of particular interest are rare decays of leptons, like $\mu \to e\gamma$ or $\tau^- \to \mu^- e^+ e^-$ for which the predicted branching ratios are so small in the SM that they probably will never be measured if the SM is the whole story. Here NP contributions could be much larger than the SM ones making their measurement possible. Observation of such decays would be then a clear signal of NP. In fact, the next years could bring discoveries in this sector of particle physics. Similar comments apply to electric dipole moments of leptons, the neutron, the proton, and atoms and anomalous magnetic moments of leptons and quarks. Also here new discoveries could take place in the coming years.

After these two grand views we are almost ready to start our expedition. It will first dominantly take place within the SM reaching distance scales of the order of 10^{-18} m (*the Attouniverse*) corresponding roughly to the mass of the heaviest SM particle, the top quark. However, already in this part we will develop a powerful technology that will allow us to go in the later parts of our book to much shorter distances scale, reaching eventually the (*Zeptouniverse*) and possibly even shorter distance scales. What we still need is an efficient strategy and corresponding outline of our expedition. The construction of such a strategy and the outline in question is our next task.

The Grand View of the Expedition and the Strategy

Writing a long book reminds me of expeditions to Himalayas, which I know only from several books. Reading books about great composers I see some similarities to composing symphonies or operas. I believe that despite differences in these three fields, it is crucial to have a plan for these expeditions and the strategy for reaching the goals. The construction of the plan for this book was rather challenging in view of many topics present in the field of weak decays. While the number of topics discussed in this book is large, some topics will be only mentioned or discussed very briefly. The principles that guided me in choosing the topics were as follows:

- I wanted to present in detail first of all the material that I know and that I developed over many years with many collaborators, in particular with many of my PhD students and postdoctoral fellows. This is quark flavor physics in the SM and beyond.
- This includes in particular calculations of QCD corrections to weak decay processes at the leading and next-to-leading order, which dominated the topics in my research group at TUM in the 1990s.
- However, I included also topics on which I wrote only few papers until now but which in my view are very important. Hopefully not only the topics but also my papers. These are in particular lepton flavor violating decays, electric dipole moments of atoms, molecules, leptons, and nucleons, and of course anomalous magnetic moments like $g-2$ of the muon. Here I benefited from many reviews that I will list in the relevant chapters.
- I put significant effort into explaining how the derivations of predictions for most important observables both within the SM and its extensions are made. This requires

often tedious Feynman diagram calculations, and the related quantum field technology had to be developed to reach this goal. However, only a subset of results could be derived here as otherwise the book would be much longer.

- But what is equally important is the development of skills in connecting the results of such calculations to observables measured by experimentalists. In this context I will stress the correlations between many observables that turn out often crucial not only for the tests of the SM but also to distinguish between its various possible extensions.
- Concerning the latter, we will first devote one chapter to the technology of the so-called *SM gauge invariant effective field theory* (SMEFT), which became very popular in recent years. This tool allows in a model independent manner to look beyond the SM, which is useful but in my view contains too many free parameters so that additional model assumptions have to be made in order to reach some definite conclusions.
- In the latter context we will discuss first the so-called *simplified models*, which contain only few parameters. Models with the so-called constrained *Minimal Flavor Violation* are the best examples here. But also models with a new heavy neutral gauge boson Z', new heavy scalars, and those with flavor changing Z couplings are sufficiently simple that we can discuss them in some detail. Already these simple models will give us some idea on what experimentalists are telling us and hopefully some hints for the construction of new theories and the identification of NP at very short distance scales beyond the reach of the LHC. Yet, we will warn the reader that very simple models can often miss the physics hidden in the complicated quantum effects described by SMEFT, and we will try to expose this problem in a few examples.
- Few chapters will be devoted to concrete nonsupersymmetric models, like models with vectorlike fermions, leptoquark models, and explicit models with new heavy-gauge bosons and scalars. In these cases no derivations will be present. Supersymmetric models will not be discussed in this book because this would require too many pages for its introduction. Instead we will give a collection of references to useful papers.
- We will also give some outlook for the future of this field.

With these goals in mind the book consists, similar to a symphony, of four parts:

PART I: Basics of Gauge Theories
PART II: The Standard Model
PART III: Weak Decays in the Standard Model
PART IV: Weak Decays beyond the Standard Model

The last part includes also observables like electric dipole moments and anomalous magnetic moments. These four parts consist of twenty chapters, which are the steps in our expedition. We are now ready to list these steps and indicate how they are related to each other. This will hopefully make our book more transparent. In this context it is advisable to read the description of a given step, presented next, before making that step. Therefore here comes an important message:

PLEASE READ THE OUTLINE OF EACH STEP BEFORE MAKING IT!

Step 1: Chapter 1

We will begin by presenting the general structure of gauge theories, stressing their symmetries and their breakdown. The structure of the Lagrangians in this step will be fundamental for the full book. We will encounter Lagrangians involving spin-0, spin-$\frac{1}{2}$ and spin-1 fields. We will also collect literature in which further details can be found. But the information provided in this step should be sufficient for following the next steps.

Step 2: Chapter 2

We will next move to discuss the SM of electroweak and strong interactions. We will present the basic SM Lagrangian, and we will discuss various properties of this simplest theory but we will not do any phenomenology at this stage. There are many excellent books on the dynamics of the SM, and we will in most cases skip derivations of well-known formulas to save space and in particular energy for the later steps that are not covered in detail in textbooks. The most important result of this step will be collection of the Feynman rules for those interactions present in the SM that we will need for explicit calculations as we proceed. For readers' convenience we will collect these rules in Appendix B.

Step 3: Chapter 3

Having the Feynman rules at hand, we will present the simplest calculations, the so-called *tree-level* calculations that do not yet require a deep knowledge of quantum field theory like renormalization and renormalization group. This step will give us leading formulas for the determination of various parameters of the SM like the Fermi constant G_F and the parameters of the CKM matrix, which will be introduced and discussed in detail. These formulas will also give us a transparent picture of the leading decays of mesons and leptons and will allow approximate determination of the parameters in question. Yet, for a serious phenomenology we will need various corrections to these leading expressions. They will be calculated or simply listed in later steps when we turn to the phenomenology at the frontiers of weak decays. In this step we will also find out that in the case of meson decays some entries in the decay amplitudes cannot be calculated in perturbation theory. These are hadronic matrix elements of quark currents and in particular hadronic matrix elements of four-quark operators. This task will be left for other steps. But seeing how these matrix elements enter the decay amplitudes will turn out to be useful when our climb will become technically more difficult, and we will have to concentrate on other matters.

Step 4: Chapter 4

This step will show us the technology of quantum field theory at the one-loop level, including general comments and results of two and higher loop calculations that we will need later on. But the first goal of this step is the detailed presentation of the dimensional regularization and the development of the technology for one-loop calculations. The next goals in this chapter are the presentation of the renormalization and the renormalization group methods. This step is rather technical but crucial. While our presentation is