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978-1-107-03350-4 - Topics in Chromatic Graph Theory

Edited by Lowell W. Beineke, Robin J. Wilson and Bjarne Toft

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